

# Optimum seismic performance estimation of integrated multiple steel pipes bridge pier connected by shear panel damper with ANN method

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**Abstract.** This paper discusses the optimum seismic performance estimation analysis of the integrated multiple steel pipes bridge pier connected by shear panel damper (SPD). The optimization result to be expected to result in an easier structural design for the practical engineer. In this study, 405 different structural configurations with 35 and more variations of SPD thickness to be analyzed by non-linear static cyclic in order to quantify its energy dissipation. The optimum energy dissipation to be assumed as analogous with best objective seismic performance in order avoiding time consumption of numerical analysis. Furthermore, the SPD's thickness related optimum energy dissipation of each structural configuration was successfully predicted by multi-parameter estimation analysis with the artificial neural network (ANN) method. As the result, the  $R$ -value and the average error value of the estimated optimum SPD's thickness compared with analytical result were 0.988 and 2.38%, respectively. With the estimated empirical equation result, the practical engineer could determine the structural configuration in the optimum seismic performance easily.

## 1 Background

Currently, the integrated steel pipe bridge pier connected by shear panel damper (SPD) has satisfied seismic performance achievement, as shown in **Fig. 1**. In the early study of Kanaji et al. [1], the behavior of an integrated pier with multiple steel pipes with damage control design of shear panel damper (SPD) was investigated by experimental study. Then, the dynamic properties effect of such kind structure related to the seismic response was investigated by Hashimoto et al. [2] with static pushover and dynamic analysis. Also, the proposal of a direct-connected pile of this typical pier can reduce the strain of steel pipes under earthquake excitation [3]. Furthermore, based on the seismic performance investigation of this typical pier built in Ebi-Junction, Osaka, can achieve Performance IIb (medium damage under Level 2 of seismic excitation) [4]. More ever, the structural performance enhancement of such structure was performed successfully by Fajar et al. [5] to achieve Performance IIa (small damage under Level 2 of the earthquake). The seismic performance enhancement was achieved by proposing a pin connection between the top of steel pipes and the head pier and redistributed the strength configuration of SPDs along the pier height, as illustrated in **Fig. 2**. However, to reach this seismic performance achievement the optimum energy dissipation should be enriched by multiple-analysis of non-linear static cyclic. It means complex-numerical analysis with exhausting of huge time and resource should be conducted by the engineer.

The other hand, the artificial neural network (ANN) have been adopted to predict the result of a phenomenon

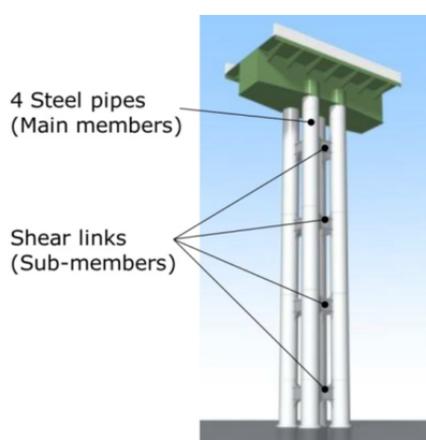
related to multiple variables of a possible reason in many major studies, including structural engineering. The prediction of PC-Girder damage was studied by Fletcher et al [6] by implementing FE-ANN analysis. Then, the behavior of Ferro-cement strengthened RC columns using was estimated with ANN method [7]. Also, the application of ANN for estimating effective parameter failure load and displacement of RC building was studied by Arslan [8]. Furthermore, the seismic performance of existing RC building was determined by ANN analysis [9]. Those estimation analysis mentioned above provide beneficial to simplify the complex-structural analysis which exhausts time and engineering resource.

This study discusses the multiple parameter estimation analysis of the optimum seismic performance on integrated bridge pier with multiple steel pipes connected by shear panel damper. The optimization results to be expected to realize an easier structural design approach for the engineer. In this study, 405 different structural configurations of 20m-height bridge pier with 35 and more variations of SPD's thickness to be analyzed by non-linear static cyclic to quantify its energy dissipation. The optimum energy dissipation to be assumed as analogous with best objective seismic performance in order avoiding time consumption of numerical analysis. Furthermore, the SPD's thickness related optimum energy dissipation of each structural configuration was successfully predicted by multi-parameter estimation analysis with the artificial neural network (ANN) method. The accuracy of estimation analysis indicated by the average and maximum error quantity of energy dissipation. The simplicity of structural design approach with the empirical formulation of ANN to be demonstrated by designing five examples of pier struc-

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(a)



(b)

**Fig. 1.** Integrated multiple steel pipes bridge pier connected by SPD [4]: (a) constructed in Ebi-junction and (b) the structural configuration

ture with and without using this formula. Finally, the seismic performance of both method to be compared.

## 2 Integrated bridge pier with multiple steel pipes connected by SPD

### 2.1 Structural configuration

The enhanced integrated bridge pier with steel pipes connected by SPD contains four steel pipes with pin connection on the top and fix connection on the bottom. The shear panel dampers are connected to the steel pipes with certain intervals to realize the integrated pier system, as shown in **Fig. 1**. During a lifetime, the vertical loads including self-weight and vehicles are supported by the axial resistance of the steel pipes, while the lateral loads are resisted by the interaction of the moment resistance of the steel pipes and the shear resistance of the SPD. When the pier structure undergoes lateral deformation, the SPD works in favor of shear forces and yields earlier than the steel pipe

column elements. The vibration energy due to the seismic excitation to be absorbed by the dissipated energy of the yielding SPD. So that the steel pipe damage due to severe earthquake excitation can be reduced, consequently, the life-cycle-cost of the structure can be improved [4].

Shear panel damper consists of a low yield strength steel material of panel plate confined by the surrounding flanges part on the all of its edges, as shown in **Fig. 2**. On the small deformations, this device quickly reaches its yielding point due to shear deformation. Series of the development of the SPD device was started from the experimental study of thin steel shear wall damper by Takahashi et.al. [11], then the analysis method of it was developed by Thorburn et.al [12]. Also, the study of the hysteretic behavior of thin plate shear wall panels was proposed [13], and more about the cyclic behavior investigation of it was examined by reference [14]. Furthermore, the SPD behavior that is subject to axial load has been put forward by references [15, 16]. Moreover, the application of low-grade SPD steel material to seismic design was presented [17] and for the seismic design on the large deformations was studied by reference [18]. In this study, the shear panel damper behavior was idealized as a bilinear material model with hysteretic behavior following the reference [19], and the ultimate deformation capacity of SPD was adopted the reference [20]. Hence, the SPD hysteretic parameters are determined by the formulation as follows **Equations (1-9)**.

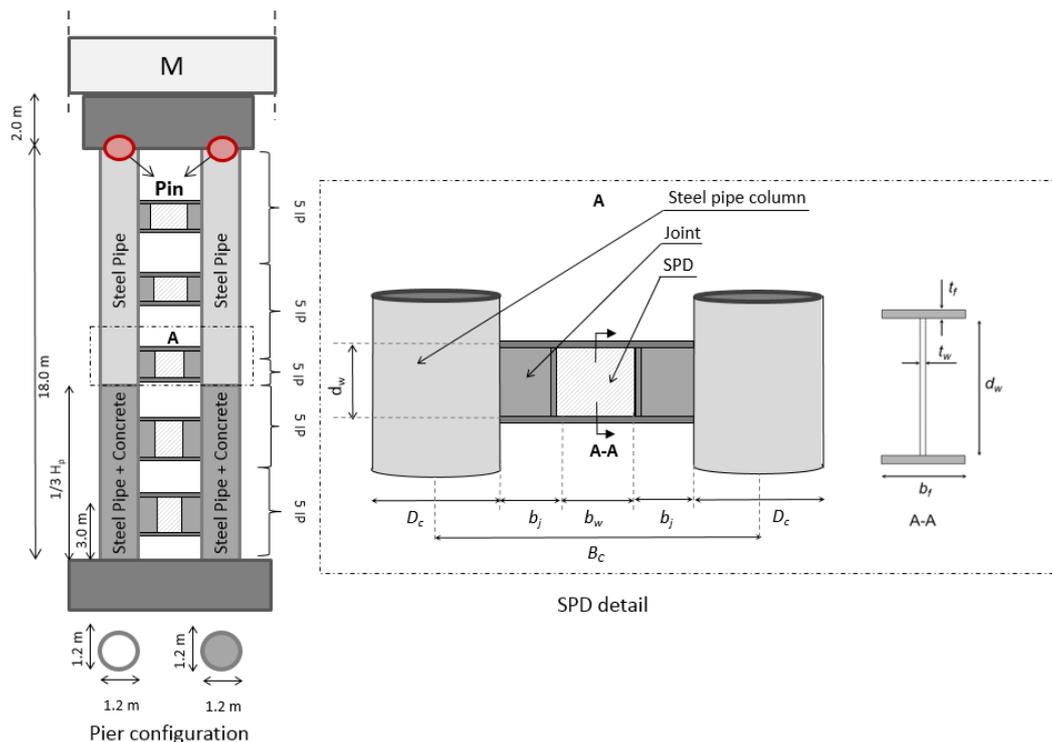
### 2.2 Seismic performance criteria

Since Kobe earthquake in 1995, the seismic coefficient for the bridge design that has been determined by the Japan Road Association (JRA) [21] contains two level of seismic design (Level 1 and Level 2). Based on the JRA criteria [21], under Level 1 of earthquake event which frequent probable occurred, the structure should remain to behave elastically (Performance I). While under Level 2 of seismic excitation, the plastic behavior of the pier member is allowed but the collapse of the structure should be avoided or the structural damage should be limited (Performance II).

Furthermore, Hanshin expressway has determined four seismic performance level of integrated steel pipes bridge pier connected by SPD [4] which contain Performance I (fully elastic behavior), Performance IIa (small damage), Performance IIb (medium damage), and Performance III (severe damage). In the proposed design of Fajar et al. [5], the new proposed configuration of the integrated multiple steel pipe bridge pier connected by SPD could achieve Performance IIa under Level 2 of earthquake excitation. In this study, achievement Performance IIa under Level 2 of ground motion was adopted for the seismic performance target.

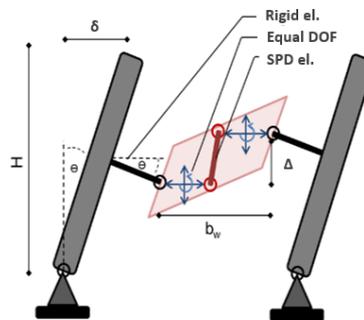
## 3 Artificial neural network (ANN)

Based on the reference [13], "artificial neural network is a parallel, distributed information processing structure

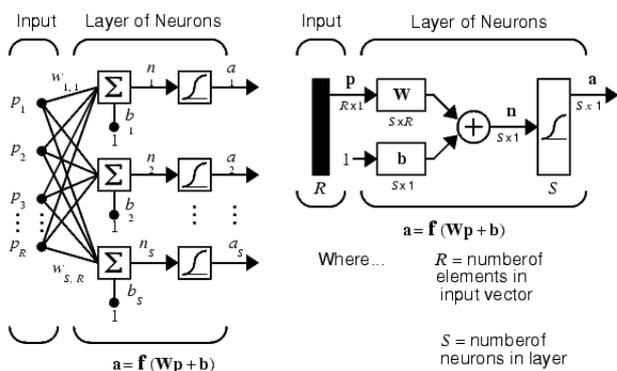


**Fig. 2.** The detail of enhanced seismic performance of integrated multiple steel pipes bridge pier connected by SPD [10]

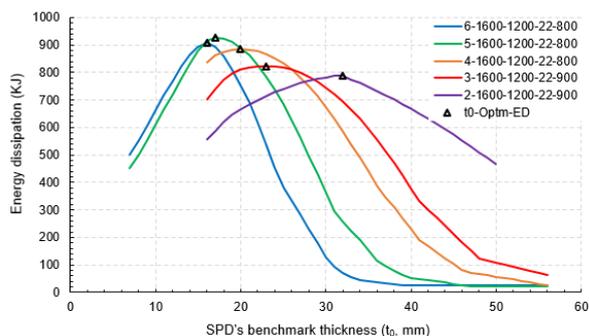
consisting of processing elements interconnected together with unidirectional signal channels called connection". Currently, a famous ANN's algorithm called feed-forward back propagation [22] has been adopted in many cases of fitting, predicting, and estimating data analysis. The architecture of feed-forward back propagation contains an input layer, hidden layers, and the output layer, as illustrated in Figure-3. By determining the appropriate variable input and output in a problem, this algorithm can determine the empiric formulation of that relation.



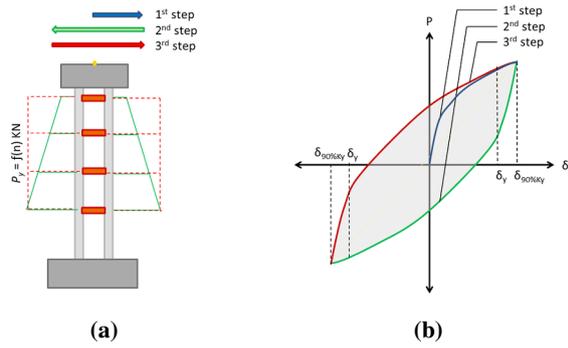
**Fig. 4.** SPD idealization on the structural configuration [10]



**Fig. 3.** Feed-forward back-propagation algorithm schematic on ANN analysis [23]



**Fig. 5.** Energy dissipation related to the SPD's benchmark thickness ( $t_0$ ) with configuration 1600-1200-22 ( $B_c = 1600\text{mm}$ ,  $D_c = 1200\text{mm}$ , and  $t_c = 22\text{mm}$ )



**Fig. 6.** Calculation method of dissipated energy on the optimized pier [10]: (a) redistribution of SPD strength and (b) cyclic loading step

## 4 Methodology

### 4.1 Structural idealization

The structure was modeled with Opensees software in two dimensional and three degrees of freedom. Then, the superstructure was idealized as lumped mass (450 ton) at an elevation position of 20m from the base in two directions (horizontal and vertical), as illustrated in **Fig. 2**. While the steel pipe column components were modeled as force-based beam-column with fiber section by applying the regularized plastic hinge with five integration points based on the references [24–28]. The steel pipe column material was modeled as smoothed bilinear (named Steel02) material by applying a combination of isotropic and kinematic hardening based on the references [24, 29]. While, the steel pipe filling material on a third of the height of the pier was modeled as a concrete (named Concrete02) material based on the reference [24]. Also, the concrete constitutive material following the reference [21].

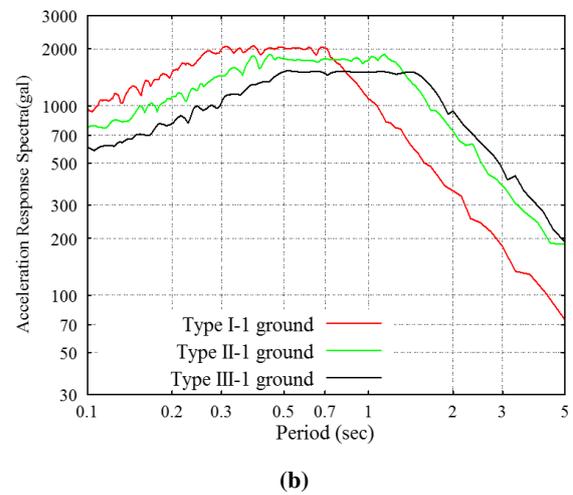
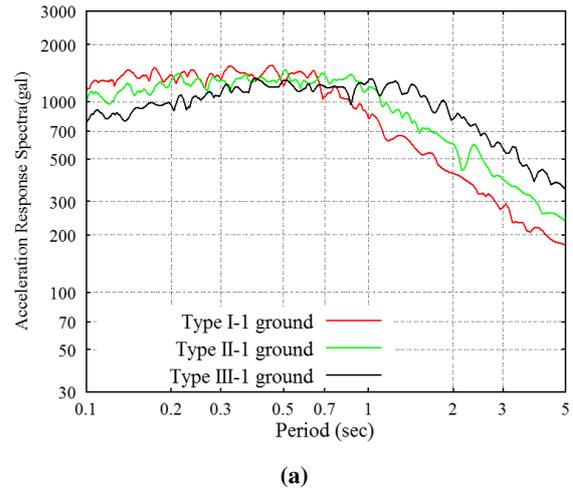
The SPD was modeled as link element based on the references [30, 31]. In the numerical model, each nodal of link element was positioned almost at one point at the center of the SPD location. Furthermore, the connection element between the SPD and the steel pipe column was idealized as a rigid elastic element. While the connection of SPD and joint element were idealized as equal-DOF in all three degrees of freedom. The connection idealization of the SPD, the joint, and the steel pipes are illustrated in **Fig. 4**. The SPD constitutive material was idealized as bi-linear (Steel01) material by applying kinematic hardening based on the reference [24]. The hysterical parameter of SPD was determined following the **Equations** ((1) - (9)) which is specified in the references [19, 20].

$$Q_n = \tau_w \cdot d_w \cdot t_w, \Delta = \gamma \cdot b_w \quad (1)$$

$$\tau_u = \tau_f + \tau_w \quad (2)$$

$$\frac{\tau_w}{\tau_{wy}} = 0.918 + \frac{0.038}{R_w^2} \leq 1.2 \quad (3)$$

$$\frac{\tau_f}{\tau_{fy}} = 0.0287 \frac{b_f}{b_w} \cdot \frac{t_f}{t_w} \left( \frac{t_f}{t_w} 4 \cdot \frac{d_w}{(n_L + 1) R_w b_w} + 2 \right) \quad (4)$$



**Fig. 7.** Level 2 ground motion excitation response spectra [21]: (a) Type I and (b) Type II

$$\tau_{wy} = \frac{\sigma}{\sqrt{3}} \quad (5)$$

$$R_w = \frac{b_w}{(n_L + 1) t_w} \sqrt{\frac{12(1 - \nu) \tau_y}{K \pi^2 E}} \quad (6)$$

$$K = 5.35 + 4 \cdot \left( \frac{b_w}{d_w} \right)^2 \text{ for } \left( \frac{d_w}{b_w} \right) \geq 1 \quad (7)$$

$$K = 5.35 \cdot \left( \frac{b_w}{d_w} \right)^2 + 4 \text{ for } \left( \frac{d_w}{b_w} \right) < 1 \quad (8)$$

$$\begin{aligned} \gamma_u &= 20 \quad (\alpha \leq 0.8) \\ &= \frac{7.034}{(R_w + 0.54)^{8.0}} + 4, \quad (0.8 \leq \alpha \leq 1.5) \end{aligned} \quad (9)$$

$$\gamma_y = \frac{\tau_w}{G} \quad (10)$$

$$G = \frac{E}{2(1 - \nu)} \quad (11)$$

With  $b$  is the component width,  $d$  is the component depth,  $t$  is the component thickness,  $n_L$  is the number of the stiffener flanges,  $E$  is the elastic modulus of steel material,  $Q_n$  is the shear force of the SPD,  $\Delta$  is the displacement of

the SPD,  $\tau$  is shear stress of the steel material,  $\gamma_s$  is a shear strain of the steel material, and  $\sigma$  is an axial stress of the steel material.

#### 4.2 Estimation of optimum seismic performance

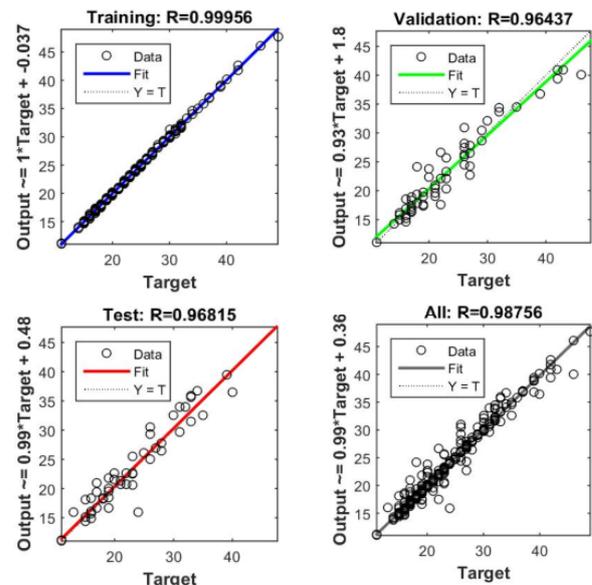
In this study, 405 different structural configurations of 20m-height bridge pier with 35 and more variations of SPD's thickness to be analyzed by non-linear static cyclic to quantify its energy dissipation. Typically, the variation of SPD thickness resulted in different energy dissipation achievement, as shown in **Fig. 5**, also different the number of SPD generate different energy dissipation trend [10]. The variable parameter of the structure including the number of SPD ( $N_{SPD}$ ), the center-to-center distance of steel pipes ( $B_c$ ), steel pipe diameter ( $D_c$ ), the steel pipe thickness ( $t_c$ ), and the depth of SPD ( $d_w$ ). The variable pattern was determined following the study of reference [10], as shown in **Table 2** which has triangular strength distribution of SPD along with the pier height. Those five input parameters were defined as the input of the ANN analysis, as shown in **Table 1**.

The optimum energy dissipation to be assumed as analogous with best objective seismic performance [10] in order to avoid time consumption of numerical analysis (the complexity of non-linear dynamic analysis). In each structural configuration with 50 variable of SPD's thickness, the energy dissipation to be calculated by measuring the area of the force-deformation curve during static cyclic analysis, as illustrated in **Fig. 6**. More ever, the SPD's thickness ( $t_w$ ) related optimum energy dissipation of each structural configuration was defined as the target of the ANN analysis. The thickness of most top location of SPD named as  $t_0$  was adopted as the benchmark in the target of estimation analysis with ANN.

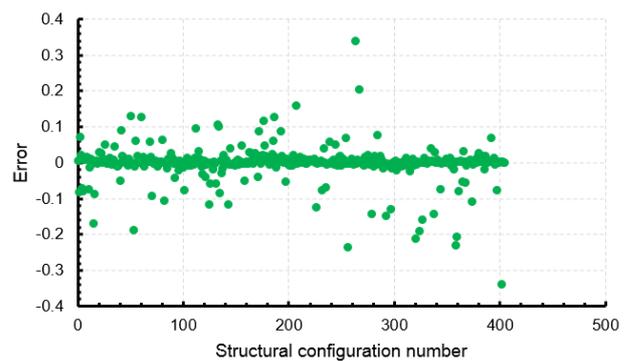
The ANN analysis with feed-forward back propagation algorithm was conducted with two layers and 80 neurons of the hidden layer. Some trial analysis with several configurations of layer and neuron were conducted to achieve a satisfied regression result which indicated by the coefficient of determination ( $R$ -value) close to 1.

#### 4.3 Seismic performance verification of the estimated optimum structure

In the term of energy dissipation and displacement ratio ( $\delta_{ratio}$ ) of the pier, the estimated optimum result to be compared with the calculated optimum and adjusted result with 22mm benchmark thickness ( $t_0$ ). The displacement ratio means the ratio of maximum structural response under ground-motion excitation divided by displacement limit of Performance Ila. To verify the seismic performance, there are six ground motions input including three of Type I and three of Type II based on the JRA code [21] to be used in the dynamic analysis, the response spectra of both are shown on the **Fig. 7**. In the dynamic analysis, direct integration with Newmark algorithm was implemented. In the Newmark algorithm, some parameters were determined including,  $\beta$  equal to 0.5,  $\gamma$  equal to 0.25, and the damping ratio ( $\xi$ ) of mass and stiffness proportional damping equal to 0.02.



**Fig. 8.** Regression curve based on the ANN analysis



**Fig. 9.** SPD's benchmark thickness ( $t_0$ ) error on the ANN analysis

## 5 Results and discussion

### 5.1 Estimated energy dissipation

Based on the regression curve, as shown in **Fig. 8**, the estimated SPD's benchmark thickness ( $t_0$ ) at optimum energy dissipation has 0.988 of determination coefficient ( $R$ ). The error data is shown in **Table 3** and **. 9**. Also, based on the monitoring five different SPD number of 1600-1200-22 structural configurations, the maximum error of  $t_0$  was about 3mm. It indicated the estimation analysis of SPD's thickness on the optimum energy dissipation with ANN could enrich good agreement. Furthermore, based on the **Fig. 10a**, the energy dissipation achievement of estimation of ANN method could be equal or almost equal with the optimum calculation. While, compared to the 22mm adjusted benchmark thickness of SPD, the estimation method could achieve larger energy dissipation, except in the case of using three SPDs.

**Table 1.** Optimized structural variable

Parameters	$N_{SPD}$	$D_c$ (mm)	$B_c$ (mm)	$t_c$ (mm)	$d_w$ (mm)	Total variables
Variable range	2 to 6 incr. 1	1200 to 1600 incr. 400	1000 to 1400 incr. 200	18 to 26 incr. 4	700 to 900 incr. 100	
Quantity	5	3	3	3	3	405

**Table 2.** SPD's dimension related the number of SPD along the pier height (from the top to bottom location)

Depth-width configuration	700mm		800mm		900mm		$t_0$ (mm)		$t_w$ (mm)
	$d_w$ (mm)	$b_w$ (mm)	$d_w$ (mm)	$b_w$ (mm)	$d_w$ (mm)	$b_w$ (mm)			
2	500 to 700 incr. 200	700 to 500 incr. -200	600 to 800 incr. 200	800 to 600 incr. -200	700 to 900 incr. 200	900 to 700 incr. -200	16 to 50 incr. 1	16 to 50 incr. 1	$t_0$ to $t_0 + (n-1)$ incr. 1
3	500 to 700 incr. 100	700 to 500 incr. -100	600 to 800 incr. 100	800 to 600 incr. -100	700 to 900 incr. 100	900 to 700 incr. -100	16 to 56 incr. 1	16 to 56 incr. 1	$t_0$ to $t_0 + (n-1)$ incr. 1
4	500 to 705 incr. 65	705 to 500 incr. -65	600 to 805 incr. 65	805 to 600 incr. -65	700 to 905 incr. 65	905 to 700 incr. -65	16 to 56 incr. 1	16 to 56 incr. 1	$t_0$ to $t_0 + (n-1)$ incr. 1
5	500 to 700 incr. 50	700 to 500 incr. -50	600 to 800 incr. 50	800 to 600 incr. -50	700 to 900 incr. 50	900 to 700 incr. -50	6 to 56 incr. 1	6 to 56 incr. 1	$t_0$ to $t_0 + (n-1)$ incr. 1
6	500 to 700 incr. 40	700 to 500 incr. -40	600 to 800 incr. 40	800 to 600 incr. -40	700 to 900 incr. 40	900 to 700 incr. -40	6 to 56 incr. 1	6 to 56 incr. 1	$t_0$ to $t_0 + (n-1)$ incr. 1

**Table 3.** Error statistic of SPD's benchmark thickness  $t_0$

Value	Average ( $\mu$ )	Standard deviation ( $\sigma$ )	Maximum
$t_0$ (mm)	0.54	1.04	8.08
Percentage (%)	2.38%	4.59%	34.13%

**Table 4.** SPD's benchmark thickness ( $t_0$ ) comparison at optimum dissipated energy with five different SPD number on the 1600-1200-22 configuration of pier

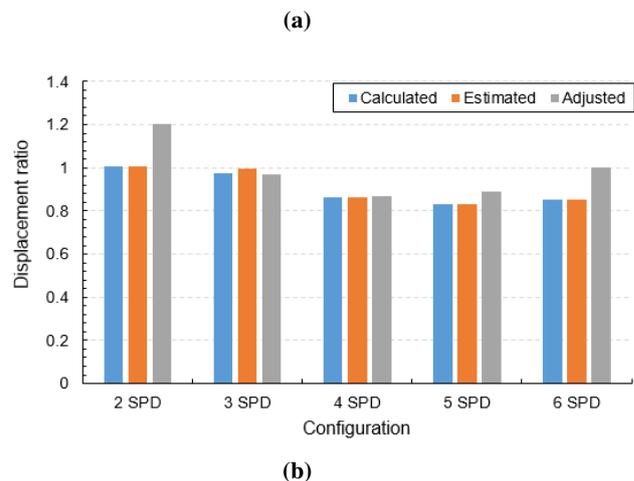
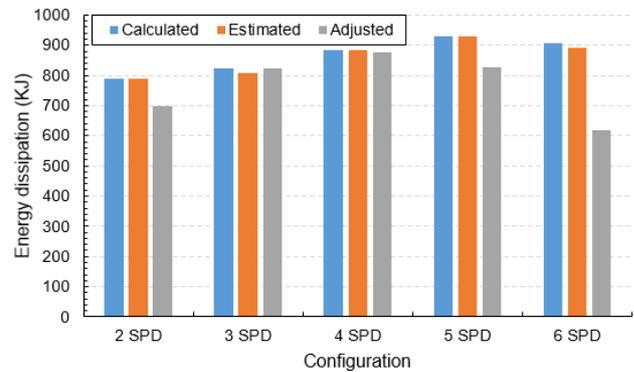
number of SPD	$t_0$ with optimum calculation (mm)	$t_0$ with estimation analysis (mm)		error (mm)
		original	rounded	
2	32.00	31.56	32.00	0.00
3	23.00	25.70	26.00	3.00
4	20.00	20.05	20.00	0.00
5	17.00	17.11	17.00	0.00
6	16.00	16.85	17.00	1.00

### 5.2 Seismic performance

As the result of **Fig. 10b**, the maximum response of pier configurations with estimated  $t_0$  almost equal with the calculated configuration result. Compared with the pier that implements 22mm of adjusted thickness, it could achieve a smaller maximum response, except the using of three SPD configuration.

### 6 Conclusions

The optimum seismic performance estimation of integrated multiple steel pipes bridge pier connected by SPD was successfully conducted with ANN analysis. The accuracy of ANN analysis in the estimating optimum energy dissipation was proved by the satisfying result of  $R$ -value and the error statistic data. By predicting the thickness of SPD related to the optimum energy dissipation of structural configuration, the best seismic performance of



**Fig. 10.** Comparison of seismic performance of the five different SPD number with 1600-1200-22 configuration: (a) energy dissipation and (b) displacement ratio

the structure could be estimated well. The structural design of estimating analysis of ANN could achieve close result with the optimization of multiple static cyclic analysis. Compared with adjustment of SPD thickness method,

it could enrich better structural performance. In the implication, with the estimation empirical function of ANN analysis, the complexity structural design process of integrated multiple steel pipes bridge connected bridge pier connected by SPD could be simplified by the estimation analysis with ANN method. Thus, a practical engineer could make the structural design of such pier structure easier.

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