

Dynamic interaction between elastic plate and transversely isotropic poroelastic medium

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Abstract. In this paper, dynamic response of an elastic circular plate, under axisymmetric time-harmonic vertical loading, resting on a transversely isotropic poroelastic half-space is investigated. The plate-half-space contact surface is assumed to be smooth and fully permeable. The discretization techniques are employed to solve the unknown normal traction at the contact surface based on the solution of flexibility equations. The vertical displacement of the plate is represented by an admissible function containing a set of generalized coordinates. Solutions for generalized coordinates are obtained by establishing the equation of motion of the plate through the application of Lagrange's equations of motion. Selected numerical results corresponding to the deflections of a circular plate, with different degrees of flexibility, resting on a transversely isotropic poroelastic half-space are presented.

1 Introduction

Biot [1] proposed a theory of wave propagations in a poroelastic material, which is a two-phase material consisting of an elastic solid with voids filled with water. Thereafter, Biot's poroelastodynamics theory has been employed by many researchers to study dynamic interaction problems between foundations and fluid-saturated porous media due to its close relevance to various practical problems in civil engineering. For example, the dynamic responses of rigid foundations under vertical loading were investigated by Kassir and Xu [2] for rigid strips, Jin and Liu [3], Zeng and Rajapakse [4] and Ai et al. [5] for rigid circular plates, and Halpern and Christiano [6], Senjuntichai et al. [7-8] and Keawsawasvong and Senjuntichai [9] for rigid rectangular plates. In addition, the influence of the flexibility of foundations on dynamic interaction between foundations and supporting poroelastic media was also investigated by Senjuntichai and Kaewjuea [10] for multiple flexible strips and Senjuntichai and Sapsathiarn [11] for a flexible circular plate.

Mechanical behaviours of natural soils and rocks commonly exhibit certain degrees of anisotropy owing to deposition or sedimentation processes. As a result, the material properties of such materials in vertical and horizontal directions are generally different. Constitutive relations for anisotropic poroelastic materials were introduced by Biot [12] to capture the anisotropy effect of the two-phase materials. Biot [13] then proposed governing equations for wave propagations in

anisotropic poroelastic materials. The interpretation of material properties of transversely isotropic poroelastic media was later presented by Cheng [14] in order to illustrate the relations between the results obtained from the laboratory measurement and the parameters in the transversely isotropic poroelastic materials proposed by Biot [13]. By adopting Biot's equations [13], Kumar et al. [15], Sahebkar and Eskandari-Ghadi [16] and Pooladi et al. [17] presented solutions of a transversely isotropic poroelastic medium subjected to time harmonic point or uniform loading. A review of literature reveals that a soil-structure interaction problem involving a flexible plate and a transversely isotropic medium has never been appeared in the past.

This paper presents vertical vibrations of an elastic circular plate of radius a under axisymmetric time-harmonic vertical loading resting on a transversely isotropic poroelastic half-space as shown in Fig. 1. The plate-half-space contact surface is assumed to be smooth and fully permeable. A classical thin-plate theory [18] is adopted for the circular plate, and the plate deflection is represented by an admissible function containing a set of generalized coordinates. A representation for contact traction is established in terms of generalized coordinates through the solution of a flexibility equation based on a fundamental solution of the half-space based on Biot's theory [13]. The equation of motion of the plate in terms of generalized coordinates are established through the application of Lagrange's equation of motion. The accuracy of the present solution is verified by comparisons with existing solutions. Selected

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numerical results are presented to portray the influence of relative flexibility and the frequency of excitation on the deflection of the circular plate.

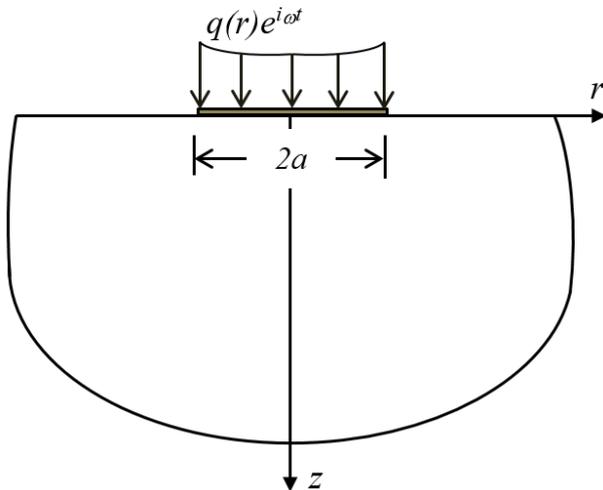


Fig. 1. A circular plate resting on a transversely isotropic poroelastic half-space.

2 Basic equations and influence functions

Consider a transversely isotropic poroelastic medium undergoing axisymmetric deformations with the cylindrical coordinate system (r, θ, z) as shown in Fig. 1. According to Cheng [14], the constitutive relations for transversely isotropic poroelastic materials can be expressed for axisymmetric deformations as,

$$\sigma_{rr} = c_{11} \frac{\partial u_r}{\partial r} + c_{12} \frac{u_r}{r} + c_{13} \frac{\partial u_z}{\partial z} - \alpha_h p \quad (1a)$$

$$\sigma_{\theta\theta} = c_{12} \frac{\partial u_r}{\partial r} + c_{11} \frac{u_r}{r} + c_{13} \frac{\partial u_z}{\partial z} - \alpha_h p \quad (1b)$$

$$\sigma_{zz} = c_{13} \left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} \right) + c_{33} \frac{\partial u_z}{\partial z} - \alpha_v p \quad (1c)$$

$$\sigma_{rz} = c_{44} \left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right) \quad (1d)$$

$$p = -M \alpha_h \left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} \right) - M \left(\alpha_v \frac{\partial u_z}{\partial z} + \frac{\partial w_r}{\partial r} + \frac{w_r}{r} + \frac{\partial w_z}{\partial z} \right) \quad (1e)$$

where σ_{rr} , $\sigma_{\theta\theta}$, σ_{zz} and σ_{rz} denote the total stress components of the bulk material; u_i and w_i denote the displacement of the solid matrix and fluid displacement relative to the solid matrix respectively in the i -direction ($i = r, z$); p is the excess pore fluid pressure (suction is considered negative); c_{11} , c_{12} , c_{13} , c_{33} and c_{44} are transversely isotropic elastic moduli; and α_i ($i = h, v$)

and M are Biot's parameters accounting for compressibility of the two-phased material. In this paper, the variables h and v are used in the subscript of material parameters to define their properties either in the plane of isotropy ($i = h$) or in the perpendicular direction to the isotropic plane ($i = v$).

The relationship of Young's modulus, Poisson's ratio, and shear modulus with the elastic moduli, c_{11} , c_{12} , c_{13} , c_{33} and c_{44} , can be expressed as follows:

$$c_{11} = \frac{E_h (E_v - E_h \nu_{vh}^2)}{(1 + \nu_h)(E_v - E_h \nu_h - 2E_h \nu_{vh}^2)} \quad (2a)$$

$$c_{12} = \frac{E_h (E_v \nu_h + E_h \nu_{vh}^2)}{(1 + \nu_h)(E_v - E_h \nu_h - 2E_h \nu_{vh}^2)} \quad (2b)$$

$$c_{13} = \frac{E_h E_v \nu_{vh}}{E_v - E_h \nu_h - 2E_h \nu_{vh}^2} \quad (2c)$$

$$c_{33} = \frac{E_v^2 (1 - \nu_{vh})}{E_v - E_h \nu_h - 2E_h \nu_{vh}^2} \quad (2d)$$

$$c_{44} = G \quad (2e)$$

where E_h is the drained Young's modulus in the plane of isotropy; E_v is the drained Young's modulus in the perpendicular direction to the isotropic plane; and G is the shear modulus. In addition, ν_h is the drained Poisson's ratio characterizing the transverse strain reduction in the plane of isotropy due to tensile stress in the same plane; and ν_{vh} is the drained Poisson's ratio corresponding to the transverse strain reduction in the plane of isotropy due to tensile stress normal to it.

In this study, the motion is assumed to be time-harmonic with the factor of $e^{i\omega t}$, where ω is the frequency of the motion, and $i = \sqrt{-1}$. The term $e^{i\omega t}$ is suppressed hereafter from all equations and expressions for brevity.

The Hankel integral transform with respect to the radial coordinates is employed to solve for the general solution. The Hankel transform and its inverse transform can be expressed respectively as [19],

$$\bar{f}(\xi, z) = \int_0^\infty f(r, z) J_n(\xi r) r dr \quad (3a)$$

$$f(r, z) = \int_0^\infty \bar{f}(\xi, z) J_n(\xi r) \xi d\xi \quad (3b)$$

where J_n denotes the Bessel function of the first kind of order n , and ξ is the Hankel transform parameter.

It can be shown that the zero-order Hankel transforms of u_z , w_z , σ_{zz} , and p , and the first-order Hankel transforms of u_r and σ_{rz} for the case of axisymmetric loading applied on the surface of a half-space (see Fig. 1) can be expressed as,

$$\bar{u}_r = \chi_1 A_1 e^{-\lambda_1 z} + \chi_2 A_2 e^{-\lambda_2 z} + \chi_3 A_3 e^{-\lambda_3 z} \quad (4a)$$

$$\bar{u}_z = \psi_1 A_1 e^{-\lambda_1 z} + \psi_2 A_2 e^{-\lambda_2 z} + \psi_3 A_3 e^{-\lambda_3 z} \quad (4b)$$

$$\bar{p} = \zeta_1 A_1 e^{-\lambda_1 z} + \zeta_2 A_2 e^{-\lambda_2 z} + \zeta_3 A_3 e^{-\lambda_3 z} \quad (4c)$$

$$\bar{w}_z = \varpi_1 A_1 e^{-\lambda_1 z} + \varpi_2 A_2 e^{-\lambda_2 z} + \varpi_3 A_3 e^{-\lambda_3 z} \quad (4d)$$

$$\bar{\sigma}_{rz} = \varphi_1 A_1 e^{-\lambda_1 z} + \varphi_2 A_2 e^{-\lambda_2 z} + \varphi_3 A_3 e^{-\lambda_3 z} \quad (4e)$$

$$\bar{\sigma}_{zz} = \eta_1 A_1 e^{-\lambda_1 z} + \eta_2 A_2 e^{-\lambda_2 z} + \eta_3 A_3 e^{-\lambda_3 z} \quad (4f)$$

where A_1, A_2 and A_3 are the arbitrary functions to be determined from appropriate boundary conditions. In addition, λ_1, λ_2 and λ_3 are the dimensionless complex wave numbers associated with the three body waves, in which,

$$\begin{aligned} \lambda_1 &= \sqrt{\Omega_3 - \frac{\Omega_1}{3\Omega_3} - \frac{\gamma_2}{3\gamma_1}}; \\ \lambda_2 &= \sqrt{\Delta\Omega_3 - \frac{\Omega_1}{3\Delta\Omega_3} - \frac{\gamma_2}{3\gamma_1}}; \\ \lambda_3 &= \sqrt{\Delta^2\Omega_3 - \frac{\Omega_1}{3\Delta^2\Omega_3} - \frac{\gamma_2}{3\gamma_1}} \end{aligned} \quad (5)$$

Other parameters appearing in Eqs. (4) and (5) are explicitly shown in the **Appendix**.

To determine the influence functions corresponding to the vertical loading on the surface of the half-space, a boundary value problem has to be solved. For the case of uniformly distributed vertical loading of unit intensity applied over an annular area, the boundary conditions are given as follows:

$$\bar{\sigma}_{rz}(\xi, 0) = 0 \quad (6a)$$

$$\bar{\sigma}_{zz}(\xi, 0) = \bar{f}(\xi) \quad (6b)$$

$$\bar{p}(\xi, 0) = 0 \quad (6c)$$

where $\bar{f}(\xi)$ is the Hankel transform of the annular loading with the external and internal radii denoted by r_{lo} and r_{li} , respectively, and it is given by,

$$\bar{f}(\xi) = \frac{1}{\xi} \left[r_{lo} J_1(\xi r_{lo}) - r_{li} J_1(\xi r_{li}) \right] \quad (6d)$$

The arbitrary functions A_1, A_2 and A_3 for the annular vertical loading on the surface of the half-space can be determined by solving the above boundary-value problem by substituting the relevant general solutions in Eq. (4) into the boundary conditions in Eq. (6). The vertical displacement in the Hankel transform domain is

then obtained by substituting those arbitrary functions into Eq. (4b), and it is later used as the required influence function to solve the dynamic interaction problem between a circular plate and a transversely isotropic poroelastic half-space as shown in Fig. 1.

3 Formulation of interaction problem

Consider an elastic circular plate of radius a subjected to axisymmetric vertical loading as shown in Fig. 1. The displacement of the plate is represented by $w(r)$ in the following form:

$$w(r) = \sum_{n=0}^N \Delta_n r^{2n}, \quad 0 \leq r \leq a \quad (7)$$

where Δ_n ($n = 0, 1, \dots, N$) denotes a set of generalized coordinates.

The strain and kinetic energies of the plate denoted by U_p and V_p , respectively, can be expressed as [18]:

$$U_p = \frac{1}{2} \int_0^a 2\pi D \left[\left(\frac{d^2 w}{dr^2} + \frac{1}{r} \frac{dw}{dr} \right)^2 - \frac{2(1-\nu_p)}{r} \frac{dw}{dr} \frac{d^2 w}{dr^2} \right] r dr \quad (8a)$$

$$V_p = \frac{1}{2} h_p \int_0^a 2\pi \rho_p [w(r)]^2 r dr \quad (8b)$$

where

$$D = \frac{E_p h_p^3}{12(1-\nu_p^2)} \quad (8c)$$

In the above equations, h_p is thickness of the plate; and E_p, ν_p , and ρ_p are Young's modulus, Poisson's ratio, and the mass density of the plate material respectively.

A fully permeable circular plate under axisymmetric vertical loading is resisted by the normal contact traction acting on bottom surface of the plate. Let S denote the circular contact area between the plate and the supporting medium. The normal traction can be represented by a traction field $T_z(r)$ acting on the circular surface S . The strain energy U_h of the transversely isotropic poroelastic half-space can be expressed in the following form:

$$U_h = \frac{1}{2} \int_0^a 2\pi T_z(r) w(r) r dr \quad (9)$$

In addition, the potential energy due to a vertical load, $q(r)$, as shown in Fig. 1 is written as,

$$W = - \int_0^a 2\pi q(r) w(r) r dr \quad (10)$$

The discretization technique based on Zeng and Rajapakse [4] is employed to study this interaction problem. The circular contact area S is discretized into a number of Ne annular elements. It is assumed that $T_z(r)$ is constant within each annular element. The unknown contact traction $T_z(r)$ is then evaluated by solving a flexibility equation based on the influence function as described in the previous section. The Lagrangian function L of the plate given by Washizu [20] is defined as follows:

$$L = V_p - U_p - U_h + W \quad (11)$$

The generalized coordinates Δ_n ($n = 0, 1, \dots, N$) are determined from the following equation:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\Delta}_n} \right) - \frac{\partial L}{\partial \Delta_n} = 0, \quad n = 0, 1, \dots, N \quad (12)$$

The solution of a linear simultaneous equation system given by Eq. (12) yields the solution of the generalized coordinates Δ_n ($n = 0, 1, \dots, N$) for a given plate-half-space system. The deflection of the circular plate can then be obtained by substituting the generalized coordinates into Eq. (7).

4 Numerical results and discussion

In this study, a globally adaptive numerical quadrature scheme [21] with a 21-point Gauss-Kronrod rule is applied to the semi-infinite integral with respect to ξ , given by Eq. 3(b), to determine the required influence functions used for this interaction problem. In all numerical results presented hereafter, a non-dimensional relative flexibility parameter defined as $\gamma = Ga^3 / D$, and a non-dimensional frequency defined as $\delta = \omega a \sqrt{\rho / G}$ are used, in which ρ is the mass density of the bulk material. In addition, a non-dimensional deflection of the circular plate is given by $w^*(r) = Gw(r) / aq_0$ where q_0 is the magnitude of uniformly distributed loading acting on the circular plate. Other non-dimensional material parameters are defined as: $E_h^* = E_h / G$; $E_v^* = E_v / G$; $M^* = M / G$; $\rho_f^* = \rho_f / \rho$; $m_h^* = m_h / \rho$; $m_v^* = m_v / \rho$; $b_h^* = b_h a / \sqrt{\rho G}$; and $b_v^* = b_v a / \sqrt{\rho G}$, where ρ_f is the mass density of the pore fluid; m_v and m_h are the density-like parameters that depend on ρ_f and the geometry of the pores; b_v and b_h are the parameters accounting for the internal friction due to the relative motion between the solid matrix and the pore fluid.

The accuracy of the present solution scheme is confirmed by comparing with existing solutions. Fig. 2 presents the comparison of non-dimensional deflections between the present solution and the solution given by Senjuntichai and Sapsathiarn [11] for the case of a circular plate, subjected to a uniformly distributed

loading of constant magnitude q_0 , resting on an isotropic poroelastic half-space. The material properties employed in the present solution are as follows: $\gamma = 100$; $E_h^* = E_h^* = 2.6$; $\nu_h = \nu_{vh} = 0.3$; $\alpha_h = \alpha_h = 0.95$; $\rho_f^* = 0.53$; $m_h^* = m_h^* = 1.1$; $M^* = 12.2$; and $b_h^* = b_h^* = 2.3$. In addition, a smooth contract surface with fully permeable condition is assumed. Convergence of the proposed solution scheme was studied with respect to the number of annular elements, Ne , and a set of generalized coordinates, N , employed in the proposed solution scheme. It can be clearly seen from Fig. 2 that the present solution, with $Ne = 32$ and $N = 8$, agrees very closely with the existing solution [11] for both real and imaginary parts of the displacement at the center of the circular plate ($r = 0$).

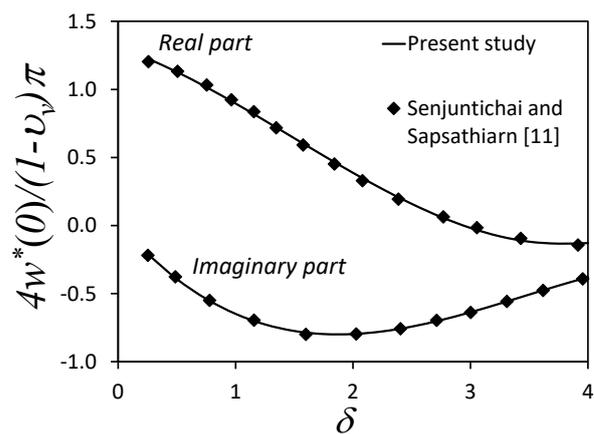


Fig. 2. Comparison with existing solutions for isotropic poroelastic half-space

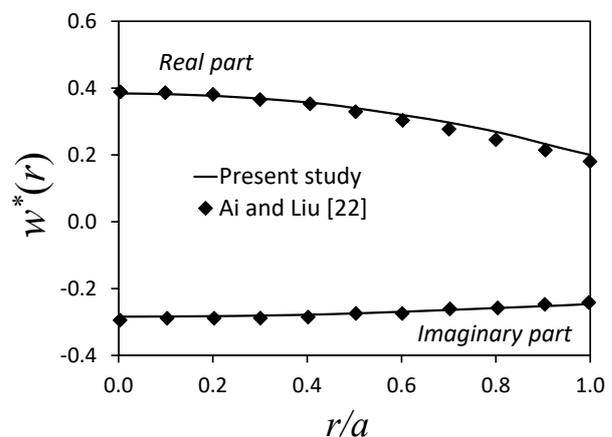


Fig. 3. Comparison with existing solutions for transversely isotropic elastic half-space

The accuracy of the present study is also examined in Fig. 3 by comparing with displacement profiles given by Ai and Liu [22] who studied the case of a circular plate, subjected to a uniformly distributed loading of constant magnitude q_0 , resting on a transversely isotropic half-space. The material properties employed in the present solution are as follows: $\delta = 1$; $\gamma = 11.25$; $E_h^* = 2.5$; $E_h^* =$

7.5; $\nu_h = \nu_{vh} = 0.3$ (other parameters are set to be negligibly small). It can be observed from Fig. 3 that a very good agreement between the presented result and that proposed by Ai and Liu [22] is obtained for both real and imaginary parts of $w^*(0)$.

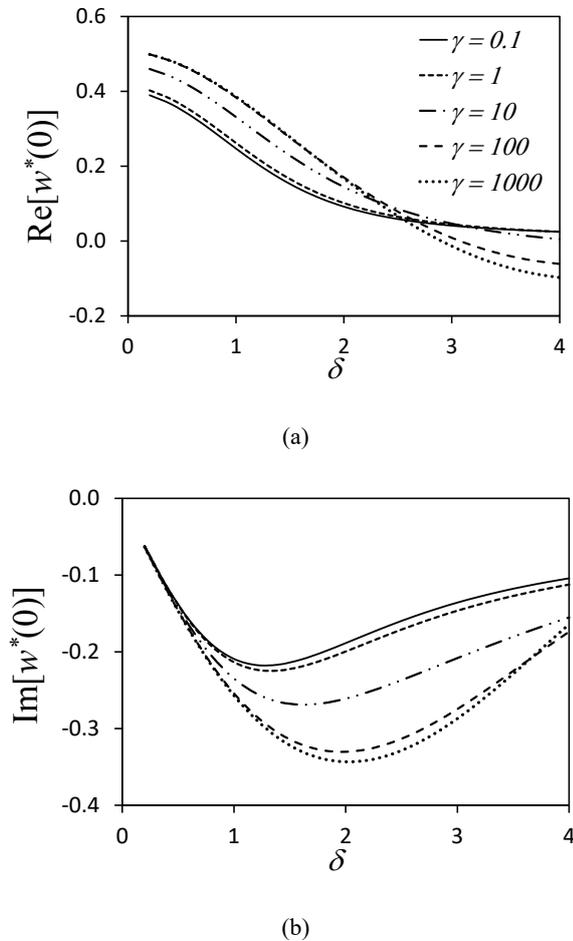


Fig. 4. Influence of relative flexibility on non-dimensional central deflection: (a) real part; (b) imaginary part

Next, vertical vibrations of a uniformly loaded circular plate resting on a transversely isotropic poroelastic half-space are shown in Figs. 4 and 5. The material properties employed in the presented results are defined as follows: $E_h^* = 2.5$; $E_h^* = 5$; $\nu_h = \nu_{vh} = 0.3$; $\alpha_h = \alpha_h = 0.95$; $\rho_f^* = 0.5$; $m_h^* = m_h = 1.1$; $M^* = 12.5$; and $b_h^* = b_h^* = 2.5$. Fig. 4 shows both real and imaginary parts of non-dimensional deflection at the center of a circular plate for the non-dimensional frequency range of $\delta = 0.2-4.0$. In addition, solutions are presented for different values of relative flexibility, i.e., $\gamma = 0.1, 1, 10, 100$ and 1000 . It can be seen from the figure that the displacement of the plate depends significantly on both γ and δ . The magnitude of the displacements decreases with decreasing the relative flexibility for both real and imaginary parts of $w^*(0)$, except for $\delta = 2.5-4.0$, where the increase of $w^*(0)$ with decreasing γ is noted.

The influence of the relative flexibility on the deflection profiles of the circular plate is shown in Fig. 5 for $\delta = 1$. It can be seen that the relative flexibility has a strong influence on the deflection profiles of the circular plate in the range $\gamma = 1-100$. In addition, it is found that the plate behaves as a rigid disk when γ is smaller than 0.1 . This implies that a flexible plate can be modelled as a rigid plate in this interaction problem when $\gamma \leq 0.1$.

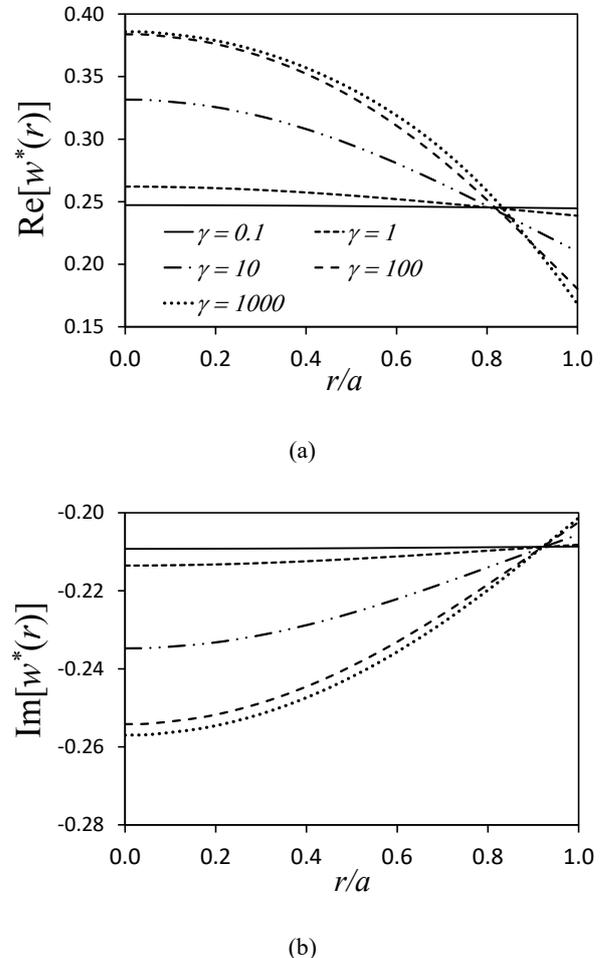


Fig. 5. Influence of relative flexibility on non-dimensional deflection profiles with $\delta = 1$: (a) real part; (b) imaginary part

5 Conclusion

This paper presents dynamic interaction between an elastic circular plate under vertical loading and a transversely isotropic poroelastic half-space. The interaction problem is investigated based on a variational principle together with appropriate influence function and discretization technique. The proposed scheme is verified with existing solutions for both isotropic poroelastic and transversely isotropic media. Selected numerical results are presented to demonstrate the influence of relative flexibility on the deflection of a circular plate. It is found that the influence of relative flexibility is significant when $\gamma = 1-100$. Both real and

imaginary parts of the deflection decrease with decreasing γ , and the maximum deflection is observed when $\gamma = 1000$. It is also found that a circular plate with a small value of γ , i.e. $\gamma \leq 0.1$, can be considered as a rigid plate in the analysis of vertical vibrations of circular plate under axisymmetric vertical loading.

6 Appendix

The parameters in Eqs. (4) and (5) are given as follows:

$$\chi_j = \xi \lambda_j \left[s_4 (s_3 + \beta_v \lambda_j^2) - s_5 s_6 \right] \quad (13a)$$

$$\psi_j = - (s_3 + \beta_v \lambda_j^2) (s_1 + c_{44} \lambda_j^2) + s_6^2 \xi^2 \quad (13b)$$

$$\zeta_j = \lambda_j \left[s_5 (s_1 + c_{44} \lambda_j^2) - s_4 s_6 \xi^2 \right] \quad (13c)$$

$$\varpi_j = (\lambda_j \zeta_j - \rho_f \omega^2 \psi_j) \beta_v \quad (13d)$$

$$\varphi_j = c_{44} (\lambda_j \chi_j - \xi \psi_j) \quad (13e)$$

$$\eta_i = \xi c_{13} \chi_j + c_{33} \lambda_j \psi_j - \alpha_v \zeta_j \quad (13f)$$

$$\Omega_1 = -\frac{\gamma_2^2}{3\gamma_1^2} + \frac{\gamma_3}{\gamma_1}; \Omega_2 = \frac{2\gamma_2^3}{27\gamma_1^3} - \frac{3\gamma_2\gamma_3}{9\gamma_1^2} + \frac{\gamma_4}{\gamma_1};$$

$$\Omega_3 = \left(-\frac{1}{2}\Omega_2 + \frac{1}{2}\sqrt{\Omega_2^2 + \frac{4\Omega_1^3}{27}} \right)^{1/3}; \quad (13g)$$

$$\Delta = -\frac{1 - \sqrt{3}i}{2}$$

$$\gamma_1 = c_{33} c_{44} \beta_v;$$

$$\gamma_2 = (s_2 c_{44} + s_1 c_{33} + s_4^2 \xi^2) \beta_v + (s_3 c_{33} + s_5^2) c_{44};$$

$$\gamma_3 = (s_2 \beta_v + s_5^2) s_1 + (s_2 c_{44} + s_1 c_{33} + s_4^2 \xi^2) s_3$$

$$- (2s_4 s_5 s_6 + s_6^2 c_{33}) \xi^2; \gamma_4 = (s_1 s_3 - s_6^2 \xi^2) s_2 \quad (13h)$$

$$s_1 = -\xi^2 c_{11} + \omega^2 \rho - \omega^4 \rho_f^2 \beta_h;$$

$$s_2 = -\xi^2 c_{44} + \omega^2 \rho - \omega^4 \rho_f^2 \beta_v;$$

$$s_3 = -\xi^2 \beta_h + \frac{1}{M}; s_4 = -(c_{13} + c_{44}) \quad (13i)$$

$$s_5 = \alpha_v - \omega^2 \rho_f \beta_v; s_6 = \alpha_h - \omega^2 \rho_f \beta_h$$

$$\beta_j = \frac{1}{(m_j \omega^2 - i b_j \omega)} \quad (j = h, v) \quad (13j)$$

References

1. M.A. Biot, J. Acoust. Soc. Am. **28**, 168-179 (1956)
2. M.K. Kassir, J. Xu, Int. J. Solid Struct. **24**, 915-936 (1988)
3. B. Jin, H. Liu, Soil. Dyn. Earthquake Eng. **18**, 437-443 (1999)
4. X. Zeng, R.K.N.D. Rajapakse, Int. J. Numer. Analyt. Meth. Geomech. **23**, 2075-2095 (1999)
5. Z.Y. Ai, C.L. Liu, J.P. Jiang, Meccanica **51**, 1887-1895 (2016)
6. M.R. Halpern, P. Christiano, Earthquake Eng. Struct. Dyn. **14**, 439-454 (1986)
7. T. Senjuntichai, S. Keawsawasvong, R. Plangmal, Mar. Georesour. Geotec. (to be published)
8. T. Senjuntichai, S. Keawsawasvong, R. Plangmal, Comput. Geotech. **100**, 121-134 (2018)
9. S. Keawsawasvong, T. Senjuntichai, IOP Conference Series: Materials Science and Engineering, **269**, 012047 (2017)
10. T. Senjuntichai, W. Kaewjuea, J. Mech. Mater. Struct. **3(10)**, 1885-1901 (2008)
11. T. Senjuntichai, Y. Sapsathiarn, The Eighth East Asia-Pacific Conference on Structural Engineering and Construction, Nanyang Technological University (2001)
12. M.A. Biot, J. Appl. Phys. **26**, 182-185 (1955)
13. M.A. Biot, J. Appl. Phys. **33**, 1482-1498 (1962)
14. A.H.D. Cheng, Int. J. Rock Mech. Min., **34(2)**, 199-205 (1997)
15. R. Kumar, N.R. Garg, A. Miglani, J. Sound Vib., **261**, 697-714 (2003)
16. K. Sahebkar, M. Eskandari-Ghadi, J. Hydrol., **537**, 61-73 (2016)
17. A. Pooladi, M. Rahimian, R.Y.S. Pak, Appl. Math. Model., **50**, 177-199 (2017)
18. S.P. Timoshenko, S. Woinowsky-Krieger, *Theory of plates and shells* (McGraw-Hill, 1959)
19. I.N. Sneddon, *Fourier Transforms* (McGraw-Hill, 1951)
20. K. Washizu, *Variational methods in elasticity and plasticity, 2nd Ed* (Pergamon, 1982)
21. R. Piessens, E. Doncker-Kapenga, C.W. Uberhuber, D.K. Kahaner, *QUADPACK, A subroutine package for automatic integration* (Springer, 1983)
22. Z.Y. Ai, C.L. Liu, Soil. Dyn. Earthquake Eng. **67**, 257-263 (2014)

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