Road train motion stability in BRT system

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Abstract. The peculiarities of organization and perspectives of mass passenger transportation in the city and beyond are considered with the use of "Bus Rapid Transport" (BRT) or Metrobus. Different aspects of study of motor vehicles (MV) controllability and stability are analyzed. It is substantiated that it is sufficient to consider the potential stability of the MV itself, in order to guarantee the stability of the "driver - MV" system with a large reserve. A mathematical model of a three-axle bus train consisting of a bus and two trains (metrobus) is developed and the factors influencing the critical speed as the main index of the stability of its movement are determined. It is established that the increase of the critical speed of the metrobus can be achieved by increasing the base of the bus, the first and the second trailer, as well as the mass of the bus and the coefficients of resistance of the drive wheels of the bus driving axle and the trailers axles. At the same time, increasing the distance from the mass center to the bus rear axle, increasing the distance from the mass center to the point of the coupling of the bus with the first trailer, increasing the mass of trailers and the resistance of the resistance of the wheel drive of the bus axis lead to a decrease in the critical speed of the metrobus. This must be taken into account both when designing metrobuses, and when operating them.

1 Formulation of the problem

Metrobus or "Bus Rapid Transport" (BRT) is the name of a rather complex system of bus traffic [1]. The BRT project involves the movement of buses on specially assigned and often fenced traffic lanes. The main advantage of the metrobus is its complete isolation on the road from other modes of transport. As a means of transportation, the latest generation of articulated buses equipped with engines up to 300 kW is selected. At that, in the salons of the metrobus the preference is given to the places for standing, as in the subway. Due to this, one double-articulated road train transports up to 200 passengers.

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The record capacity metrobus, the AKIA superlong double-articulated metrobus, designed to carry nearly three hundred passengers, was presented at Busworld Turkey 2016 in Istanbul. The AKIA Ultra LF25 metrobus has a length of 25 meters and can carry a record (in this class) number of passengers, namely 290. There are 29 seats in it. This vehicle is specifically designed for the assigned lane of high-speed bus service BRT, which has been operating successfully in some of the largest cities in the world, including Istanbul. The technical characteristics of the metrobus are still few - it is only known that it is equipped with a turbodiesel engine Mercedes-Benz of the environmental standard Euro 6.

Superlong metrobus Ultra LF25 can be produced not only with a diesel but also with a hybrid power unit. Moreover, the trolleybus option is also possible. In the latter case, it is equipped with two traction electric motors with a capacity of 160 kW and with rods on the roof [2]. However, the world's largest metrobus was shown in Sweden. The Swedish automaker Scania introduced a double-articulated bus model, powered by natural gas. The model complies with Euro-6 standards. It was presented at the Busworld Latin America international exhibition in Medellin, Colombia. The bus chassis has a length of 28 meters. The body was manufactured by Brazilian company Busscar. The passenger compartment of the vehicle can accommodate up to 300 passengers.

The BRT system has a number of undeniable benefits:

- High passenger capacity and efficient payment systems provide low-cost travel.
- High speed of movement allows the metrobus to transport a significant share of passenger traffic, which contributes to reducing the number of motor transport on the city roads and, accordingly, reducing emissions of waste gases.
- An expanded information system informs passengers about the schedules of the routes.

Currently, more and more cities are choosing a system of the bus rapid transport for important reasons such as cost and convenience. The cost of construction of such a wide-gauge railroad transport, as a subway, is 10 times higher than that of BRT. Moreover, installing this system in the cities that choose the BRT system can take 2 years, and the construction of the subway can be lingered for decades.

Today, metrobuses operate in several countries: USA, Brazil, Venezuela, Colombia, Guatemala, Canada, Mexico, Australia, New Zealand, Japan, Iran, Turkey, France, Czech Republic, etc. This list is quickly updated, as more cities are taking decisions in favor of the BRT system. In different cities and countries, the system may look differently: somewhere they build special stopping points with the platform, while others use small stop stations. There is no single standard anywhere, each city solves its problems in its own right, taking into account the geographical location of the city, the flow of passenger traffic, and the development of the city. But all agree on one thing - BRT is an excellent assistant in solving ecological and transport problems [3]. But for its implementation it is necessary to solve a number of problems associated with the choice of the highway for the metrobus, the choice of rolling stock, etc.

In the works [4, 5] the indicators of maneuverability of a double-articulated road train are determined both on rigid and elastic laterally oriented wheels. In particular, it is shown that the overall lane of such a train with a total length of 25 m is about 10 m, that is, in order to move, such train needs its own fenced lane, on which the metrobus can move with the maximum speed provided with both the traction capabilities of the leading link and the stability of the movement.

2 Analysis of literary sources

In the works known today, issues of controllability and stability are considered in two aspects [6-15]:

1. The research taking into account the characteristics of all elements of the "driver-car-road" system, which is considered as a closed-loop system of automatic control [14].
2. The research of the stability and controllability of a vehicle (road train), in which the driver's influence is excluded.

In the first group of works, the problem is most commonly considered in general, that is, it explores simultaneously both controllability and stability in this complex system of obtaining, processing and transmitting of information, the formation of control impacts, aimed at reducing the deviations of both the tractor unit and trailer cars from the desired process. In this case, the meaning used in the theory of automatic control is added to the concepts of controllability and stability. Studied in such a way, the controllability consider one or another characteristic of transient processes under the simplest typical control influences. Stability characterizes the behavior of the system in the transition mode and relates primarily to the system's own movements, generated by the initial conditions (perturbations) and its internal properties, but not by external influences. Stability is considered in relation to any process, both controlled and uncontrolled [14].

In the second group of works, the vehicle is considered isolated as an object of controlling, and the stability of the driver - motor vehicle (MV) system is determined by the stability of the vehicle and the driver's psychophysical capabilities, as well as by the level and nature of the existing perturbations. As shown by the calculations performed for different types of MV, the presence of a closed control scheme can increase their critical speed of motion by 1.5-2 times and increase the number of the links, the stability of which can be secured [14].

Thus, if the stability of the MV is ensured, then the stability of the driver - MV system is also provided with a large margin. Therefore, it is sufficient to consider the motion at the open-loop control, that is, the potential stability of the vehicle itself.

Stability research is carried out in two main ways [15]:

- Numerical integration of the obtained equations of motion for given perturbations in their right-hand sides.
- Thus, the experimental indices are determined.
- Qualitative study of stability in solving equations of motion by one of the methods of academician Oleksandr Lyapunov or his followers, while finding the critical speed, which determines the conditions for the transition from stable motion to unstable.

The use of either method requires the presence of differential equations of motion of a double-articulated road train.

The purpose of the work is to develop a mathematical model of a double-articulated road train consisting of a bus and two trailers (metrobus) and to determine the factors that influence the critical speed as the main indicator of the stability of its movement.

3 Research results

In the paper [8] the system of equations of a double-articulated trailed road train (Fig. 1) is presented in the form (where m_i are the masses of individual parts of the road train; M_i are the moments of resistance to the rotation of individual parts of the road train; c is the distance from the rear axle of the truck tractor to the coupling point; a is the distance from the center of mass of the truck tractor to its front axle; b_{li} is the distance from the center of mass of the truck tractor to its middle and rear axles; d_i , l_i is the distance from the center of mass of individual modules to the front axle and the coupling points respectively; Y_1 , Y_{1i} , Y_{2j} , Y_{4p} are the lateral forces acting on the wheels of the front, rear axle of the truck tractor, the first and the second trailers respectively; b_{2j} , b_{4p} is the distance from the center of mass of the first and the second trailers to their rear axles respectively):

$$\left\{ \begin{array}{l} (m + m_1 + m_2 + m_3 + m_4) \omega V = Y_1 + \sum_{i=1}^2 Y_{1i} + \sum_{j=1}^3 Y_{2j} + \sum_{\rho=1}^3 Y_{4\rho}; \\ -c(m + m_1 + m_2 + m_3 + m_4) \omega V = \alpha Y_1 + b_{1i} \sum_{i=1}^2 Y_{1i} - c \left(\sum_{j=1}^3 Y_{2j} + \sum_{\rho=1}^3 Y_{4\rho} \right) + M_1; \\ \left[(m_1 d_1 + m_2 l_1 + (m_3 + m_4) l_2) \right] = l_1 \sum_{j=1}^2 Y_{2i} + l_2 \sum_{\rho=1}^3 Y_{4\rho} + M_1 - M_2 - M_3; \\ m_2 d_2 \omega V = \sum_{j=1}^3 Y_{2j} (d_2 + b_{2j}) + M_2; \\ (m_3 d_3 + m_4 l_3) \omega V = l_3 \sum_{\rho=1}^3 Y_{4\rho} + M_3 - M_4; \\ m_4 d_4 \omega V = \sum_{\rho=1}^3 Y_{4\rho} (d_4 + b_{4\rho}) + M_4. \end{array} \right. \quad (1)$$

Fig. 1. The scheme of unsteady rotation of the metrobus

The stability criterion is the critical speed V_{kr} that depends on the design parameters of the metrobus and operational factors. The exceeding of the critical speed leads to the loss of stability of the motor vehicle. By setting the law of changing the angle of rotation of the controlled wheels of the bus, one can find, by numerical integration of the equations of motion, the values of the parameters of the metrobus movement when performing various maneuvers. However, to solve the stability problem, the conditions of the existence of the movement parameters are more important. If the metrobus was perturbed at some moment, then the initial conditions will change. The shape of the movement will also change, and unlike the original, it will also become perturbed. The comparison of the equation systems of unperturbed and perturbed conditions allows to obtain the differential equations of perturbed

motion, in which, instead of the variables v , there are their perturbations, and the left sides of all the equations are zero.

Analytical expressions for the variables describing the movement of the metrobus $U(\theta_i)$, $\omega(\theta_i)$, $\varphi_i(\theta_i)$ and $\dot{U}$ in the function of the angle of rotation of the controlled wheels of the bus corresponding to the steady motion can be obtained from the system of equations of stationary motions of the metrobus with four kinematically independent elements on a circular trajectory of a sufficiently large radius, provided that $V = \text{const}$ [16].

By the linearity of the lateral deviation forces in the function of the deviation angle we obtain:

$$\begin{aligned} Y_{ij} &= k_{ij} \delta_{ij}; \quad \delta_i = \theta_i - \frac{U + \alpha \omega}{V}; \quad \delta_{li} = -\frac{U}{V} + \frac{\omega}{V} b_{li}; & (i=1,3), \\ \delta_{2j} &= -\theta_{2j} - \frac{U}{V} + \frac{\omega}{V} (c + l_1 + d_2 + b_{2j}) - \varphi_1 - \varphi_2; & (j=1,2), \\ \delta_{4\rho} &= -\theta_{4\rho} - \frac{U}{V} + \frac{\omega}{V} (c + l_2 + l + d_4 + b_{4\rho}) - \varphi_1 - \varphi_3 - \varphi_4; & (\rho=1,2), \end{aligned} \quad (2)$$

Taking into account the expressions that determine the lateral forces and the deviation angles of the axes of the road trains, the system of equations (1) is written in the form (3).

$$\left\{ \begin{aligned} (m + m_1 + m_2 + m_3 + m_4) \omega V &= k_1 \theta_1 - \sum_{j=1}^3 k_{2j} \theta_{2j} - \sum_{\rho=1}^3 k_{4\rho} \theta_{4\rho} - \frac{U}{V} \left(k_1 + \sum_{i=1}^2 k_{li} + \sum_{\rho=1}^3 k_{4\rho} \right) - \frac{\omega}{V} \left[k_1 a - \sum_{i=1}^2 k_{li} b_{li} - \sum_{j=1}^3 k_{2j} (c + l_1 + d_2 + b_{2j}) - \sum_{\rho=1}^3 k_{4\rho} (c + l_2 + l_3 + d_4 + b_{4\rho}) \right] - \varphi_1 \left(\sum_{j=1}^3 k_{2j} + \sum_{\rho=1}^3 k_{4\rho} \right) - \varphi_2 \sum_{j=1}^3 k_{2j} - (\varphi_3 + \varphi_4) \sum_{\rho=1}^3 k_{4\rho}; \\ -c(m + m_1 + m_2 + m_3 + m_4) \omega V &= c \left(\sum_{j=1}^3 k_{2j} \theta_{2j} - \sum_{\rho=1}^3 k_{4\rho} \theta_{4\rho} \right) - \frac{U}{V} \left[\left(k_1 a - \sum_{i=1}^2 k_{li} b_{li} - c \left(\sum_{j=1}^3 k_{2j} \theta_{2j} - \sum_{\rho=1}^3 k_{4\rho} \theta_{4\rho} \right) \right) - \frac{\omega}{V} \left[k_1 a^2 + \sum_{i=1}^2 k_{li} b_{li}^2 + c \sum_{j=1}^3 k_{2j} (c + l_1 + d_2 + b_{2j}) + c \sum_{\rho=1}^3 k_{4\rho} (c + l_2 + l_3 + d_4 + b_{4\rho}) \right] + \varphi_1 \left[c \left(\sum_{j=1}^3 k_{2j} + \sum_{\rho=1}^3 k_{4\rho} \right) + q_1 \right] + c \left[\varphi_2 \sum_{j=1}^3 k_{2j} + (\varphi_3 + \varphi_4) \sum_{\rho=1}^3 k_{4\rho} \right]; \right. \\ [m_1 d_1 + m_2 l_1 + (m_3 + m_4) l_2] \omega V &= -l_1 \sum_{j=1}^3 k_{2j} \theta_{2j} - l_2 \sum_{\rho=1}^3 k_{4\rho} \theta_{4\rho} - \frac{U}{V} \left(l_1 \sum_{i=1}^2 k_{2i} + l_2 \sum_{\rho=1}^3 k_{4\rho} \right) + \frac{\omega}{V} \left[l_1 \sum_{j=1}^3 k_{2j} (c + l_1 + d_2 + b_{2j}) + l_2 \sum_{\rho=1}^3 k_{4\rho} (c + l_2 + l_3 + d_4 + b_{4\rho}) \right] + \\ &\quad \varphi_1 \left(l_1 \sum_{j=1}^3 k_{2j} + l_2 \sum_{\rho=1}^3 k_{4\rho} - q_1 \right) - \varphi_2 \left(l_1 \sum_{j=1}^3 k_{2j} + q_2 \right) - \sum_{\rho=1}^3 k_{4\rho} l_2 \varphi_4 + \varphi_3 \left(l_2 \sum_{\rho=1}^3 k_{4\rho} - q_1 \right); \end{aligned} \right. \quad (3)$$

$$\begin{aligned}
 & \left[m_2 d_2 \omega V = - \sum_{j=1}^3 k_{2j} \theta_{2j} (d_2 + b_{2j}) - \frac{U}{V} \sum_{j=1}^3 k_{2j} \theta_{2j} (d_2 + b_{2j}) + \frac{\omega}{V} \left[\sum_{j=1}^3 k_{2j} (d_2 + b_{2j}) (c + \right. \right. \\
 & \left. \left. + l_1 + d_2 + b_{2j}) \right] - \varphi_1 l_1 \sum_{j=1}^3 k_{2j} - \varphi_2 \left(l_1 \sum_{j=1}^3 k_{2j} - q_2 \right); \right. \\
 & \left(m_3 d_4 + m_4 l_3 \right) \omega V = - l_3 \sum_{\rho=1}^3 k_{4\rho} \theta_{4\rho} - \frac{U}{V} l_3 \sum_{\rho=1}^3 k_{4\rho} + \frac{\omega}{V} l_3 \sum_{\rho=1}^3 k_{4\rho} (c + l_2 + l_3 + d_4 + b_{4\rho}) - \\
 & - \varphi_1 l_3 \sum_{\rho=1}^3 k_{4\rho} - \varphi_3 \left(l_3 \sum_{\rho=1}^3 k_{4\rho} - q_1 \right) - \varphi_4 \left(l_3 \sum_{\rho=1}^3 k_{4\rho} + q_4 \right); \\
 & m_4 d_4 \omega V = - \sum_{\rho=1}^3 k_{4\rho} \theta_{4\rho} (d_4 + b_{4\rho}) + \frac{\omega}{V} \left[\sum_{\rho=1}^3 k_{4\rho} (d_4 + b_{4\rho}) (c + \right. \\
 & \left. + l_2 + l_3 + d_4 + b_{4\rho}) \right] - (\varphi_1 + \varphi_3 + \varphi_4) \sum_{\rho=1}^3 k_{4\rho} (d_4 + b_{4\rho}) - \varphi_4 q_2.
 \end{aligned} \tag{3}$$

In the system of equation (3) the following symbols are taken:

$$\beta_{ij} = \alpha_{ij} = a_{ij} \left(i = \overline{1, 6}; j = 1, 3, 4, 5, 6 \right), \text{ if } j = 2, \text{ then}$$

$$\beta_{12} = m + m_1 + m_2 + m_3 + m_4;$$

$$\beta_{22} = -c(m_1 + m_2 + m_3 + m_4); \beta_{42} = m_2 d_2;$$

$$\beta_{52} = m_3 d_4 + m_4 l_3; \beta_{62} = m_4 d_4;$$

$$\alpha_{12} = k_1 a - \sum_{i=1}^2 k_{1i} b_{1i} - \sum_{j=1}^2 k_{2j} (c + l_1 + d_2 + b_{2j}) - \sum_{\rho=1}^3 k_{4\rho} (c + l_2 + l_3 + d_4 + b_{4\rho});$$

$$\alpha_{22} = k_1 a^2 + \sum_{i=1}^2 k_{1i} b_{1i}^2 + c \sum_{j=1}^3 k_{2j} (c + l_1 + d_2 + b_{2j}) + c \sum_{\rho=1}^3 k_{4\rho} (c + l_2 + l_3 + d_4 + b_{4\rho});$$

$$\alpha_{32} = l_1 \sum_{j=1}^3 k_{2j} (c + l_1 + d_2 + b_{2j}) + l_2 \sum_{\rho=1}^3 k_{4\rho} (c + l_2 + l_3 + d_4 + b_{4\rho});$$

$$\alpha_{42} = \sum_{j=1}^3 k_{2j} (d_2 + b_{2j}) (c + l_1 + d_2 + b_{2j});$$

$$\alpha_{52} = l_3 \sum_{\rho=1}^3 k_{4\rho} (c + l_2 + l_3 + d_4 + b_{4\rho});$$

$$\alpha_{62} = - \sum_{\rho=1}^3 (d_4 + b_{4\rho}) (c + l_2 + l_3 + d_4 + b_{4\rho}).$$

The solution of the system of equations (3) will be the values of variables that correspond to the stationary regimes, namely:

$$U = \frac{\Delta_U}{\Delta}; \omega = \frac{\Delta_\omega}{\Delta}; \varphi_1 = \frac{\Delta_{\varphi_1}}{\Delta}; \varphi_2 = \frac{\Delta_{\varphi_2}}{\Delta}; \varphi_3 = \frac{\Delta_{\varphi_3}}{\Delta}; \varphi_4 = \frac{\Delta_{\varphi_4}}{\Delta}. \tag{4}$$

The roots of the characteristic equations can be determined by numerical methods. It should also be noted that the description of the motor vehicle movement, which is really a non-linear object, by linear equations is a substitution of one task by another, with which the former may not have anything in common (due to the non-consideration of the nonlinearity of deviation and the members of higher than the first order of motion equations).

Hence the following problem arises: to establish the necessary and sufficient conditions of stability at the first approximation. According to Oleksandr Lyapunov's theorem on the stability of the stable motion at the first approximation [15], if all the roots of the characteristic equation of the system of the first approximation of the perturbed motion equations have negative valid parts, then the unperturbed motion is stable and, moreover, asymptotically stable, no matter what the members of higher orders in the differential equations of perturbed motion are.

The conditions under which all the roots have negative valid parts are determined by the Lyenar-Shippar criterion [16]: in order for the characteristic equation to have all the roots with negative valid parts, it is necessary and sufficient that:

1. All the coefficients of the characteristic equation were positive.
2. The main diagonal minors of the Gurvits matrix, compiled for this characteristic equation, were positive.

These conditions are satisfied in the case of the affinity of the denominator ω (4), that is, the affinity of the denominator of the principal determinant of the system, which has the form

$$V < V_{kr} = \beta / (-\alpha). \tag{5}$$

To determine the coefficients α and β , consider the column $a_{i2}(i = \overline{1,5})$ in Table 1, which contains the coefficients at angular velocity ω . Each of them consists of two addend: in one of them the speed is in the numerator, in the second one - in the denominator.

Table 1. The coefficients of the system of equilibrium equations solved with respect to variables.

The coefficients of the left part of the equilibrium equation							Right parts
U/V	ω		φ1	φ2	φ3	φ4	
	V	V ⁻¹					
1	2	3	4	5	6	7	8
$k_1 + \sum_{i=1}^2 k_{1i} +$ $+ \sum_{j=1}^3 k_{2j} +$ $+ \sum_{\rho=1}^3 k_{4\rho}$	$m +$ $+ \sum_{s=1}^4 m_s$	$ak_1 - \sum_{i=1}^2 k_{1i} b_{1i} +$ $-\sum_{j=1}^3 (c + l_1 + l_{2j}) k_{2j}$ $-\sum_{\rho=1}^3 (c + l_2 +$ $+ l_3 + l_{4\rho}) k_{4\rho}$	$\sum_{j=1}^3 k_{2j} +$ $+ \sum_{\rho=1}^3 k_{4\rho}$	$\sum_{j=1}^3 k_{2j}$	$\sum_{\rho=1}^2 k_{4\rho}$	$\sum_{\rho=1}^2 k_{4\rho}$	$k_1 \theta_1 +$ $-\sum_{j=1}^3 k_{2j} \theta_{2j}$ $-\sum_{\rho=1}^3 k_{4\rho} \theta_{4\rho}$
$k_1 + \sum_{i=1}^2 k_{1i} +$ $+ \sum_{j=1}^3 k_{2j} +$ $+ \sum_{\rho=1}^3 k_{4\rho}$	$m +$ $+ \sum_{s=1}^4 m_s$	$a^2 k_1 - \sum_{i=1}^2 k_{1i} b_{1i}^2 +$ $c \left[\sum_{j=1}^3 (c + l_1 + l_{2j}) k_{2j} \right.$ $+ \sum_{\rho=1}^3 (c + l_2 +$ $+ l_3 + l_{4\rho}) k_{4\rho} \left. \right]$	$q_1 +$ $+ c \left(\sum_{j=1}^3 k_{2j} \right.$ $+ \sum_{\rho=1}^3 k_{4\rho} \left. \right)$	$-c \sum_{j=1}^3 k_{2j}$	$-c \sum_{\rho=1}^3 k_{4\rho}$	$-3 \sum_{\rho=1}^3 k_{4\rho}$	$c \left(\sum_{j=1}^3 k_{2j} \theta_{2j} \right.$ $+ \sum_{\rho=1}^3 k_{4\rho} \theta_{4\rho}$ $+ \alpha k_1 \theta_1 \left. \right)$

1	2	3	4	5	6	7	8
$l_1 \sum_{j=1}^3 k_{2j} +$ $+l_2 \sum_{\rho=1}^3 k_{4\rho}$	$m_1 d_1 +$ $+m_2 l_1 +$ $+(m_3 +$ $m_4) l_2$	$-l_1 \sum_{j=1}^3 (c + l_1 + l_{2j}) k_{2j}$ $-l_2 \sum_{\rho=1}^3 (c + l_2 + l_3 +$ $+l_{4\rho}) k_{4\rho}$	$l_1 \sum_{j=1}^3 k_{2j}$ $-q_1$ $+l_2 \sum_{\rho=1}^3 k_{4\rho}$	$l_1 \sum_{j=1}^3 k_{2j}$ $+k_2$	$l_2 \sum_{\rho=1}^3 k_{4\rho}$ $+q_3$	$l_2 \sum_{\rho=1}^3 k_{4\rho}$	$l_1 \sum_{j=1}^3 k_{2j} \theta_{2j} +$ $-l_2 \sum_{\rho=1}^3 k_{4\rho} \theta_{4\rho}$
$\sum_{j=1}^3 k_{2j} l_{2j}$	$m_2 d_2$	$-\sum_{j=1}^3 (c + l_1 + l_{2j}) k_{2j}$	$\sum_{j=1}^3 k_{2j} l_{2j}$	$l_1 \sum_{j=1}^3 k_{2j} l_{2j}$ $-q_2$	—	—	$-\sum_{j=1}^3 k_{2j} \theta_{2j} l_2$
$\sum_{\rho=1}^3 k_{4\rho}$	$m_3 d_3 +$ $m_4 l_3$	$-l_2 \sum_{\rho=1}^3 (c + l_2 + l_3 +$ $+l_{4\rho}) k_{4\rho}$	$l_3 \sum_{\rho=1}^3 k_{4\rho}$	$l_3 \sum_{\rho=1}^3 k_{4\rho}$ $-q_3$	$l_3 \sum_{\rho=1}^3 k_{4\rho}$ $-q_2$	$l_3 \sum_{\rho=1}^3 k_{4\rho}$ $-q_3$	$-l_3 \sum_{\rho=1}^3 k_{4\rho} \theta_{4\rho}$
$\sum_{\rho=1}^3 k_{4\rho} l_{4\rho}$	$m_4 d_4$	$-\sum_{\rho=1}^3 (c + l_2 + l_3 +$ $+l_{4\rho}) k_{4\rho} l_{4\rho}$	$\sum_{\rho=1}^3 k_{4\rho} l_{4\rho}$	—	$l_3 \sum_{\rho=1}^3 k_{4\rho}$ $+q_4$	$l_4 \sum_{\rho=1}^3 k_{4\rho}$	$-\sum_{\rho=1}^3 k_{4\rho} \theta_{4\rho} l_{4\rho}$

If you present the main determinant in the form of the sum of two determinants:

$$\|a_{ij}\|_1^6 = V^2 \times \|\beta_{ij}\|_1^6 + \|\alpha_{ij}\|_1^6, \tag{6}$$

then α and β in (5) will be defined as:

$$\beta = \|\beta_{ij}\|_1^6; \; a = \|\alpha_{ij}\|_1^6. \tag{7}$$

The solution of the equation (5) is found using the Mapple 14 software. Taking into account the selected parameters of the road train, the speed of its movement, which has been steadily increasing, was set. In this case, the roots of the characteristic equation were found and the stability conditions were checked (5), (6). Thus, at the speed of 37.9 m/s, the roots of the characteristic equation were:

$$\begin{aligned} eigv := & -10.18 \quad -5.35 \quad -3.04 \quad -1.72 \quad -1.99 \\ & -1.72 \quad +1.99 \quad -1.03 \quad -13.76 \\ & -1.03 \quad +13.77 \quad +0.64 \end{aligned} \tag{8}$$

However, at a much lower speed $V^*=34,6\text{ m/s}$, a couple of complex conjugate roots $\lambda_{7,8}$ are transferred into a positive half-plane, which means the appearance of oscillatory instability when moving with velocities $V>V^*$. The emergence of this type of instability is associated with fluctuations of the links, which lead to the stability loss of the entire system, i.e. the stability loss of the metrobus.

It was shown in [17, 18] that the critical speed of a road train is significantly influenced by its design parameters. Therefore, further by means of the solution of the system of differential equations of the metrobus movement, the influence of pressure in the tires of the wheels of trailers axles, the location of the trailers center of mass, distances from the center of mass of the bus c and the trailer c1 to the coupling points, the base of trailers and the mass of the bus and trailers on the magnitude of the critical speed of movement were analysed.

In order to extend the results of the study of the influence of constructive factors and modes of motion on the stability indicators to the study of double-articulated metrobuses, it was suggested to make calculations using dimensionless parameters. The range of change of dimensionless geometric parameters has been chosen taking into account the features of the structures of existing metrobuses and is within the range of 0.8...1.2. It was considered from the following geometric factors:

$$a, c, c_1 = \frac{a, c, c_1}{L} = 0.2...0.8; \quad L = \frac{L}{D} = 0.2...0.8; \quad L_1, L_2 = \frac{L_1, L_2}{D_1, D_2}, \quad (9)$$

where a is the distance from the bus center of mass to the front axle; c is the distance from the bus center of mass to the coupling point with the first trailer; c_1 is the distance from the center of mass of the first trailer to the coupling point with the second trailer; L, L_1, L_2 are, respectively, the base of the bus, the first and the second trailer; D, D_1, D_2 are, respectively, the length of the bus, the first and the second trailer.

The results of calculating the influence of geometric factors on the overall traffic lane are shown in Fig. 2.

Fig. 2. The influence of relative parameters of the bus (a) and trailers (b) on the critical speed of the metrobus

Data analysis, Fig. 2, allows us to draw the following conclusions:

- An increase in the base of the bus, the first and the second trailers leads to an increase in the critical speed of the metrobus.
- An increase in the distance from the center of mass to the rear axle of the bus, an increase in the distance from the center of mass to the coupling point of the bus with the first trailer leads to a decrease in the critical speed of the metrobus, while an increase in the distance from the center of mass of the first trailer to the coupling point with the second trailer increases this speed.

The influence of the mass factors of the metrobus links and the coefficients of resistance to the lateral deviation of the metrobus links is shown in Fig. 3, 4. The mass of the bus, trailers and coefficients of resistance to the lateral deviation of the link wheels varied from 0.8 to 1.2:

$$m_1, m_2, m_3 = \frac{m_1, m_2, m_3}{m_1 + m_2 + m_3} = 0.8...1.2, \quad (10)$$

$$k_1, k_2, k_3, k_4 = \frac{k_1, k_2, k_3, k_4}{k_0} = 0.8 \dots 1.2, \quad (11)$$

where: m_1, m_2, m_3 are, respectively, the mass of the bus, the first and the second trailer, k_1, k_2, k_3, k_4, k_0 are, respectively, the coefficients of resistance of the deviation of the wheels of the driving and guided axles of the bus and the axles of the first and the second trailers.

Data analysis (Figures 3, 4) allows us to draw the following conclusions:

- An increase in the critical speed of the metrobus is caused by an increase in the mass of the bus and the coefficients of resistance to the deviation of the wheels of the bus driving axle and the trailers axles.
- A decrease in the critical speed of the metrobus is caused by an increase in the mass of the trailers and the coefficient of the resistance to the deviation of the wheels of the bus controlled axle.

The obtained dependencies should be taken into account both when designing metrobuses, and when operating them.

Fig. 3. Dependence of the critical speed of the metrobus on the relative mass of the bus and the trailers

Fig. 4. Dependence of the critical speed of the metrobus on the relative coefficients of the resistance of the lateral deviation of the axles of the bus and the trailers

4 Conclusions

The mathematical model of the double-articulated road train consisting of a bus and two trailers has been developed and the factors influencing the critical speed as the main index of stability of its movement have been determined. It has been established that an increase in the critical speed of the metrobus can be achieved by increasing the base of the bus, the first and the second trailers, as well as the mass of the bus and the coefficients of resistance of the deviation of the wheels of the bus driving axle and the trailers axles. At the same time, an increase in the distance from the center of mass to the bus rear axle, an increase in the distance from the center of mass to the coupling point of the bus with the first trailer, an increase in the mass of trailers and the coefficient of resistance to the deviation of the wheels of the bus controlled axle leads to a decrease in the critical speed of the metrobus. This must be taken into account both when designing metrobuses, and when operating them.

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