

# Dynamic response of long-span bridges subjected to non-uniform excitation: a state-of-the-art review

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**Abstract.** In recent years, considerable attention has been paid to the research of dynamic response of long-span bridges with particular emphasis on seismic behavior. Cable-stayed and suspension bridges are the most popular types. Since long-span bridges have multi-supports and extreme lengths, due to the spatial variation effects, the ground motions at different supports might be non-uniform. A state-of-the-art update review of the response of long-span bridges subjected to non-uniform excitation is presented. The review mainly focuses on the theoretical aspects of non-uniform excitation, numerical studies, and experimental studies to verify some of the theoretical findings. In this paper, a review of the use of shake-table in experimental studies of long-span bridges is also presented. The non-uniform cases considered include a time delay with the same support excitations, multiple support excitations, and the combination of the first and the later. The results are discussed and summarized in comparison to the cases of uniform support excitation.

## 1 Introduction

In general, structural analysis of seismic loads is based on the assumption that earthquake excitation at all structural supports is the same, known as uniform excitation. Spatial variability of earthquake ground motion is usually not taken into account in the seismic analysis of the bridge due to the simplification of the design method. Since the length of the long-span bridge is comparable to the seismic wavelength and the seismic waves propagate from the source to the surface with limited speed, it is intuitively clear that all bridge support cannot be moved simultaneously.

During the few decades, many researchers have carried out numerical analysis of the seismic behavior of long-span bridges subjected to non-uniform ground motion [1-2]. However, several experimental studies have been developed in this field. The experimental method plays an important role in civil engineering research. Both small-scale modeling tests and full-scale modeling tests are considered robust research methods that can be applied to study the real behavior of structures due to various loading cases and to verify the theoretical analysis.

However, it is challenging to conduct experimental studies of long-span bridges that have earthquake loads. The payload limit of the shake table and the limited space for testing in the laboratory, as well as strict similarity

requirements for dynamic testing, significantly increased the level of difficulty in this study.

Several tests using shake table on long-span bridge models have been published in the last few decades [3-5], while none of them considers the consequences of non-uniform excitation. Testing using a multiple shake table system on the two-girder bridge model was completed at the University of Nevada, Reno by considering non-uniform earthquake excitation case [6-7].

An updated state-of-the-art-review of the dynamic response of long-span bridges due to non-uniform excitation is presented here. The review briefly covers the theoretical aspects, numerical studies, and experimental studies, but strongly emphasizes the shaking table tests. The types of bridges presented here are long-span bridges, but the work associated with simple models to understand the behavior of the response structure subjected to non-uniform excitations are also summarized. The results of some necessary experimental tests are also included.

## 2 The theoretical aspect of non-uniform excitation

The first investigation of the theory of multiple support excitation (MSE) submitted by Bogdanoff et al. [8] and Clough and Penzien [9] who discussed the effects of MSE for multi-degree-of-freedom structures.

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After installation of dense instrument arrays around the world which record earthquake data, the causes of MSE have been explored and identified [10-13]. Group of researchers has previously studied spatial variation of ground motion phenomena. The effect of the wave passage (the difference in the arrival of the seismic wave to different supports of the structures) has been investigated by Kiureghian and Neuenhofer [10], Kiureghian [11], and Harichandran [12]. The incoherence effect studied the loss of coherency of earthquake waves due to reflections and refractions [10-13]. Further study on the local site effect (frequency content and amplitudes of earthquake influenced by different geological condition) and attenuation effect (decay of amplitudes and energy of earthquake wave propagating through the ground) have been investigated intensively[10-12].

### 3 Analytical studies

Wu et al. [14] presented a seismic analysis of multiple supporting structural systems that undergo nonuniform excitations. The time history method was used for this analytical study.

In matrix form, the governing equation of motion for the multi-degree of freedom system with multi-support excitation can be formulated as:

$$\begin{bmatrix} M_a & 0 \\ 0 & M_b \end{bmatrix} \begin{Bmatrix} \ddot{U}_a \\ \ddot{U}_b \end{Bmatrix} + \begin{bmatrix} C_{aa} & C_{ab} \\ C_{ab} & C_{bb} \end{bmatrix} \begin{Bmatrix} \dot{U}_a \\ \dot{U}_b \end{Bmatrix} + \begin{bmatrix} K_{aa} & K_{ab} \\ K_{ab} & K_{bb} \end{bmatrix} \begin{Bmatrix} U_a \\ U_b \end{Bmatrix} = \begin{Bmatrix} \bar{F}_a \\ F_b \end{Bmatrix} \quad (1)$$

where  $\bar{U}_a$  and  $\bar{U}_b$  denotes the displacements relating to non-support and support points;  $M_a$  and  $M_b$  are the lumped mass matrices corresponding to non-support and support points;  $C_{aa}$  and  $K_{aa}$  are the damping matrix and elastic stiffness matrix respectively denoting the forces occurred in the unsupported points due to the motion of the unsupported points;  $C_{bb}$  and  $K_{bb}$  are the damping matrix and elastic stiffness matrix respectively indicating the forces occurred in the support points due to the motion of the support points;  $C_{ab}$  and  $K_{ab}$  are the damping and stiffness matrices respectively expressing the coupling forces happened in the non-support points by the motion of the supports and, vice versa,  $\bar{F}_a$  is the specified external forces applied to the unsupported points;  $F_b$  is the reaction force at the structural support points. Total time derivative is denoted by ( $\dot{\quad}$ ). Also, the participation of the fixed points has been eliminated from the above formula. Formula (1) can be divided become two rows of formula. Because of nothing external forces, the first row of formulas can be expressed as:

$$\begin{aligned} [M_a]\{\ddot{U}_a\} + [C_{aa}]\{\dot{U}_a\} + [K_{aa}]\{U_a\} \\ = - [C_{ab}]\{\dot{U}_b\} - [K_{ab}]\{U_b\} \end{aligned} \quad (2)$$

and the second row of formulas can be expressed as:

$$\begin{aligned} [M_b]\{\ddot{U}_b\} + [C_{bb}]\{\dot{U}_b\} + [K_{bb}]\{U_b\} + [C_{ab}]\{\dot{U}_a\} \\ [K_{ab}]\{U_a\} = \{F_b\} \end{aligned} \quad (3)$$

The solving of the two rows of formula can be obtained by the standard solution procedure for a specified time history of displacement and velocity at every support points.

In the seismic analysis, the stress occurred in the structure can be detailed become a primary section and a secondary section. Both sections have different allowable limit rules for each other. The pseudostatic displacement component,  $U_a^s$ , may be gained from the static equilibrium formula by eliminating the dynamic force of formula (2). This situation gives guidance to

$$[K_{aa}]\{U_a^s\} = -[K_{ab}]\{\bar{U}_b\} \quad (4)$$

alternatively,

$$\{U_a^s\} = [H_{ab}]\{\bar{U}_b\} \quad (5)$$

where  $[H_{ab}]$  is the pseudostatic influence matrix and can be found from the solving of the following formulas:

$$[K_{aa}][H_{ab}] = -[K_{ab}] \quad (6)$$

Furthermore, the displacement response for the dynamic component is stated as

$$\{U_a^d\} = \{U_a\} - \{U_a^s\} \quad (7)$$

Substituting formula (7) and (5) into formula (2) leads to the following motion formulas that are related only to the dynamic displacement component:

$$\begin{aligned} [M_a]\{\ddot{U}_a^d\} + [C_{aa}]\{\dot{U}_a^d\} + [K_{aa}]\{U_a^d\} \\ = - [M_a][H_{ab}]\{\ddot{U}_b\} - ([C_{ab}] + [C_{aa}][H_{ab}])\{\dot{U}_b\} \end{aligned} \quad (8)$$

If the damping matrix is comparable to the stiffness matrix, or if the damping contribution to the effective support force is neglected, the term of  $\ddot{U}_b$  on the right part of formula (8) will disappear. It will be obtained

$$\begin{aligned} [M_a]\{\ddot{U}_a^d\} + [C_{aa}]\{\dot{U}_a^d\} + [K_{aa}]\{U_a^d\} \\ = - [M_a][H_{ab}]\{\dot{U}_b\} \end{aligned} \quad (9)$$

For the system with free vibration situations that may exist without excitation support, the formula for the undamped system can be expressed as

$$[M_a]\{\ddot{U}_a^d\} + [K_{aa}]\{U_a^d\} = 0 \quad (10)$$

The solving of the formula (10) can be stated as

$$\{U_a^d\} = \{\phi\} e^{i\omega t} \quad (11)$$

where  $\omega$  represents the natural circular frequency;  $\phi$  denotes mode shape of the structure. Both of the constants can be determined from

$$(\omega^2 [M_a] - [K_{aa}])\{\phi\} = 0 \quad (12)$$

For the analysis of the forced vibration, solution of the formula (9) is first modified become normal mode coordinates as

$$\{U_a^d\} = \sum_{n=1}^n \{\phi\}_n \zeta_n(t) \quad (13)$$

Substituting formula (13) into formula (9) yields in rows of uncoupled formulas. Appropriate to the  $n$ th mode,

$$M_n \ddot{\zeta}_n + 2M_n \eta_n \omega_n \dot{\zeta}_n + M_n \omega_n^2 \zeta_n = q_n \quad (14)$$

where,

$$M_n = [\phi]_n [M_a] \{\phi\}_n \quad (15)$$

$$\eta_n = \frac{[\phi]_n [C_{aa}] \{\phi\}_n}{2M_n \omega_n} \quad (16)$$

$$q_n = [\phi]_n \left( -[M_a] [H_{ab}] \{\ddot{U}_b\} \right) \quad (17)$$

The damping matrix condition was considered orthogonal. Thus, the uncoupled modal formula can be obtained either numerically or analytically.

## 4 Numerical studies

There had been several numerical studies in the past to study the effects of the non-uniform excitations for the long-span bridge. Harichandran et al. [15] presented random vibration analysis for the stationary and transient linear stochastic response of three long-span bridge excited by non-uniform excitations. By using two-dimensional (2D) finite-element models, all three original long-span bridges were evaluated. The studies were directed on : (1) the 213 m Cold Spring Canyon arch bridge (CSCB) in California and the 518 m New River Gorge arch bridge (NRGB) in West Virginia to find out behavior of the longitudinal and lateral response; and (2) the Golden Gate suspension bridge (GCB) in California to study behavior of the lateral response. It was drawn conclusions that transient effects should be supposed for typical suspension bridges, but may be neglected for typical deck arch bridges. It was also presumed that the use of uniform excitations is inadmissible for three prototype long-span bridges, whereas the use of non-uniform excitations is admissible for the lateral response of short suspension bridge and the longitudinal response of short arch bridges. However, it is inadmissible for the lateral response of long suspension bridge main spans, the lateral response of short and long arch bridges, and the longitudinal response of long arch bridges.

Li and Yang [16] evaluated the seismic response of an existing long-span prestressed-concrete continuous-rigid-frame bridge subjected to multi-support excitations. Incoherence effect, local site effect, and the wave passage effect were considered in the numerical simulation using the displacement time history method. The Harichandran-Vanmarcke model correlation function was considered to study incoherence effect on the bridge model due to spatially varying of earthquake ground motion. Furthermore, the acceleration peak value of Elcentro earthquake with an interval of 0.01 s was given to location design with intensity from 0.15 g to 0.307 g to evaluate local site effect. The bridge model was subjected to the longitudinal earthquake with and without regard to the traveling wave effect. Apparent velocity was 1000m/s, 1500 m/s, and 2000 m/s respectively. The contrast of results between multi-support excitations and uniform excitation was wave passage effect more significance than others effect. Consequently, the influence of the multi-support excitations on the seismic response of the long-span prestressed-concrete continuous rigid-frame bridge must be considered.

Savor et al. [17] investigated seismic analysis of deck-type concrete arch bridge to the spatially varying of earthquake ground motion. The seismic wave passage effect and the loss of coherence effect were performed on the bridge of similar layout but different spans by time-history analysis. Results of the longitudinal response analysis showed that spatial variability of earthquake ground motion should be taken into account for the seismic design of long-span arch bridges. It may be seen that the wave passage effects were generally more unfavorable than the loss of coherency effects, but neither of the consequences can be ignored.

## 5 Experimental studies

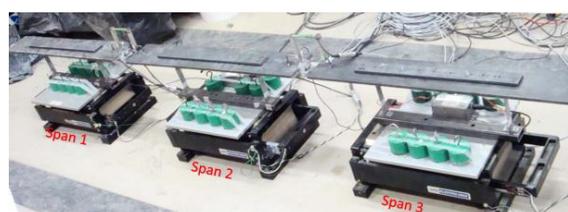
Some experimental studies have been reported on non-uniform excitations effects, starting from a simple model the two degrees of freedom structure (2 DOF) [18] to a relatively complex model of cable-stayed bridges [20-21]. Norman et al. [18] investigated a simple two degrees-of-freedom (2 DOF) structure to develop an approach for numerically and physically modeling the effects of MSE (schematic diagram in Figure 1).



Fig. 1. The simple experimental model [18]

The effect of MSE was investigated on this 2DOF system by applying different frequency sine wave motions. It was possible to determine the frequencies of motion by holding the frequency of motion at one end of model constant and varying the frequency of motion at the other end. The result showed that the peak response of the system might be significantly underestimated if multiple support excitation was ignored.

Sun et al. [19] conducted shake table tests on a scaled bridge model with three identical spans constructed using polyvinyl chloride (PVC) to investigate pounding between two identical single-degree-of-freedom bridge models. The three-span bridge model was executed by three separately controllable shake tables considering non-uniform excitation, time-delay, loss of coherency, and different stiffness of site soil. The detail of the testing setup for the case of pounding can be shown in Figure 2.



**Fig. 2.** Setup experimental study considering the pounding effect [19]

The results showed relative displacement between girders increases so that pounding force increase. The experimental behavior of the column bending moments were found to decrease with the pounding. Opening and closing relative displacements were observed to decrease as harder soil ground motion.

Furthermore, Yang and Cheung [20] and Yang et al. [21] carried out an experimental study using dual shake table system for a long-span cable-stayed bridge model due to non-uniform excitation. The 1:120 cable-stayed bridge model was designed and constructed for dynamic testing by considering the similitude laws. The summary of the scale factors can be found in Table 1.

There are two types of tower-girder connections used in experimental testing to study the influence of non-uniform excitations. Scheme of instrumentations consists of 33 channels of displacement transducers, accelerometers, and strain gauges were planned for the experimental test using the dual shake table. Photograph of instrumentation scheme and finished model can be seen in Figure 3.

The result of the shake table test showed the effect of non-uniform excitation on different structural components and different structural form has a complex and inconsistent response. Yang and Cheung [20-21] suggested more data from an experimental study should be analyzed for the further study.

**Table 1.** The similitude ratio for the scaled testing model [20, 21]

Quantity	Scale Factor
Length	1: 120
Acceleration	1: 1
Time	1: 10.954
Frequency	1: 0.0913
Modulus	1: 11.182
Strain	1: 1
Model Weight	1: 161022



**Fig. 3.** The completed model with instrument scheme [21]

## 6 Shaking Table Test

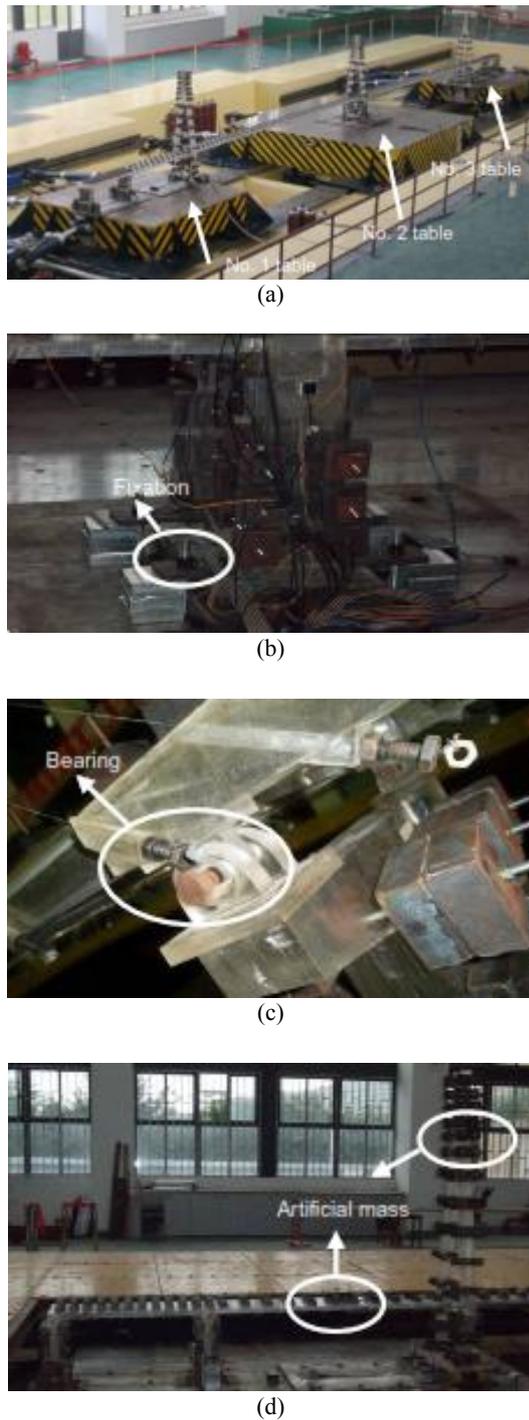
In the case of long-span bridges such as cable-stayed bridges and suspension bridges, experimental testing is usually done using a shaking table. Most of the studies primarily attempt to investigate the dynamic behavior of long-span bridges including the identification of their maximum response.

Zong et al. [22] explored the seismic response characteristics of a 1:100 multi-span cable-stayed bridge model under uniform and non-uniform excitations. The prototype of the cable-stayed bridge was The Wuhan Erqi Yangtze River Bridge (WEYRB) connecting Wuchang and Hankou Towns in Wuhan City. The overall length of the bridge is 1732 m and arrangement of the spans is 90 m+160 m+ 616 m+ 616 m+ 160 m+ 90 m. The total width of the bridge deck is 31.4 m.

The design of the scale model was based on dynamic similitude laws considering four requirements. First, the proposed scale model should be simple regarding fabrication and construction. Second, the geometric scale was chosen according to the arrangement of the shaking table system and the prototype length. Third, the scale model should be as large as possible to eliminate irregularity effects in structural configurations and variations caused by material properties. The last, the relations between a scale model and prototype bridges for tensile stiffness and bending stiffness of the main girder,

tower and cable, and the mass of these components, should be similar.

Furthermore, the details of the shaking table tests were found in Figure 4.



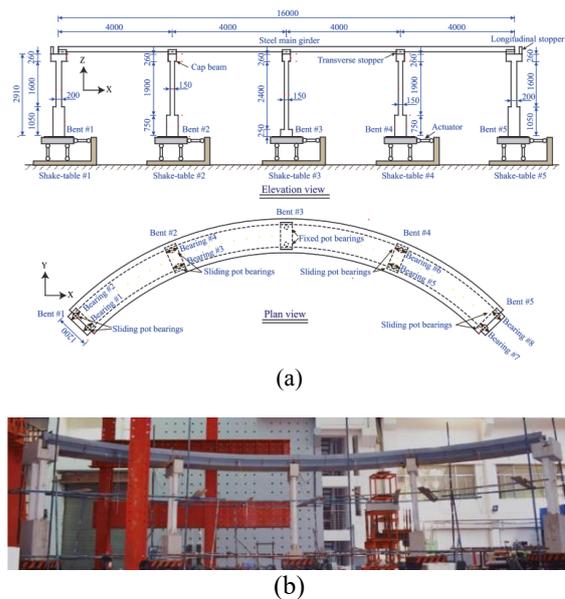
**Fig. 4.** Detail the scaled model [22]: (a) three tower cable-stayed bridge; (b) fixation of the tower; (c) cable and bearing; (d) artificial mass

Experimental test results showed that the seismic response of the scaled model was most apparent due to uniform excitation of Jiangxin wave. When the wave velocity exceeded 616 m/s, the seismic performance of the bridge scaled model, especially the towers and the main girder, were unfavorably affected by traveling wave

effects. Also, the shear failure of the middle tower bearing first appeared when the maximum acceleration of the El Centro wave reached  $4 \text{ m/s}^2$ . This study explains the results that have been achieved to provide an understanding of the dynamic response of multi-span cable-stayed bridges due to non-uniform excitations.

Li et al. [23] evaluated the effects of non-uniform excitation using shake-table tests for a 1/10-scale typical curved bridge. Non-uniform excitations include wave passage effect, local site effect, and ground motion multidimensionality. The prototype bridge was a four-span curved continuous girder bridge with 100 m curvature radius and  $4 \times 40 \text{ m}$  layout. This bridge has concrete box-section for main girder and double columns (connected by a cap beam) for bents.

The schematic representation of the curved bridge test is shown in Figure 5.



**Fig. 5.** The curved bridge (in millimeters) [23]: (a) schematic display of the curved bridge; (b) layout of the actual curved bridge

The results of the experimental test showed the wave passage effect and the site effect significantly influence the seismic response of curved bridges. Further, the wave passage effect is mainly owing to the higher asymmetric modes and pseudo-static displacements. The straight bridge may be less damaged than the curved bridges during the same earthquake. The structures with a fundamental frequency close to the predominant frequency of the site may be significantly affected. The radius of curvature causes the curved bridge to be more sensitive to ground motion spatial variations, especially the effects of local sites.

## 7 Concluding Remarks

The review of the literature on the topics explaining that works on the following area are still inadequate and deserve the attention of future research to understand the problem better and to provide specific guidelines for the design:

- Investigation of the types of material for bridge model that appropriate to the types of material for the prototype, especially on the material of cable element, pylon, and girders.
- Research on the dynamic response of long-span bridges during construction due to non-uniform excitation in the bridge and also the effects of the direction of the earthquake.
- Study on how to measure effectively earthquake response of cable element of long-span bridge subjected to a spatial variation of earthquake ground motion.

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