Numerical study of the relation between chosen statistical parameters of input stresses and cumulative fatigue damage provided rainflow decomposition

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Abstract. The aim of the article is to present a series of numerical tests of the cumulative fatigue damage solution under assumption of random stress input. It is realised study of the influence of the statistical moments or power spectral density on the cumulative fatigue damage using well-known rainflow method. The distribution of the cumulative fatigue damage is determined for pre-defined material parameters.

Keywords: Statistical moments, power spectral density, rainflow decomposition, cumulative fatigue damage, stress analysis

1 Introduction

Engineers from different fields have been interested in the prediction of the cumulative fatigue damage of construction components from various materials for almost 180 years. The description of the fatigue process has gone through its development until it reached the mathematical formula of the fundamental principles. However, some axioms have remained almost unchanged, such as the description of the fatigue curve or the different application of a linear Malmgren-Miner approach that is widely used to count cumulative damage [1, 2]. With the development of new materials and computational techniques, more feasible fatigue criteria have been developed that are based on, for example, multiaxial stressing or tracking the stress history [1, 3].

The machine components are often subjected to stochastic stress during real operation [3]. The first attempts to apply common probability theory tools and statistics failed. Furthermore, the calculation of cycle numbers and amplitudes using Fourier transform did not correspond to experiments and reality [4]. Only in 1968 Matsuishi and Endo came up with the suggestion of nonstandard cycle counting and the method was called the rainflow method. This/Their computational algorithm compiles closed hysteresis loops of stress or

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stress-strain response by calculating the amplitude of movement and mean values on the principle of rain that flows down the roof. In practice, the algorithm based on this principle can be used only by computing. Those who are interested will find a more detailed description in the relevant literature [2, 3]. There are numerous publications that deal with this topic. Some of these topics were solved, but many of them are still waiting for their discovery. The aim of the authors is to present a non-traditional numerical study of the relationship between the stochastic stress input characterized by the first two statistical moments and the power spectral density of different character on one side and the cumulative fatigue damage calculation by using rainflow decomposition on the other side. In order to fulfil our aim, we chose one type of material with predefined characteristics when carrying out the numerical experiment.

2 Theoretical aspects of random signal modelling and fatigue damage prediction

A popular numerical testing method of the stochastic process is the approach based on the well-known Monte Carlo method [5, 6, 7]. This method is becoming more attractive because of current computing capabilities. The results are gained from a series of numerical analyses with approximately 1000 to 10000 realizations of random wake while it is advised to generate 5000 to 20000 random values of the analysed function [7], for example, defined by power spectral density (PSD)) $S_{ff}(\omega)$ for each realization. The simulation of a stationary (normal) centred Gaussian process $f(t)$ is usually rewritten [6] as

$$f(t) = \sqrt{2} \cdot \sum_{k=1}^{N} \sqrt{S_{ff}(\omega_k) \cdot \Delta\omega \cdot \cos(\omega_k \cdot t - \varphi_k)}, \tag{1}$$

where φ_k is a random number with the uniform distribution ($0 \leq \varphi_k \leq 2.\pi$), t is time, ω_k is circular frequency, $\Delta\omega$ is the increment of circular frequency (constant throughout the frequency range calculation). If $S_{ff}(\omega)$ is constant, $f(t)$has the character of white noise. This approach is appropriate if PSD function $f(t)$is defined by discrete values [5]. Theoretically, the most common case is already mentioned white noise whose PSD is constant. To generate this process, one can use a classical random number built-in generator that can be found, for example, in Matlab. However, if the process of PSD is defined by mathematical function, it is more appropriate to use special numerical filters which are able to model the process state [8, 9]. The most common filter is a Kanai-Tajimi differential filter that is used, for example, in earthquake modelling. Its mathematical formula is as follows:

$$m_e \cdot \ddot{f}(t) + b_e \cdot \dot{f}(t) + k_e \cdot f(t) = w(t) \tag{2}$$

where *w(t)*is a well-known Gaussian white noise process with a constant power spectral density S_0=1. Parameters m_e, b_e, k_e are gained by comparing with the experiment and subsequently by using one of the identification approaches [10, 11]. The generated random character of the stress is the input to the cumulative fatigue damage calculations following the principles of rainflow decomposition of the monitored signal and subsequent application of the given fatigue curve.

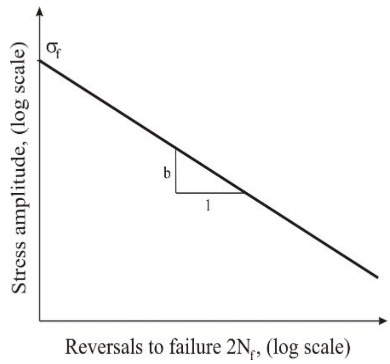

Fig. 1. *S-2N* curve

It is well-known that the Wöhler curve (Fig. 1, sometimes called S-2N curve) is the basic source of getting information of the material fatigue life [12, 13]. Generally, the S-2N curve is statistically evaluating by experimental fatigue curve [13-15]. This is a graph of the magnitude of a cyclical nominal stress σ_A against the logarithmic scale of cycles to failure $2N_f$. It is advantage to show it in logarithmical or semi logarithmical coordinates. The $\sigma_A - 2N_f$ relation can be written as follows

$$\sigma_A = \sigma_f \cdot \left(2N_f\right)^b, \tag{3}$$

where σ_f is the fatigue stress coefficient, $2N_f$ is number of cycles to failure, b is fatigue strength exponent and σ_A is stress amplitude to failure. Some researches, the relationship (1) rewrite into the following form

$$\sigma_A^m \cdot \left(2N_f\right) = K, \tag{4}$$

where $m = -(1/b)$ and $K = \sigma_f^{(-1/b)} = \sigma_f^m$. Considering mean stress modified version of the stress amplitude (using Goodman, Soderberg, Geber), eq. 2 can be rewritten as follows

$$\left\{\sigma_A \cdot \left[1 - \left(\frac{\sigma_M}{R_F}\right)^k\right]^{-1}\right\}^m \cdot \left(2N_f\right) = K. \tag{5}$$

If $k=1$ and $R_F=R_E$ (yield stress) the Soderberg´s model is used, if $k=1$ and $R_F=R_M$ (strength limit) the Goodman´s model is used and if $k=2$ a $R_F=R_M$ the Geber´s model is used [3]. Using the linear Palmgren-Miner law, we can calculate fatigue damage for stress amplitude σ_{Ai} as follows [15-20]

$$d_i = \frac{1}{2N_{f\_i}} = \left\{\frac{\sigma_{Ai}}{\sigma_f} \cdot \left[1 - \left(\frac{\sigma_M}{R_F}\right)^k\right]^{-1}\right\}^m. \tag{6}$$

3 Numerical tests

Based on the theoretical assumptions stated above, the computational procedures were programmed in Matlab language to simulate selected cases. The material selected for testing had the following mechanical parameters: Young‘s modulus E = 2.0·10^5 MPa, elastic limit R_e = 247 MPa, point of S/N curve N_A = 10^4 *cycles,* σ_{Amax} = 180 MPa, fatigue limit σ_C = 58 MPa, exponent of S/N curve m = 5.2. The value σ_f was obtained using point [N_A, σ_{Amax}] of S/N curve in eqs. 3 or 4. The input signal of stress was modeled as follows

$$\sigma(t) = \sigma_m + \sigma_A \cdot f(t) \tag{7}$$

with parameters:

- mean stress value: σ_m=30[MPa],
- stress amplitude: σ_A=30 [MPa],
- maximum frequency applied: f_{max}=20 [Hz],
- white noise with PSD: $S_{ff}=1$ [Pa^2·s],
- Kanai-Tajimi model with parameters: m_e=0, b_e=3, k_e=100, $S_{ff}=1$[Pa^2·s],
- Kanai-Tajimi model with parameters: m_e=1, b_e=20, k_e=2000, $S_{ff}=1$[Pa^2·s].

For selected material (fatigue curve parameters), the authors decided to analyze the following cases:

a) *The effect of random input signal with a constant mean value, amplitude, and PSD with the character of white noise* (Figs. 2, 3).

The results of the model study shown in Figures 2 and 3 describe the random character of cumulative fatigue damage *D* even when the same statistical parameters of stress are used. This led us to the conclusion that in the calculation, it is necessary to take into account also the changes in the randomness of the signal that cannot be easily identified and which can be detected by the rainflow decomposition of this signal and it appears in the uncertainty of the resulting fatigue damage value.

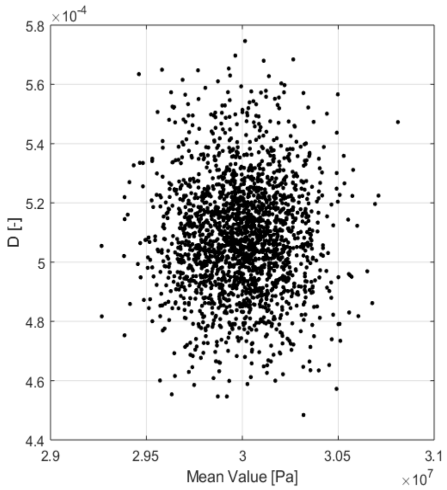

Fig. 2. Damage vs. mean value - distribution of fatigue damage in the case of white noise PSD and constant mean value and amplitude

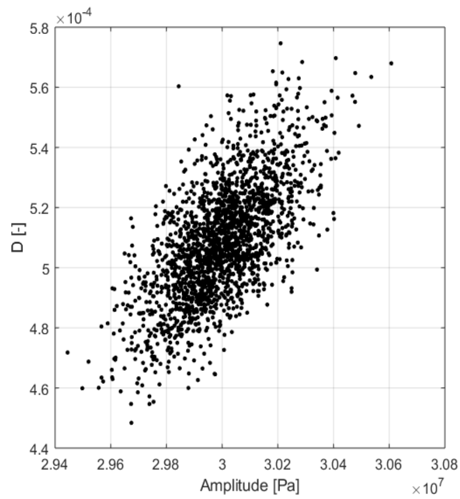

Fig. 3. Damage vs. amplitude - distribution of fatigue damage in the case of white noise PSD and constant mean value and amplitude

b) *The effect of change of mean stress value on the fatigue damage assuming a constant stress amplitude and a normal distribution with the character of white noise.*

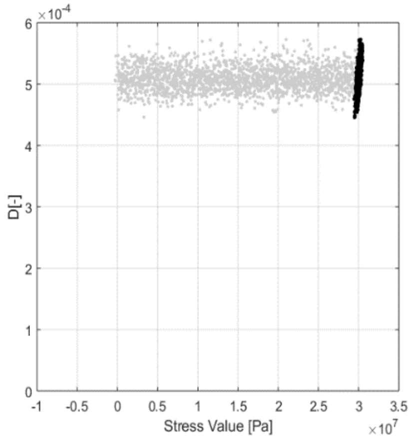

Fig. 4. Fatigue damage vs. mean stress value

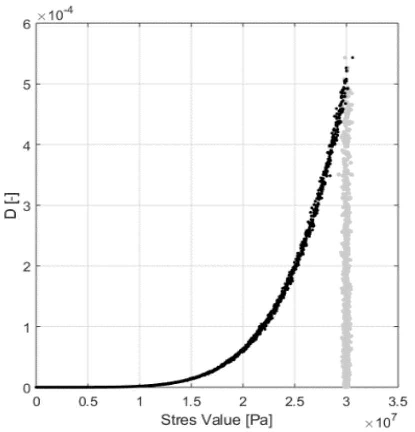

Fig. 5. Fatigue damage vs. stress amplitude

On the basis of a series of analyses, it is possible to state the independence of damage D degree at the mean stress value with respect to the application of the Goodman correction relationship (3). However, a certain degree of randomness has remained similar to the previous case and has the character of a normal distribution (See Figure 4).

c) *The effect of change of stress amplitude on the fatigue damage assuming a constant mean value and a normal distribution with the character of white noise.*

The numerical experiment proved the dependence of fatigue damage degree from the stress amplitude in the exact correlation with the relationship to the fatigue curve. Due to this dependence, the randomness is non-significant, respectively; it follows the previous cases. However, it plays a minimal role in the overall context of the computational model as the course of the observed function $D(\sigma_A)$ in Figure 5 shows.

Other numerical tests have been aimed at generating a random signal with the same statistical moments (first two moments) but different PSD character and the effect of different PSD on PDF cumulative fatigue damage.

d) The effect of PSD with a white noise character of the input signal with a constant mean value and amplitude on PDF cumulative fatigue damage.

The results of the numerical study shown in Figures 6 and 7 document the random nature of cumulative fatigue damage *D* with the same statistical parameters and the "classical" constant course of PSD stress. PDF function of cumulative damage *D* has the character of a normal Gaussian distribution. The distribution parameters for the test case are shown in Table 1.

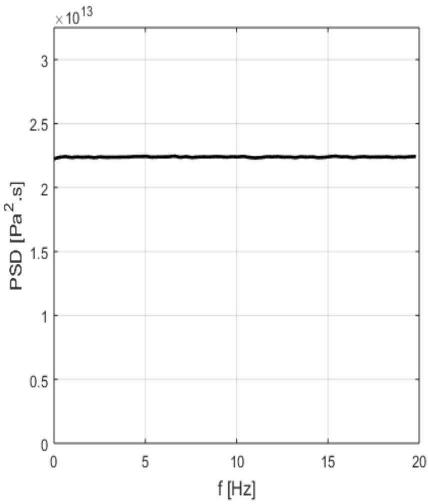

Fig. 6. Modelled white noise PSD of the random stress input

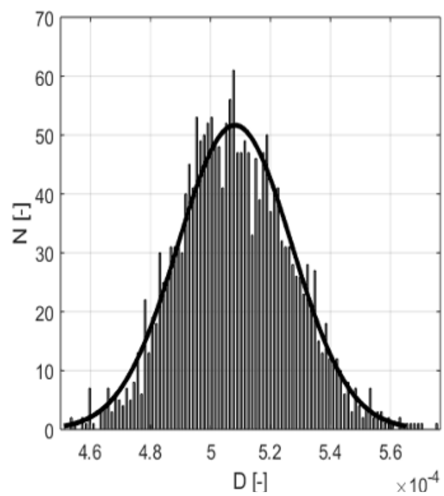

Fig. 7. Fatigue damage histogram of the D and PDF model of the normal distribution

e) The effect of modelling function PSD (exponential character) of an input signal with a constant mean value and amplitude on PDF cumulative fatigue damage D.

The results of the study document the random character of cumulative fatigue damage *D*, but the PDF has the character of a lognormal distribution with a different mean value and a standard deviation (Figures 8 and 9). The distribution parameters for the test case are shown in Table 1.

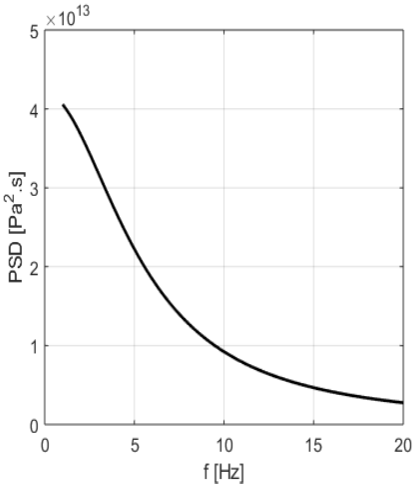

Fig. 8. Modelled PSD of the random stress input

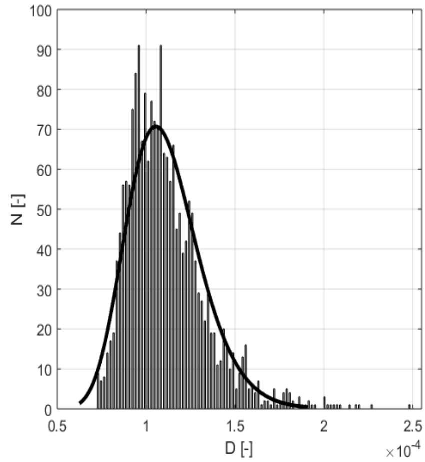

Fig. 9. Fatigue damage histogram and PDF model of the lognormal distribution

f) *The effect of modelling function PSD (with a resonant peak) of an input signal with a constant mean value and amplitude on PDF cumulative fatigue damage.*

On the basis of the tests carried out, the random character of cumulative fatigue damage *D* can be again stated. However, PDF has again the character of the lognormal distribution with different mean value and standard deviation as in the previous cases (Figures 10, 11). The distribution parameters for the test case are shown in Table 1.

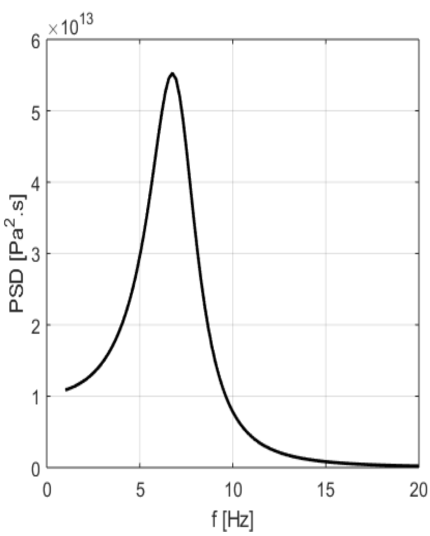

Fig. 10. Modelled PSD of the random stress input

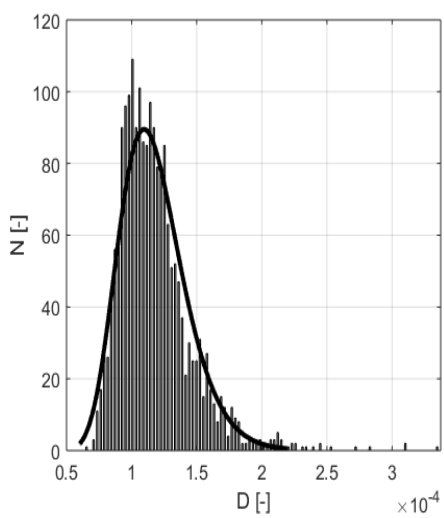

Fig. 11. Fatigue damage histogram and PDF model of the lognormal distribution

Table 1. Overview of statistical parameters

character of PSD	PDF of *D*	mean Value E[*D*]	amplitude or E[D^2]
White noise	normal	$5.093 \cdot 10^{-4}$	$1.947 \cdot 10^{-5}$
1. Kanai–Tajimi model	lognormal	$1.108 \cdot 10^{-4}$	$2.219 \cdot 10^{-5}$
		-9.125	0.1815
2. Kanai–Tajimi model	lognormal	$1.176 \cdot 10^{-4}$	$2.810 \cdot 10^{-5}$
		-9.069	0.2177

Since a lognormal distribution is not commonly used, we will provide some information about this distribution. The lognormal distribution is a probability distribution whose logarithm has a normal distribution. The lognormal distribution is applicable when the quantity of interest must be positive, since log(x) exists only when x is positive. The lognormal distribution is closely related to the normal distribution. If x is distributed lognormally with parameters μ and σ, then log(x) is distributed normally with mean μ and standard deviation σ. The probability density function of the lognormal distribution is

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(\frac{-(\ln x - \mu)^2}{2\sigma^2}\right); x > 0 \tag{8}$$

The mean value is

$$\mu = \exp\left(\mu + \frac{\sigma^2}{2}\right) \tag{9}$$

and the variance is

$$\sigma^2 = \exp(2\mu + \sigma^2)(\exp(\sigma^2) - 1) \tag{10}$$

4 Conclusion

The presented short study focuses on the analysis of the relationships between the statistical parameters of the random stress and the resulting degree of fatigue damage gained by traditional computational tools, such as the rainflow decomposition of the signal and the application of the standard fatigue curve. Based on a series of numerical experiments on pre-selected models, it was found out that the randomness of the analysed damage rate is significantly affected by the PSD character. The PSD's power over the entire applied frequency range has a significant effect on the result. On the one hand, tests have shown that PSD with the character of white noise has the most adverse effect on fatigue damage. On the other hand, PSD modelling cases using the Kanai-Tajimi differential model, which describe real stress cases, have generally more favourable results - lower prediction of fatigue damage. The traditional and well-known impact of stress amplitude was not a priority of interest and had no justification for further research in this area.

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