

Study of Strain-Softening Stage in Structurally Inhomogeneous Materials

Vladimir Mironov^{1,2} and Olga Lukashuk^{2,a}

¹*Institute of Engineering Science, RAS (Ural Branch), 620049, 34 Komsomolskaya st., Yekaterinburg, Russia*

²*Ural Federal University, 620002, 19 Mira Street, Yekaterinburg, Russia*

Abstract. The paper considers – in terms of modeling and experimental testing – the softening stage manifested in structurally inhomogeneous materials under quasi-static and cyclic load. The method which is used to build the theoretical basis is typical for mechanics. It starts with defining a new material property – strain-softening phase. The stage is realized in the form of a descending branch of the curve on computer diagrams recorded during simple tension or torsion tests on non-reusable specimens in a reasonably rigid testing machine. The type of such a diagram is determined by the structural inhomogeneity of a material, which is defined by distribution law for strength and plasticity properties of structural elements. Complete stress-strain curves or diagrams (CSSD) with a branch descending to zero have been plotted for materials of various classes. Degradation of parameters of a descending branch on a computer diagram recorded for trained specimens provides a new angle on the subject of relation between static and cyclic material properties. The experiments and modeling which were carried out provide basic understanding of softening in terms of deformation.

1 Introduction

The whole variety of material properties and states used in strength calculations on solid bodies is reflected in correlations which define boundary-value problems. As the research on mediums with complex properties progressed, more and more models appeared. Those models allowed to factor in various aspects of body behavior under external load. It was how the theories of elasticity, plasticity, prolonged strength, and many others were formed. According to the internal logic of mechanics, when you try to solve a problem concerning body failure, a new material property should also be defined – strain-softening stage. It is even more important to do when one considers the fact that many attempts to solve the fracture problem by means of elastic-plastic analysis have lead to incorrect results, coming into a conflict with initial experiments [1,2]. Determining such functions of a material which could describe its softening stage is quite promising for the purposes of bridging a known gap between problems and criteria of fracture mechanics and disseminated damage mechanics [2,3] and developing, on that basis, new resource-saving techniques for calculating strengths of machine parts.

2 Modeling of material softening stage

^a Corresponding author : oldim96@mail.ru

Using even the simplest structural models of Daniels and Masing suffices to single out the stage of material softening from the whole lot [4, 5]. Virtual “dead” tension applied to a bundle of variable-strength parallel rods reveals a branch descending to zero on diagrams plotting relations between nominal stress and strain (stress-strain diagrams). The type of such a diagram is determined by the structural inhomogeneity of a material, which is defined by distribution law for strength and plasticity properties of structural elements. It should be stressed that the more inhomogeneous the material is used, the flatter the descending branch becomes. The importance of this fact can be fully understood only when the behavior of the machine-specimen system is studied (Fig. 1a) using the method of catastrophe theory [6]. Substituting a uniform distribution law for ultimate strengths of elements with a normal law, one is left with a “crease” catastrophe (Fig. 1).

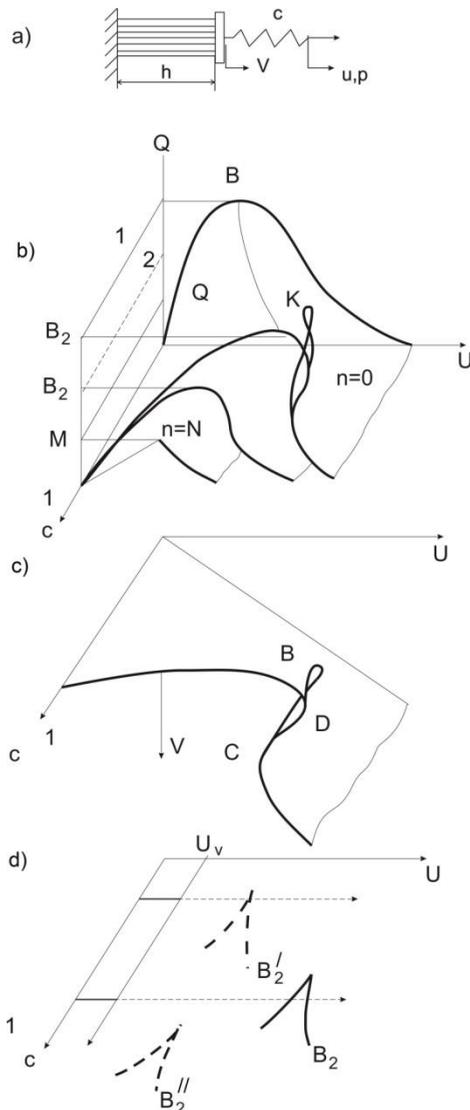


Figure 1. Fold catastrophe of the machine-specimen system.

Analyzing how the surface of equilibrium states of the machine-specimen system behaves in the neighborhood of a bifurcation point (or a set of points) provides a complete scenario of deformation (the cases of diminishing stresses included). Specific surfaces in the form of a crease or fold bear the names of related catastrophes. Figures 1b (for load Q) and 1c (for a preset displacement u of the

machine's movable grip) show crease surfaces which are typical of a computer diagram $Q(u)$ with a bifurcation point located on the descending branch. Mapping those catastrophes in the control space reveals areas of stable and unstable behavior of the system (divided by bifurcation lines B_2 in Fig. 1d).

The instability manifested in the system's deformation in the form of a catastrophic jump on the surface of equilibrium states is related to a complete or partial failure of a specimen. For example, when a specimen is loaded with a certain force, it leads to its dynamic failure at the maximum of the computer diagram regardless of the rigidity of the loading device (line $B-K$ in Fig. 1b). To the contrary, when kinematic loading is used, the machine's rigidity plays a crucial role in providing stability of the system's deformation. At a sufficiently high rigidity, there are ways of achieving stable deformation, or even equilibrium (without dynamics) division of a specimen into two parts (path 1 in Fig. 1d). When the rigidity of the loading system is lowered, bifurcation points come up in the strain path, which leads to a jump from one plane of the surface Π to another (path 2 in Fig. 1d). Theoretically, in the case of an infinitely small rigidity, the system would lose its stability at the maximum of the computer diagram – similar to the above case of loading with a specified force. Obviously, the fracture pattern for specimens made of the same material would vary depending on the rigidity of the loading system – dynamic (or brittle) when the rigidity is low, and equilibrium (or ductile) when it is high.

The deformation instability of the system regarded as a criterion of a specimen's fracture also takes into consideration the impact which the material's properties have on the fracture pattern. It shows up in the dependence of the system's stability conditions on the material's structural inhomogeneity. As model-based calculations demonstrate, the structural inhomogeneity determines the slope of the descending branch, which, in turn, determines the location of bifurcation lines B_2 in the plane of control parameters (Fig. 1d). The more homogeneous the material is, the more abrupt the descending branch of the relation diagram is, and the higher the rigidity of the loading system should be to ensure an equilibrium fracture of a specimen. Thus, to correctly solve problems of fracture mechanics, instead of a material constant, a material function describing the softening stage is needed, along with a methodology which would take the conditions of loading into consideration.

A material function in the form of a diagram which relates stress and strain (deformation) to the descending-to-zero branch is called a function of resistance or a complete stress-strain diagram (CSSD) [6]. One important practical area of usage for such functions is connected to description of prolonged fracture. When parameters of a static resistance function change due to cyclic material damage, it leads to changes in bifurcation pattern of a studied machine-sample system. Depending on controlled parameters of the cycle and behavior of the CSSD degeneracy, there are several scenarios of fatigue failure. The catastrophe could draw closer to the path of loading (B_2' in Fig. 1d) or move away from it (B_2'' in Fig. 1d). In spite of those two different scenarios of fatigue process, the final failure of a specimen happens when the machine-specimen system loses its stability. Therefore, the instability of strain could be defined as a unified cause of static and cyclic failure of specimens. Only conditions of its occurrence are different.

Modeling turned out to be quite effective in terms of finding patterns of change in material properties at the stage of softening. Structural material models were additionally supplemented with conditions of component failure. It was assumed that an element which has been fractured at the point of its ultimate strength moves apart, without pushing, those elements which have not been affected, thus creating a void. When load is withdrawn, this void could close up, with the fracture element working in compression. Such models were used to study instantaneous or tangent material properties under active added loading E^p, ν^p and unloading E^u, ν^u , necessary to form incremental physical equations (E is a modulus, and ν is a lateral deformation coefficient). Fig. 2 shows the results of numerical experiments when the normal law of distribution is maintained for deformations of elements transition into the plastic ε_T and fracture ε_Z states.

For a model specimen as a unified system of N elements,

$$\varepsilon_k = \varepsilon, \sigma = \frac{1}{N} \sum_{k=1}^N \sigma_k, \eta = \frac{1}{N} \sum_{k=1}^N \eta_k \quad (1)$$

is true in the case of strain, and

$$\varepsilon_k^u = \varepsilon^u, \sigma^u = \frac{1}{N} \sum_{k=1}^N \sigma_k^u, \eta^u = \frac{1}{N} \sum_{k=1}^N \eta_k^u \quad (2)$$

- when unloading takes place.

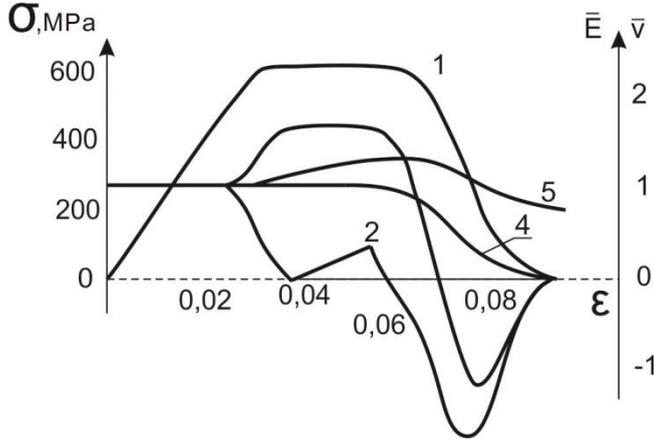


Figure 2. Properties of model material.

These expressions helped to find, first of all, original dependences $\sigma(\varepsilon), \eta(\varepsilon), \sigma^u(\varepsilon^u), \eta^u(\varepsilon^u)$, and then, via numerical differentiation, target functions $E^u(\varepsilon^u) = \frac{d\sigma^u}{d\varepsilon^u}, \nu^u(\varepsilon^u) = -\frac{d\eta^u}{d\varepsilon^u}$ and $E^u(\varepsilon^u) = \frac{d\sigma^u}{d\varepsilon^u}, \nu^u(\varepsilon^u) = -\frac{d\eta^u}{d\varepsilon^u}$, manifested in the form of deformation functions at the start of unloading ($\varepsilon = \varepsilon^*$).

The dependence $\sigma(\varepsilon)$ in the Fig. 2 (curve 1) has a descending branch which comes up as a result of separate elements of the system being fractured and characterizes the strain-softening of the model material. Graphs E^u, ν^u , which are drawn in relative coordinates (line 4), match and change only when structural elements are fractured. To the contrary, current values of E^p, ν^p begin to change when plasticity appears, and by the start of fracture ($\varepsilon = \varepsilon_{z\alpha}$), would have taken values of $E_p = 0, \nu_p = 0.5$. One could point out at the similar pattern of change for the graphs E_p and ν_p (lines 2' and 3 in the Fig. 2) when $\varepsilon > \varepsilon_{z\alpha}$. The tangent modulus becomes negative, and the lateral strain coefficient diminishes and falls down to negative values, as well, which reflects the fact of considerable loosening of the material. Line 5 (Fig. 2) corresponds to the lateral strain coefficient, which could be defined as a correlation between complete lateral and axial strains. If residual plastic deformations (typical of this model) are present, the finite value of the coefficient is above zero.

The modified Masing model could be extended by adding cyclic properties of materials – for example, the law of random longevity distribution and the pattern of ultimate strength degeneracy [7]. In that case, when stationary loading is applied to a model specimen, one can witness gradual degeneracy of resistance function, along with every kinetic curve mentioned here.

3 Experimental Study of Strain-Softening Stage

Phenomenological approach to the problem of fracture predetermines the methods of studying material properties based on mechanical testing of specimens. At the same time, the specifics of

carrying out the tests aimed at recording CSSDs touch upon fundamental notions of experimental mechanics: hypothesis of continuity and macroscopic definability. The conditions for specimen testing and their geometry contradict rationality of established experimental principles. New demands are made not only on technology and tools of experiments but also on analysis and interpretation of the results obtained.

Strictly speaking, an object to be tested should have a typical or representative volume of material [4]. Departure from this requirement in the field of elastic vibrations is justified by the fact that one can manage to obtain such characteristics of material which would be invariant to the size of the object in question. But even in typical tests – for example, in uniaxial tensile tests – their output characteristics does depend on the size of a specimen [8,9]. Plotting CSSDs aggravates this problem due to inevitable localization of deformations by the length of the specimen. The working section of the specimen which is outside the area of localization becomes a part of the loading system, largely diminishing its rigidity. As a result, one fails to plot the descending branch on the computer diagram when typical specimens are used, even in the case of a high-rigidity loading device.

In accordance with modeling principles, it is necessary to test specimens of smaller sizes, if possible. In uniaxial tensile tests, one-time specimens of 2 mm in diameter and 0,3 mm in radius of fillet transition were utilized as having close-to-standard parameters of the ascending branch. A ring-like loading device of variable rigidity allowed to draw a CSSD with a branch descending to zero for materials of various classes [10,11]. Recalculating the functions of resistance using stress-strain coordinates (based on Koshi) of computer diagrams allowed to plot CSSDs for materials of various classes (Fig. 3) as specified by Russian standards.

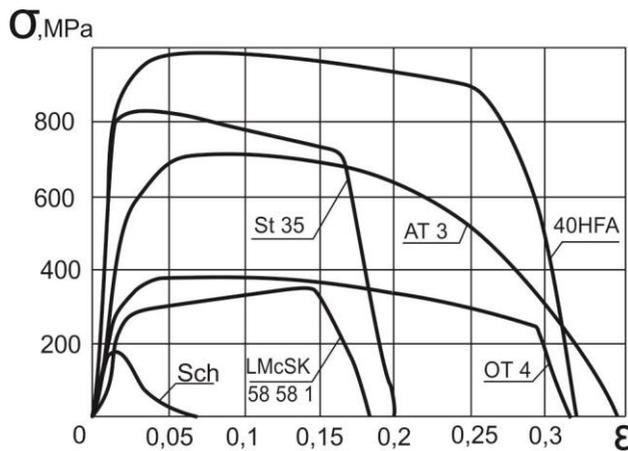


Figure 3. Complete Stress-Strain Diagrams.

Building one model or another of a material in the state of softening predetermines selection of characteristics which would reflect the destructive processes taking place within [12-16]. One could assume free-energy release and load redistribution among elements of the medium to be typical signs of fracture. Two mechanisms, at least, are in charge of energy dissipation in materials: plastic deformation and formation of structural micro-defects. The mechanical characteristics corresponding to those would be the ultimate strain ϵ_z (determined by the length of a CSSD) and the modulus of unloading E'' (diminishing as formation and development of defects progress). Synergetic material properties, which are related to redistribution of stresses when defects appear and grow, are determined by structural inhomogeneity. The slope of the descending branch of a CSSD is also connected to the structural inhomogeneity. Besides, it closely correlates with crack growth resistance of a material [13]. The corresponding mechanical characteristic would be a $E^P(\epsilon)$ function. Another indicator of the degree by which the material is damaged could be a $\nu^P(\epsilon)$ function. All functional

material characteristics noted above are derived from a basic experiment on plotting a CSSD with interim unloading points.

The function of resistance with a descending-to-zero branch gives the most comprehensive and logically complete idea of static material properties. It also provides a unifying concept of the limiting state of a material, relating it to the loss of stability in the process of deformation. In tests run on cast-iron specimens, which would manifest no neck until their fracture, patterns revealed were similar in terms of quality to the ones in Fig. 2 [11].

4 Conclusion

Regarding fracture as a loss of deformation stability leads to conceiving a new notion of the limiting state of a material. The descending branch of the resistance function provides a full set of marginal material states for a specified path of loading. In certain experimental conditions, all testing runs in equilibrium, without a loss of stability of the machine-specimen system. When conditions of stability fail in a specific case, only a part of the descending branch is followed in equilibrium. The state of the material at a critical point when the body loses its deformation stability could be called a critical limiting state.

Experimental studying of the phenomenon of material softening mainly follows major principles of experimental mechanics, but certainly it has its own peculiarities and methodological issues. This statement is true at least for structural materials which have no neck when strained.

Degradation of the parameters of the descending branch of a computer diagram for specimens which have undergone cyclic training provides a new angle on the subject of relation between static and cyclic material properties. In both cases, the loss of deformation stability plays the roles of both a cause and a criterion of fracture. The results cited here give a certain basis for further modeling and experimental studies into the softening stage in structural materials.

References

1. G.P. Cherepanov, *Mechanics of Brittle Fracture* (Science, Moscow, 1974)
2. S.D. Volkov, V.P. Stavrov, *Statistical Mechanics of Composite Materials* (BSU Publishing, Minsk, 1978)
3. J.A. Callins, *Failure of Materials in Mechanical Design: Analysis, Prediction, Prevention*, (World Moscow, 1984)
4. S.D. Volkov, *Statistical Theory of Strength*, (Machine Building State Publishing, Moscow, 1960)
5. S.D. Volkov, *Function of material strength and statement of boundary problems of fracture mechanics*, (USSR Academy of Sciences, Sverdlovsk, 1986)
6. V.I. Mironov, O.A. Lukashuk, *J. Key Eng. Mater.*, **735**, 89-94 (2017)
7. V.I. Mironov, A.V. Yakushev, *J. Bull. of USTU*, **22** (2004)
8. N.A. Shaposhnikov, *Mechanical Testing of Materials*, (Mashinostroenie, Moscow, 1954)
9. V.T. Troshhenko, L.A. Sosnovskiy, *Fatigue Strength of Metals and Alloys* (Naukova Dumka, Kiev, 1987)
10. A.S.G 01 N 3/08 *Device and method of testing material specimens on strain*, publ. 10.05.2005. bul. 13, 2005.
11. V.I. Mironov, O.A. Lukashuk, D.I. Vichuzhanin, *J. Sol. St. Phen.*, **815**, 815-820 (2017)
12. A.A. Lebedev, N.G. Chausov, *J. Strength Problems*, **2**, 6-10 (1983)
13. I.G. Emel'yanov, V.I. Mironov, *Durability of Shell Structures* (UrO RAN Publishing, Yekaterinburg, 2012)
14. V.F. Terentiev, *Fatigue Strength of Metals and Alloys* (Intermetals Engineering, Moscow, 2002)
15. G.V. Uzhik, *Fatigue and Endurance of Metals* (Foreign Literature Publishing, Moscow, 1963)
16. V.V. Struzhanov, V.I. Mironov, *Strain-softening of Materials in Structural Elements* (UrO RAN Publishing, Yekaterinburg, 1995)