Dynamics of the mobile platform with four wheel drive

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Abstract. The dynamics problem of motion of the mobile platform with four wheel drive under the unsteady conditions have been formulated and analysed. The mobile platform prototype have been equipped with four independently driven and steered electric drive units. The theoretical model have been formed for the proposed design concept of the platform. The relations between friction forces in longitudinal and transverse directions in reference to the active forces have been considered. The analysis of the motion parameters for different configurations of the wheel positions has been included. The formulated initial problem has been numerically solved by using the Runge-Kutta method of the fourth order. The sample simulation results for different configurations of the platform elements during its motion have been included and the conclusions have been formulated.

1 Introduction

The description of motion of the three wheeled mobile platform has been presented in [5]. The preliminary studies of the four wheeled mobile platform have been conducted, and the results have been gathered in [4]. The research about systems such as wheeled mobile robots have been performed by scientists representing different point of view. An adaptive robust trajectory tracking controller for a Mecanum-wheeled mobile robot with Newton-Euler approach has been described in [1]. The robust controller for tracking the trajectory of the four wheeled vehicles with description of the robust control design has been presented in [2]. The output-feedback control strategy for the path following of autonomous ground vehicles has been proposed in [3]. The mathematical model of a 4-wheel skid steering mobile robot with an extension of the kinematic control law at the dynamic and motor levels using the Lyapunov analysis has been developed in [6]. Formulation of the dynamic optimization problem with the kinematic principles has been described through differential equations in [7].

The non-linear dynamics model of the robot with four steered and driven wheels has been established and described in [8], in order to define an accurate observer of the sideslip angle in a high dynamics context (designing the controller).

In this work the dynamics of motion of the mobile platform with four wheel drive is considered and the formulated initial problem has been solved by using the Runge Kutta

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method of the fourth order. The simulation results have also been included in the further sections.

2 Model of the mobile platform motion

The model of dynamics of motion of the four wheeled mobile platform in Cartesian coordinates has been formulated. The characteristics of the design of the platform is schematically presented in Fig. 1.

Fig. 1. Position of the center of mass of the platform in the reference frames.

Fig. 2. Schematic model of the system in reference frame.

Geometric features have been identified as follows: a, b – lengths between wheels in the x direction, c, d – lengths between the wheels in the y direction, h_s – distance between the ground and the center of mass of the platform, h – height of the platform, r_k – radius of the platform's wheel. The model of the mobile platform constituting the base of all considerations under the dynamics of motion is presented in Fig. 2. $OXYZ$ represents the reference frame. S point, according to Fig. 2, is the centre of mass of the platform, and it has been chosen as the origin of the local coordinate system connected with the platform. The $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ vectors are unit vectors in global coordinate system. The local coordinate $O_iX_iY_iZ_i$ ($i = 1,2,3,4$) have also been introduced in the model. The directions of the respective axes of all local coordinate systems have been adopted as parallel so the $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors in local coordinate systems, respectively. The forces taken into account in the analysis have been shown in Fig. 3.

Fig. 3. Drive unit with force distribution during motion

In Fig. 3 the designations considering the active forces: F_{ci} and the passive forces: F_{oi} , T_{pi} and T_{wi} ($i = 1,2,3,4$) have been adopted. The M_{ni} represents the drive torque and it is treated as the source of forced motion of the platform.

Model of the platform has been designed as the system with four steered wheels with electric drive units. The possibility to realize motion in any desired direction of motion have been provided. The design concept assumes that each wheel can be steered independently. The dynamics description have been determined with taking into account the acting forces on each wheel of the platform. The forces have been transformed in between the coordinate systems in order to obtain the motion parameters and to present the results in global coordinates. The description has been established by considering the mobile platform as the rigid structure.

The method of determining the motion parameters is gathered in this work and the set of the equations is presented. The Coulomb friction model has been adopted in this work.

The mathematical description of dynamics in a planar motion has been presented in previous work [4]. The model of the dynamics has been developed with taking into account the description of the coordinates transformation. The possibility to fall into the skid has been also included. The active force F_{ci} can be calculated from the formula:

$$\mathbf{F}_{ci} = \frac{M_{ni}}{r_k} \cdot \mathbf{e}_1^i \quad (1)$$

where: M_{ni} – the drive torque, r_k – radius of the wheel.

The friction forces have been set in the form of the Eq. 2 and 3.

$$\mathbf{T}_{wi} = -\mu_w \cdot N_i \cdot \text{sign}(v_{wi}) \cdot \mathbf{e}_1^i \quad (2)$$

$$\mathbf{T}_{pi} = -\mu_p \cdot N_i \cdot \text{sign}(v_{pi}) \cdot \mathbf{e}_2^i \quad (3)$$

where: μ_w, μ_p – the coefficients of friction in the longitudinal and transverse direction, v_{wi}, v_{pi} – the velocity components in the longitudinal and transverse direction, respectively.

All of the forces applied to the wheels occurring during motion can be boiled down to the resultant forces according to the Eq. 4.

$$\mathbf{W}_i = \mathbf{F}_{ci} + \mathbf{F}_{oi} + \mathbf{T}_{wi} + \mathbf{T}_{pi} \quad (4)$$

where: $\mathbf{F}_{oi}$ – the other resistance force occurring during motion.

The vector equation of progressive motion of the center of mass of the mobile platform can be written in the form:

$$m\mathbf{a} = \sum_{i=1}^4 \mathbf{W}_i \quad (5)$$

where: m – the total mass of the platform, $\mathbf{a}$ – the acceleration of the center of mass of the platform,

$\mathbf{W}_i$ – the resultant force obtained by considering the active and passive forces.

The vector equation of the rotational motion around the center of mass for the platform can be written in the form:

$$\frac{d\mathbf{K}}{dt} = \sum_{i=1}^4 \mathbf{s}_i \times \mathbf{W}_i + \sum_{i=1}^4 \mathbf{M}_i \quad (6)$$

where: $\mathbf{K}$ – the angular momentum vector of the platform, $\mathbf{s}_i$ – the location vectors of each of the drive wheels, $\mathbf{M}_i$ – moment occurring during the rotational motion of the wheels.

Considering the planar motion of the platform on the plane OXY and neglecting the moments $\mathbf{M}_i$, on the bases of Equations 5 and 6, the equations of motion can be written in the form:

$$\ddot{X} = \frac{1}{m} \sum_{i=1}^4 W_{ix} \quad (7)$$

$$\ddot{Y} = \frac{1}{m} \sum_{i=1}^4 W_{iy} \quad (8)$$

$$\ddot{\beta} = \frac{1}{I_z} \sum_{i=1}^4 (s_{ix} \cdot W_{iy} - s_{iy} \cdot W_{ix}) \quad (9)$$

where: $\ddot{X}$ – acceleration of center of mass on the X-axis in reference frame,

$\ddot{Y}$ – acceleration of center of mass on the Y-axis in reference frame,

$\ddot{\beta}$ – angular acceleration around center of mass of the platform.

The equations of motion, written in the form of differential equations, together with the initial conditions have been used for determination of the trajectory, velocity and acceleration of the platform.

The solution of the formulated initial problem the Runge-Kutta method of the fourth order has been used. The sample simulation results have been presented in the next section.

3 Sample simulation results

The simulation results regarding the chosen cases of motion of the designed mobile platform have been determined on the basis of the potential possible situation. The total observation time of platform motion work is 20 s. During work time of similar systems, the different type of possible unexpected situations may have happened.

In this work every wheel of the mobile platform at first four seconds of motion drives on the same surface, represented in the same coefficients of friction. After four seconds of motion, the left side wheels invaded the slippery surface, e.g. because of frozen puddle, poured oil or ride on a different pavement.

In mathematical description the different value of the coefficients of friction between the left and right side wheels have been introduced. The initial values of the following parameters are gathered in Table 1.

Table 1. Initial parameters in the analysis.

Parameter	Symbol	Value
Drive torque without slippage [Nm]	M_{nl}	15
Coefficient of friction in the longitudinal direction	μ_w	0.2
Coefficient of friction in the transverse direction	μ_p	0.1
Mass of the platform [kg]	m	200
Acceleration of gravity [m/s ²]	g	9.81
Radius of wheel [m]	r_k	0.2
Total worktime [s]	t_c	20
Angle of inclination the platform [rad]	β	$\pi/9$

The course of the initial drive torque is presented in Fig. 4.

The coefficients of friction have been changed from the initial values. The coefficient of friction in the longitudinal direction $\mu_w = 0.1$, and in the transverse direction $\mu_p = 0.05$ after four seconds of motion.

Fig. 4. Representation of the drive torque M_{ni}

The dependence between the active and passive forces have been gathered and have been graphically presented in Fig. 5.

Fig. 5. Representation of dependence between active and passive forces during motion

The linear parameters of progressive motion have been determined and have been presented in Fig. 6 and 7.

Fig. 6. Motion parameters in the X axis in reference frame

Fig. 7. Motion parameters in the Y axis in reference frame

The angular parameters of motion have been determined by using the equation of motion around the center of mass of the platform.

The results of the changes between the angle and the angular velocity and acceleration have been presented in Fig. 8.

Fig. 8. Motion parameters around the center of mass

The trajectory of motion of all the wheels and the trajectory of the center of mass of the platform have been presented in Fig. 9.

Fig. 9. Trajectory of motion of the mobile platform

Fig. 10. The progressive motion velocity

After fourth seconds value of the developed friction have been overcome by the value of the active force. In the consequence of this situation, the motion around center of mass have

been occurred. The course of the linear velocity of the progressive motion of the platform is presented in Fig. 10.

The velocity of the progressive motion of the platform, after switching off the drive torque, remains constant. This is a consequence of simplification, that all the resistant forces have been neglecting at this period of motion time. Without this assumption the platform would lose velocity until it stops.

4 Conclusions

The presented model of dynamics is useful in investigation of the motion of wheeled mobile robots. The consequences of the unexpected situations that may occur during working such machines can be analyzed. The representation of relations between the motion parameters have been included. The developed model is also useful to examine different configurations of the drive units.

The formulated initial problem has been solved numerically by using the Runge – Kutta method of the fourth order. The sample simulation results can be treated as the cases of the platform motion in possible circumstances of platform work. Directions of further studies concern the extension of the model with introducing to the mathematical description other elements of the real object in order to prevent the platform from falling into a skid.

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