Analysis of the state of prestressed structure using data collection simulation technique

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Abstract. The paper presents a method of analysing the impact of basic variables on parameters characterizing the state of prestressed structures in transient and persistent design situation based on computer simulations and DOE (Design Of Experiments) data collection technique. The approach enables us to identify the key variables and to finde the limits of compromise area in which the state of structures are considered to be acceptably safe. Desing basis, procedures and results of the DOE technique use were presented on the example of a post-tensioned beam designed in accordance with the requirements of Eurocode 2. The study comprised the "what-if" analysis of the variation of total load, the losses of prestressing force in individual cables and the design depth of the beam section considering their influence on the ultimate load-bearing capacity and maximum deflection of the beam. The results obtained in the form of sensitivity and trade-off charts allow for the determination of the order and range of influence of the variables in the analysis, and the areas of unfavourable interactions of the studied variables on parameters characterizing safety of the structure.

1 Introduction

The design of structural elements and structures is a process of deliberate or intuitive optimal modelling, and its aim is to fulfil the assumed requirements to the extent possible. Theory and optimization methods are the subject of many years of basic research and are used in all fields of science and engineering widely understood [1, 2]. In recent years, the increase in the complexity and size of designed systems is followed by rapidly developing methods and techniques providing solutions close to optimal. However, the examples of application of these methods in engineering practice sometimes raise objections. This is due to both the approximate nature of the solutions obtained, and the uncertainty associated with the selection and specification of criteria, parameters, variables, optimization and limitations on the range of their variations and correlation omission. The widespread availability and the exponential growth of computing power as well as development of software for automated design and optimization of general systems and those adapted to the engineering design now enable the design of engineering structures fairly close to the optimal ones. The designer can carry out the "what-if" parametric analysis (i.e. simplified sensitivity analysis) to verify how the structure responds to changes in specific parameters and to the extent of these changes. There can also be performed a probabilistic analysis to estimate the reliability measurement and the structure safety, to identify multiple correlations between input and output variables. It is also possible to designate by the use of Monte-Carlo simulation method an approximate response surface of the structure for the adopted plan of virtual experiment and obtain trade-off charts showing the correctness of solutions obtained in the adopted design space.

The paper presents a brief description of a method of analysing the effect of changes in the values and uncertainties of considered design variables on the characteristics of the structure's state based on the "what-if" parametric sensitivity analysis and trade-off charts, by using simulation data collection techniques (DOE - Design Of Experiments) for the construction of the structure's response surface. There was also indicated the possibility of using this technique for estimating the correlation matrix between design variables and the structure characteristics, that is, between inputs and outputs of the analysed system. Design basis, procedures and results of the applied DOE technique for the evaluation of sensitivity and reliability of the structure were presented on the example of a post-tensioned concrete beam designed according to the requirements and procedures specified in Eurocode 2. Taking into account the random nature of the load, the total losses of prestressing force in each cable and deviations in the calculated depth of the beam cross-section from the nominal value, there was carried the "what-if" analysis of the impact of these variables on the ultimate load-bearing capacity and maximum deflection of the beam. The results obtained in the forms of a response surface,

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covariance matrix, sensitivity and trade-off charts allow for the determination of the order and range of influence of the analysis variables on parameters characterizing the safety of structure and the areas of unfavourable interactions of the studied variables.

2 Parametric analysis of structure sensitivity

In order to search for an engineering solution that sufficiently meets the numerous requirements of the European Community Council Directive 89/106 / EEC concerning products and civil structures, and the Polish Construction Law and current design standards, it is required to take into consideration many compromises. Reaching an optimal solution, or even close to optimal, involves adequate knowledge, specialized and expensive software and a lot of work and time. Taking advantage of the most widely used FEM software, a designer focuses mostly on the analysis of several intuitively selected variants and on choosing a solution that meets the basic standard requirements to a varying degree, without a comprehensive assessment of the extent and consequences of deviations from the optimal solution.

Applied to the engineering structures, the formal sensitivity analysis is used to determine changes in the considered characteristics of the structure state $\Delta y(\boldsymbol{x})$ (e.g. load-bearing capacity, displacement, vibration frequency, etc.) for a determined value of design variables vector $\boldsymbol{x}_o$ and the assumed increase in their value $\Delta\boldsymbol{x}$. With regard to the structural elements it generally involves the sensitivity analysis of the destructive impacts or transverse forces, deflections, frequency and mode of eigen vibrations, and the like, against the variation of design variables vector describing the mechanical and geometrical properties of the element. The most widely practised application is called parametric sensitivity analysis consisting in calculating the value of the selected characteristics of the structure state for a few or several arbitrarily chosen design variables and matching their increments to changes in the structure. This results in obtaining several points on the response surface of the structure which enables a rough assessment of the structure. However, the use of such a description of the structure's responses in attempts to optimize or build trade-off charts can lead to serious errors.

In the design of a structure the knowledge of response surface allows for determining the acceptable range of variables (also random variables) for which the design requirements are met, as well as for the choice of an optimal combination of these variables, in terms of their meeting established requirements and criteria [3]. In the case of real structures, for the approximation or interpolation of the response surfaces of acceptable accuracy, it is required to make multiple calculations of the structure responses (deflections, load-bearing capacity, etc.) for different combinations of increments in the structure's state variables, which amounts to performing a very large number of virtual experiments. Calculations are performed using appropriate FEM software, but due to the non-linearity and multidimensionality of applied structural models and a random nature of most of the state variables, they are very expensive. In order to reduce the time and costs of necessary calculation there are used Monte Carlo (MC) simulation methods with reduction techniques developed within the framework of the experiment theory. In order to determine the relationship between the design variables and the structure's reaction the researchers used in the ANSYS DesignXplorer program module [4] a technique of data collecting called DOE (Design Of Experinets) which is based on the experimental design known as Latin Hypercube Sampling (LHS) that allows considerable reduction in the number of results required to obtain a determined accuracy of the response surface, comparing to what offers the basic MC method [5,6]. Knowledge of the structure's response surface allows for a clear presentation of the relationship between the design and output variables by way of charts enabling us the visualization and correct interpretation of the structure's responses in different situations and for determining the necessary changes to be made to meet specific requirements.

A very useful design practice method of using the results of calculations made to determine the structure's response surface is providing trade-off charts representing the acceptability of solutions in a particular area of the design. Each design variable x_i , or a group of such variables $x_1,x_2,..,x_n$ from the design area of variation, is correlated with the result of the structure response calculations y_j in the form of one point. The set of all calculation results $y_1,y_2,\ldots,y_k$ indicates the area of possible states of structure where it is possible to make a division between acceptable solutions meeting the established criteria to a varying degree and those solutions that are close to the optimum[12-14].

The use of experimental plan based on Latin Hypercube Sampling method, i.e. a random selection of design variables from the design area of variations for which the structure's responses are computed allows for designing the correlation matrix between all the variables indicated in the calculation. Knowledge of the measure value of correlation allows for the assessment of the significance of the increasing or decreasing effect of the design variables on the characteristics of the structure state and the synergy effect [7]. This facilitates decision-making on the modification of the design in order to get close to the optimal solution [17].

3 Example

The subject of DOE analysis is a post-tensioned beam of the span of 18m, made of concrete strength class C60/75. The beam is prestressed by means of 3 seven-strand cables, of the total cross-section area of 3150mm2. The characteristic value of tensile strength of the prestressing steel is f_{pk}=1850MPa, and the characteristic value of the yield stress of the reinforcing steel f_{yk}=500MPa [8,9]. The total load of the beam consists of dead loads and uniformly distributed live load.

The numerical calculatations presented in this chapter were done using the „Ansys" program [4]. The *„SOLID65"* element was used for the 3D modelling of concrete. The element is defined by 8 nodes having 3 degrees of freedom and for the considered analysis. Four parameters to defined concrete were used, namely: shear transfer coefficients equal for an open crack 0.3 and for a dosed crack 0.8, uniaxial tensial cracking stress 3.5 Mpa and uniaxial crushing stress (positive) 3.5 Mpa. For modelling reinforcing bars „LINK 180", that is the uniaxial tension-compression element with 3 degrees of freedom at each node. The connection between bars and concrete is via constrain equations that connect the selected nodes of one region to selected elements of the other region. „SHELL 181" element was used for modelling of cables by extruding circular section along the path creating cables. It is a 4 nodes element with 6 degrees of freedom at each node. The shell element was also used for modelling anchorage plates at ends of the beam.

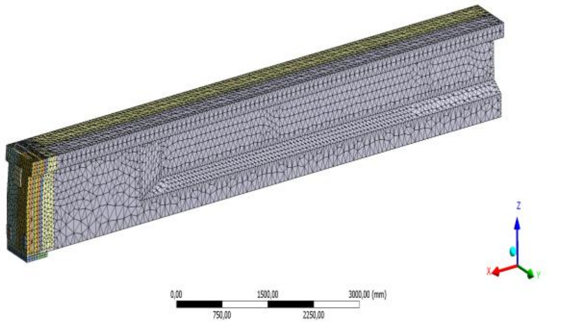


Fig. 1. a. The FEM model of the beam -view.

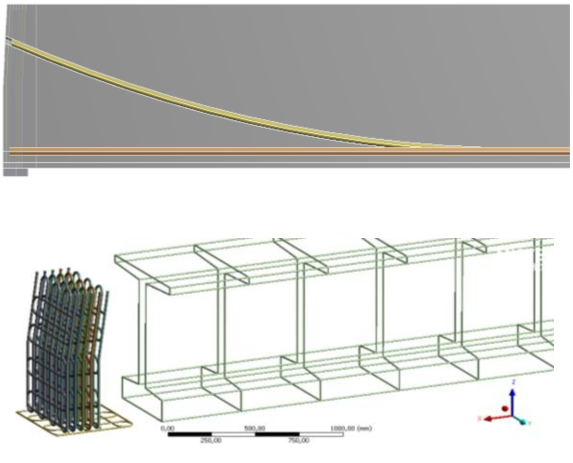


Fig. 1. b. The FEM model of the beam – cable routes and reinforcement of the anchoring area

3.1 Numerical analysis of the structure

The set of all calculation results $y_1,y_2,\ldots,y_k$ indicates the area of feasible states of the post-tensioned concrete beam structure, where it is possible to make a division between acceptable solutions meeting the established criteria to a varying degree and those solutions that are close to the optimum. [10,15]. Optimization criteria are usually limited to the standard requirements related to the ultimate states of load-bearing capacity and serviceability. Acceptable solutions are based on the restrictions on the maximum value of the prestressing force in cables, tensile and compression stresses in concrete and deflections [8]. In the worked example, the acceptable initial value of the prestressing force resulting from these limitations was P_0=4500 kN (with 1500 kN in each of the 3 cables). Calculated value of immediate losses of prestress for the assumed values of mechanical properties of the materials, deformations of concrete, prestressing steel and cable profiles. On this basis, it was assumed in the simulation design that the extent of prestressing losses is within the range of 9.2-15.25% (ARG1, 2, 3 fig.2). The GQ total load applied at the top flange consists of the dead load of the beam and reinforced concrete floor slab and uniformly distributed live load. For the calculations it was assumed that the GQ load is a Gaussian random variable with parameters: m_{GQ}=55 kN/m; v_{GQ}=0.20. The fifth input variable included in the analysis was depth of the bottom flange cross-section $ARG4 \equiv d = d_{nom} \pm 20$ mm.

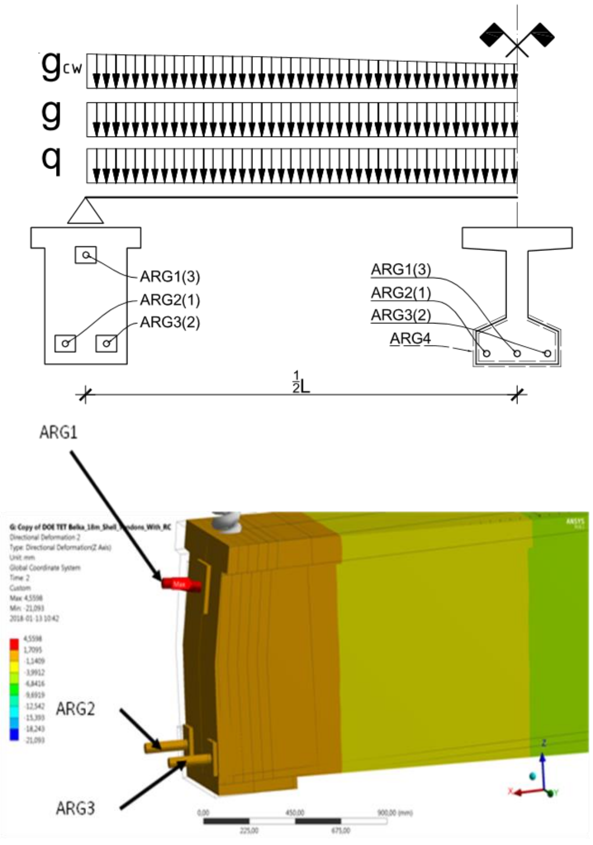


Fig. 2. Input variables for the analysis of the beam state (GQ; ARG1, ARG2, ARG3,ARG4)

The FEM numerical calculation shows that the greatest deflection (upwards) immediately after prestress is 26.89 mm (acc. to analytical calculations - 22.19 mm). Maximum deflection in a persistent situation is -21.09 mm (acc. to analytical calculations - 17.84 mm). The maximum shear stress in the concrete immediately after prestress is 21,12 MPa, and in the persistent design situation – 17,98 MPA. Figure 3 shows deflections in (a) the transient (b) and persistent design situation - fig. 3

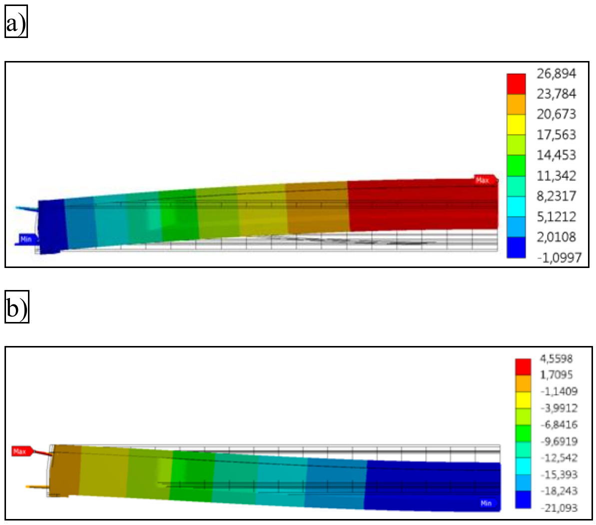
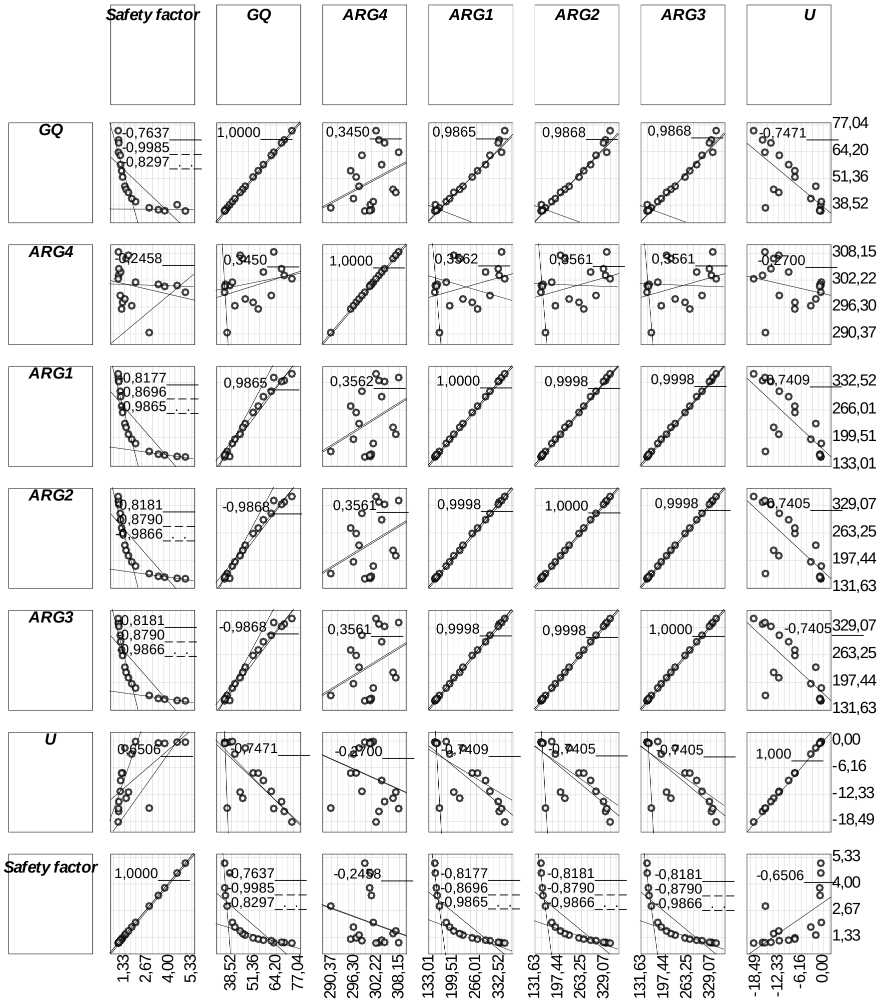


Fig. 3. Deflection of the beam in the transient (a) and persistent design situation (b)

3.2 Correlation analysis

When applying DOE data collection techniques [4] which uses the MC simulation calculation method, the modulus ANSYS DesignXproler of the „ANSYS" [4] programme can be used for the selection of required number calculation repetitions increases dramatically with the increasing number of the computing model variables, which results in significantly longer calculation time. Design points are chosen according to the experimental plan based on the Latin Hypercube Sampling method [5, 6]. Sampling of this type is one of the most effective the correlation matrix comprising Pearson linear correlation coefficients ρ_{XY} and graphs of the second type regression lines against the points representing the results of simulation calculations for all pairs of input and output variables considered in the analysis: the total load (GQ), the values of prestressing force losses in the cable1 (ARG1), 2 (ARG2) and 3 (ARG3) and the deflection of the beam (U) and the global safety factor (s). The value of the global safety factor defined as the ratio of the average values of load-bearing capacity and the effect of loads effect

Table 1. Correlation matrix. Input variables: total load (GQ), the prestressing force including losses in the cable 1 (ARG1), 2 (ARG2) and 3 (ARG3) and output variables: beam deflection (U) and the global safety factor (s).



The values of the coefficients of correlation between the depth of the bottom beam's flange and the other variables indicate a low or negligible correlation ρ_{XY} = 0.246÷0.36. For the other variables, the results indicate a significant linear correlation ρ_{XY} = 0.70÷0.90 or a very strong one $\rho_{XY} \geq 0.90$. Correlations between the global safety factor ***s*** and other variables are clearly non-linear in nature but for values ***s*** less and more than 2.5 they are linear and very strong. Noteworthy is also moderate but significant correlative relationship between the maximum deflection of the beam and the other variables ρ_{XY} = 0.65÷0.74.

3.3 Sensitivity analysis

Knowledge of the response surface allows for the analysis of parametric sensitivity of the structure to changes in the design variables, determining the acceptable range of input variables for which design requirements are met, as well as for the choice of optimal combination of these variables in terms of established requirements and criteria. In the case of the analysed cable-concrete beam, the seven-dimensional response hypersurface for the adopted range of 5 input and 2 output variables was approximated using the MC simulation method. The global safety factor ***s*** and the maximum deflection of the beam were assumed as output variables, which are decisive for meeting the basic requirements for safety and serviceability of the structure. In order to use a multi-dimensional hyper-surface of the response surface to the practical sensitivity analysis, i.e. the choice of design solution close to optimum or for verification of a particular solution we can use 2 or 3 three-dimensional cross-sections, so called "what-if" analysis or "trade-off charts". Fig. 4 shows a three-dimensional cross-section of the response hyper-surface of the analysed beam for the variables s-GQ-Arg1 (safety factor - total load – loss of prestressing force in cable 1). Based on "what-if" analysis we check, for example, how the value of the safety factor **s** changes if for a fixed load GQ = 47 kN/m, the prestressing force loss in the cable 1 ARG1 = 215 kN changes to ARG1 = 180 kN. The graph in Fig. 4 shows that the safety factor will increase in this situation from **s**=2.30 to **s**= 3.80. A strong tendency of the safety factor to increase matched with the decrease in prestressing force loss in the cable 1 is confirmed by the coefficient of correlation between the variables **s** i ARG1, **ρ** = -0.986 recorded in Table 1. It can also be noted that for a load greater than approx. GQ=68 kN/m and/or the loss of prestressing force in the cable 1 exceeding approx. 335 kN, the safety factor is always lower than the required **s**=2.60. The cross-section of the response hyper-surface of the analysed beam for the variables **u**-**s**-GQ (maximum deflection-safety factor-total load) is shown in Fig. 5. A designated response surface of the structure can also be used to seek solutions close to the optimum in terms of a variable or output variables (multi-criteria optimization). The more design points were used to determine the response surface the closer are the resulting solutions to this optimal design.

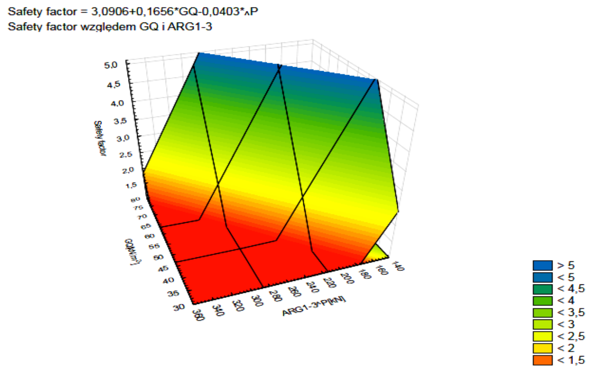


Fig. 4. Cross-section of the response hyper-surface of the analysed beam for variables **s**-GQ-Arg1 (safety factor - total load - loss of prestressing force in the cable No. 1).

It follows from this that a beam with parameters adopted in the analysis will not meet safety requirements if the load exceeds 68 kN/m and/or the loss of prestressing force in the cable 1 exceeds 335 kN, irrespective of the values of the other design variables. Assuming that the required minimum safety factor is **s**=2.60, and the acceptable deflection of the beam should not exceed the value of **u**=10 mm, the total load of the beam is about GQ=68/kN/m. For load values exceeding 68 kN/m and/or safety factor values lower than 2.60 the analysed beam deflection will exceed the limit value of 10 mm.

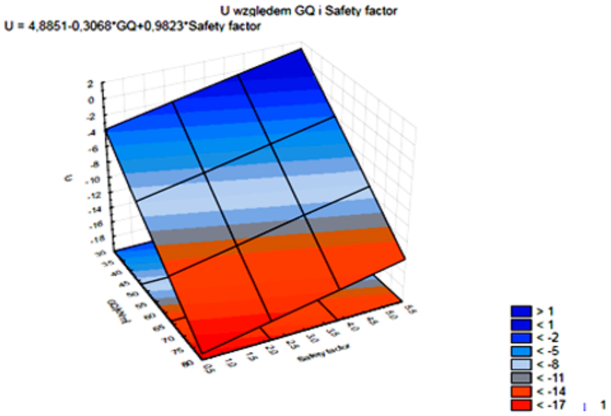


Fig. 5. The cross-section of the response hyper-surface of the analysed beam for the variables **u**-**s**-GQ (maximum deflection-safety factor-total load).

Another, simpler way to use the response surface of the structure is to draw "trade-off" charts. They represent the sets of points in the output variable- input variables plane divided into the area of acceptable and unacceptable solutions, regarding the values of output variables. Coordinates of the points which represent possible structure solutions for the specific ranges of variations of the analysis variables can be determined from the response surface.

Trade-off charts can be used, among others, for a quick verification of input design variables that have been assumed and modyfied in the design process.

Fig. 6 shows a trade-off chart for the correlation between a global safety factor and the prestressing force in the cable 3, adjusted for of the losses of prestressing force (in a persistent design situation). The dividing line between the safe and dangerous area corresponds to the safety factor value ***s***=2.60.

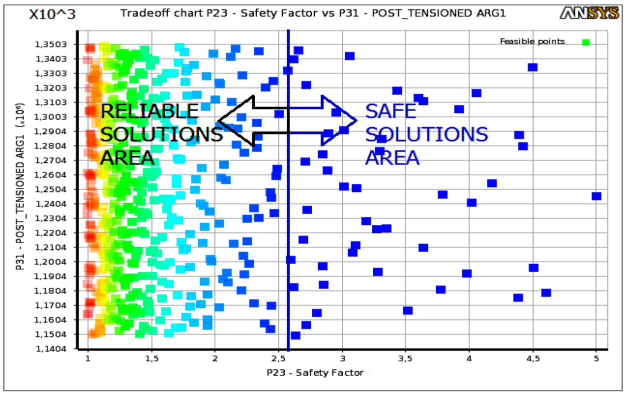


Fig. 6. Chart of trade-off between a global safety factor and the prestressing force in the cable 3 in a persistent design situation.

The chart of the trade-off between the global safety factor ***s*** and the total load beam GQ has been shown in Fig. 7. Here again the dividing line between the safe and dangerous area corresponds to the safety factor value ***s*** = 2.60.

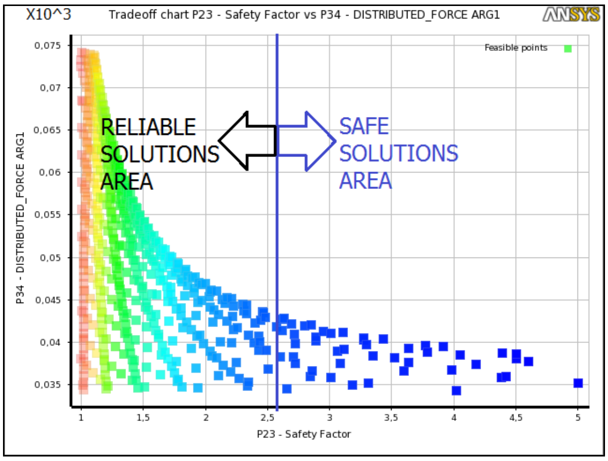


Fig. 7. The chart of the trade-off between the global safety factor s and the total load beam GQ in the persistent design situation.).

Conclusions

The strife for optimization of the structure at the design stage is present in engineering since ancient times. Development of the theory of design and optimization as well as automation of calculations raise expectations of designers and investors regarding the implementation of effective and transparent procedures for the design of civil structures that meet numerous detailed requirements relating to the construction, function, durability, environment, economy and other aspects of design.

The multifaceted nature of the requirements and the associated large number of design variables and optimization criteria which are subject to considerable uncertainties of a different nature and difficult to quantify all cause the design practice to be dominated by the approach based on verification and correction of preliminary design basis and results in view of obtaining a solution intuitively close to optimal design.

The presented approach to the analysis of the structure's state consists in using simulation techniques of data collection to approximate a response hyper-surface of the structure permitting to identify the key design variables determining the safety of structures, to define acceptable limits and those which are close to optimal states of structures and to choose sub-optimal values of considered variables.

The use of response hyper-surface of structures to develop the three-dimensional sections, and the two-dimensional trade-off charts facilitates the assessment of the range of changes of random design variables permitted by reason of certain criteria, and the choice of their values that are close to the optimum.

Design basis, procedures and results of using the presented considerations for the evaluation of sensitivity and reliability of the structure were shown on the example of the post-tensined concrete beam designed in accordance with the requirements and procedures of the Eurocode 2.

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