The possibility of simplified modelling of radiation heat transfer within a steel porous charge

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Abstract. The article refers to the problem of calculating the effective thermal conductivity k_{ef} of a steel porous charge. In proposed approach for each heat transfer mechanism, which occurs in the considered medium, the corresponding thermal resistance is assigned. The model values of the k_{ef} coefficient were determined twice for the same input data (geometric dimensions, emissivity, temperature, thermal conductivity of steel and gas). The difference between successive calculations depended on the use of two different methods to determine the thermal radiation resistance R_{rd} . In the first approach the resistance R_{rd} was calculated using the exact method and in the second approach the simplified, approximate method was used. The discrepancy between the obtained results of k_{ef} in both approaches provides the evidence to use the approximate method to determine the resistance R_{rd} . A bundle of square steel sections were used to demonstrate the challenge. In the exact method, data regarding the temperature distribution within a single profile was used. These data were obtained based on experimental research using a guarded hot plate apparatus. The calculations were performed using a temperature range between 200°C and 800°C for two sections: 60×60mm and 80×80mm and three emissivities: 0.5, 0.7 and 0.9.

1 Introduction

Heat treatment operations of steel products have significant bearing on all key performance metrics of the plant, i.e.: productivity, energy consumption, product quality and emission of pollutants. This causes that heat treatment parameters should be selected very carefully. In the metallurgical industry for control and design of heat treatment parameters special numerical models have been used for more than three decades [1-3]. Such models predict spatial and temporal changes in the temperature of the charge. One of the challenges of this prediction is having required knowledge about the thermal properties of the heated charge. This issue becomes particularly complicated when the charge with a porous structure (e.g bundles of long elements or wire coils) is treated. Such kind of charge is a granular medium with the gas phase filling the gaps in between the steel elements. Therefore, during the heat treatment the following complex processes take place simultaneously: conduction in steel, conduction in gas, contact conduction, free convection and thermal radiation. As a result, the effective thermal conductivity k_{ef} becomes the key thermal property of the porous charge. This parameter is commonly used in the theory of porous media [4, 5].

The effective thermal conductivity of the porous material can be calculated by applying the model which

is based on the analysis of thermal resistance for individual modes of heat transfer [6, 7]. If the high-temperature process is applied to the above scenario, one of the heat transfer modes to be considered in the k_{ef} model is thermal radiation. The thermal resistance of radiation R_{rd} in this model can be calculated in two ways: exact and approximate. In the exact approach, resistance R_{rd} is determined using the radiosity method. The main challenge comes down to solving the system of equations. Obtaining a solution by this method requires knowledge on the temperature values of all the surfaces that close the space of radiative heat transfer. To obtain information on changes in the temperature distribution within the heated porous charge, experimental investigations are necessary. Therefore, this is a significant drawback of this method. This problem is eliminated in the approximate method as this utilises one simple equation in which only the average temperature is applied.

This paper compares the two methods of calculating the effective thermal conductivity of a porous charge. The first, thermal radiation resistance is determined using the exact method, the second the approximated method. The analysis was carried out for the porous charge of a steel square section bundle which is illustrated in Fig. 1. Due to the high porosity of this charge, the fraction of thermal radiation in the total heat transfer is particularly high in this case.

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Fig. 1. Bundle of square sections as an example of a steel porous charge.

2 Analysis and modelling

When analyzing the thermal radiation in the area of the bundle, the process within each profile needs to be considered. This is radiation heat transfer in a three-surface enclosure as illustrated in Fig 2. It is assumed that the temperature of the bottom surface A_1 is T_1 , the temperature of the top surface A_2 is T_2 and the temperature of the lateral surface A_3 is T_3 and $T_1 > T_3 > T_2$.

Fig. 2. The layout of the surfaces considered in the analysis of radiation heat transfer in the square section.

Since the enclosure is square the following relationship takes place: $A_1 = A_2 = 0.5 A_3$. It is also assumed that the surfaces of the enclosure are opaque, diffuse, grey and its radiative properties are the same and expressed by the emissivity ε ($\varepsilon_1 = \varepsilon_2 = \varepsilon_3$). The net heat flux of radiation q_{rd} in this system is [8]:

$$q_{rd} = J_1 - J_2 \quad (1)$$

were J_1 and J_2 represents radiosity of the surfaces A_1 and A_2 respectively. The radiation resistance R_{rd} for this system is expressed by the following relationship:

$$R_{rd} = \frac{T_1 - T_2}{q_r} = \frac{T_1 - T_2}{J_1 - J_2} \quad (2)$$

Therefore, the challenge in calculating the resistance R_{rd} in the exact method is to determine the radiosities J_i of all surfaces in the considered enclosure, which are described by the following equations:

$$J_1 = E_1 - F_{2-1}\rho J_2 - F_{3-1}\rho J_3 \quad (3a)$$

$$J_2 = E_2 - F_{1-2}\rho J_1 - F_{3-2}\rho J_3 \quad (3b)$$

$$J_3 = E_3 - F_{1-3}\rho J_1 - F_{2-3}\rho J_2 - F_{3-3}\rho J_3 \quad (3c)$$

where $\rho = 1 - \varepsilon$ is surface reflectivity.

The emissive power per unit area of each surface E_i is described by the equation [8]:

$$E_i = \varepsilon \sigma T_i^4 \quad (4)$$

where σ is Stefan-Boltzmann constant.

The view factors F_{i-j} used in equations (3) for the surface layout under consideration have the following values:

$$F_{1-2} = F_{2-1} = F_{3-3} = 0.4 \quad (5a)$$

$$F_{1-3} = F_{2-3} = 0.6 \quad (5b)$$

$$F_{3-1} = F_{3-2} = 0.3 \quad (5c)$$

In order to determine each radiosity, the system of equations (3) comes down to the matrix form:

$$\mathbf{E} = \mathbf{C} \times \mathbf{J} \quad (6a)$$

where

$$\mathbf{E} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad (6b)$$

$$\mathbf{C} = \begin{bmatrix} 1 & -F_{2-1}\rho & -F_{3-1}\rho \\ -F_{1-2}\rho & 1 & -F_{3-2}\rho \\ -F_{1-3}\rho & -F_{2-3}\rho & 1 - F_{3-3}\rho \end{bmatrix} \quad (6c)$$

$$\mathbf{J} = \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} \quad (6d)$$

In the approximate approach, the resistance R_{rd} is calculated using a single equation. In this equation there is only one temperature T_m and its value is the average for the whole enclosure:

$$R_{rd} = \frac{X_{rd}}{4\sigma T_m^3} \quad (7)$$

where X_{rd} is a dimensionless coefficient with the value depending on the emissivity and the shape, as well as the relative position of the surfaces that represent the boundaries for the space [9]. For square enclosure $X_{rd} = 1/\varepsilon$.

The effective thermal conductivity of the analyzed charge is calculated using the definition of conduction resistance for the flat layer with dimension l [10]:

$$k_{ef} = \frac{l}{R_{to}} \quad (8)$$

where R_{to} is the total thermal resistance of the considered medium.

For the section bundle the l represents the sum of section dimension in the direction of heat flow and the width of the gap between the individual sections. While resistance R_{to} is the sum of the section thermal resistance R_{st} and thermal resistance of the gap R_{gp} :

$$R_{to} = R_{st} + R_{gp} \quad (9)$$

The methodology for calculating the section thermal resistance R_{st} is described in the publication [11]. While calculating this parameter, the R_{rd} resistance needs to be taken into account. Whereas the gap thermal resistance R_{gp} is calculated by the polynomial [12]:

$$R_{gp} = (B_1 t^2 + B_2 t + B_3) \cdot 10^{-4} \quad (10)$$

The values of coefficients B_i from this polynomial depend on the surface physical state of the adjacent sections.

3 Results and discussion

The calculations presented below were performed for two square sections: 60 mm and 80 mm with the wall thickness of 3 mm and three emissivities: 0.5, 0.7 and 0.9. The temperatures of the individual section surfaces required for the calculation of the resistance R_{rd} in the exact method were obtained from experimental research. These measurements were taken while determining the effective thermal conductivity of the section bundles using the guarded hot plate apparatus [13]. During these experiments, temperatures t_1 and t_2 were captured using thin jacked thermocouples within the selected sections. Measurements points were located on opposite surfaces of the section, perpendicular to the direction of the heat flow. The results of the measurements were used to calculate the temperature differences $\Delta t = t_1 - t_2$, which corresponds to the mean temperature $t_m = 0.5 \cdot (t_1 + t_2)$. The values of the parameter Δt in the function of the temperature t_m obtained for the sections 60 and 80mm are shown in Fig. 3.

Fig. 3. Temperature differences within the 60 and 80mm sections obtained from the experimental research.

The data presented in Fig. 3 was then approximated by linear regression equations. The following relationships were obtained for each section:

$$\Delta t_{60} = 0.1094 t_m - 8.45 \quad (11a)$$

$$\Delta t_{80} = 0.1681 t_m - 8.94 \quad (11b)$$

The equations (11) were used to determine the temperatures for each surfaces of the section (T_1 , T_2 and T_3) which are necessary for calculating the R_{rd} resistance in the exact method:

$$T_1 = (t_m + 273) + 0.5\Delta t \quad (12a)$$

$$T_2 = (t_m + 273) - 0.5\Delta t \quad (12b)$$

$$T_3 = (t_m + 273) \quad (12c)$$

Fig. 4 presents the results of radiation resistance calculations for the considered scenarios, which were obtained by both methods (exact and approximate). It was observed that the results for both sections using the same method (Figs. 4a and 4b) were very similar with only a very minor difference of 0.1%. However, there are significant differences between the exact and the approximate methods for the same sections, with the exact method producing higher values. These differences in the results of both methods depend on the emissivity. When the emissivity decreases greater differences are observed.

In order to analyze the deviations in detail, the percentage excess of radiation resistance ΔR_{rd} were calculated:

$$\Delta R_{rd} = \frac{R_{rd-ex} - R_{rd-ap}}{R_{rd-ap}} \cdot 100\% \quad (13)$$

where R_{rd-ex} is the resistance obtained using the exact method, whereas R_{rd-ap} is the resistance obtained in the approximate method.

Fig. 4. Resistance R_{rd} calculated by exact and approximate method for: a) section 60 mm, b) section 80 mm.

The results of the ΔR_{rd} parameter obtained for the 80mm section are presented in Fig. 5. It can be noted that this parameter is almost constant for each emissivity in a temperature function. Very similar results were obtained for the 60 mm section. This implies that for a given emissivity the average value of this parameter can be applied.

Fig. 5. Percentage excess of resistance R_{rd} for the 80 mm section.

Thus, the difference in R_{rd} resistance between the exact and the approximate method is reduced linearly with the increase in emissivity, reaching zero for the black body surface. However, for a square enclosure with an emissivity of 0.5, the R_{rd-ex} value is about 20% greater than the R_{rd-ap} value.

Fig. 6. The percentage excess of radiation resistance for the considered sections versus emissivity.

In line with the purpose of this article, it was decided to evaluate how the observed difference in R_{rd-ex} and R_{rd-ap} values affects the value of the effective thermal conductivity of the square section bundle. In these calculations it was assumed that the thermal conductivity of steel k_s and the thermal conductivity of gas phase k_g change with temperature in the following relationships:

$$k_s = 1.2 \cdot 10^{-8} t^3 - 3.2 \cdot 10^{-5} t^2 - 1.2 \cdot 10^{-2} t + 51.3 \quad (14)$$

$$k_g = -2.88 \cdot 10^{-8} t^2 + 8.05 \cdot 10^{-5} t + 0.024 \quad (15)$$

Equation (14) describes changes in thermal conductivity of low-alloy steel with carbon content of 0.2%, while equation (15) describes changes in thermal conductivity of air. These equations were determined through approximation of the literature data [14, 15].

The calculations of the coefficient k_{ef} were performed for the same scenarios as for the radiation resistance. To determine the gap thermal resistance R_{gp} , in equation (10), the following values of B_i coefficients were used:

- $B_1 = 2.31 \cdot 10^{-5}$;
- $B_2 = -0.0534$;
- $B_3 = 58.16$.

Calculations results for the effective thermal conductivity for the analyzed bundles are presented in Fig. 7. The obtained k_{ef} values for the 60 mm sections are within the 3.9÷8.8 W/(m²·K) range, whereas for the 80 mm sections the k_{ef} values are within the 3.5÷11.3 W/(m²·K) range. This coefficient increases as a function of temperature, and the dynamic of it increases with the increase in the emissivity. At the same time, higher values were obtained when the radiation resistance R_{rd-ap} was taken into account in the calculations.

Fig. 7. The coefficient k_{ef} calculated with both the exact and the approximate methods of the radiation resistance, results for: a) 60 mm section bundle, b) 80 mm section bundle.

In order to demonstrate the influence of the radiation resistance value on the effective thermal conductivity, the percentage difference between the k_{ef} values obtained for both scenarios was calculated. This parameter is defined as follows:

$$\Delta k_{ef} = \frac{k_{ef-ap} - k_{ef-ex}}{k_{ef-ex}} \cdot 100\% \quad (16)$$

where k_{ef-ap} denote the k_{ef} value calculated taking into account the resistance R_{rd-ap} , and k_{ef-ex} denote the k_{ef} value calculated taking into account the resistance R_{rd-ex} . The results of the calculations for the Δk_{ef} for each bundle are presented in Fig. 8.

The value of the Δk_{ef} for the 60mm section bundle in 200°C temperature was not greater than 1.5%. However, when the temperature increased to 800°C the obtained values were between 1.7% and 8.7%. The highest values were observed for the emissivity of 0.5. For the emissivity of 0.7 the maximum value was 5.3%. A similar trend was observed for the 80mm sections. For 200° the maximum value obtained was 2.6%, whilst for 800°C and the successive emissivities obtained values were 10.6%, 6.1% and 1.9% respectively.

Fig. 8. Values of the Δk_{ef} parameter for: a) 60 mm section bundle, b) 80 mm section bundle.

The values of the Δk_{ef} parameter averaged for the whole temperature range obtained for all the analyzed scenarios are summarized in Table 1.

Table 1. The averaged values of Δk_{ef} for each bundles.

Emissivity	Sections 60 mm	Sections 80 mm
0.5	5.0%	7.2%
0.7	3.4%	4.7%
0.9	1.2%	1.6%

As it can be seen, the use of the approximate method for calculations the radiation resistance in the model of the effective thermal conductivity, in relation to the entire temperature range, exceeds the maximum value of the k_{ef} coefficient by approximately 7%. However, this applies to the emissivity of 0.5, while the surface of steel profiles subjected to heat treatment is usually characterized by the emissivity above 0.7. Therefore, the overestimation of the value of k_{ef} using the approximate method will be less than 5%. It can be stated that for industrial needs, the accuracy of calculations at this level is sufficient.

However, in a situation where more accurate calculations of the k_{ef} coefficient are required, the radiation resistance can be calculated using the approximate method. In order to apply this approach, the use of an appropriate correction term is required, which takes into account the linear impact of the emissivity:

$$R_{rd-cr} = (1.39 - 0.39\varepsilon) \frac{X_{rd}}{4\sigma T_m^3} \quad (17)$$

Due to the use of equation (17), the mathematical model of the effective thermal conductivity avoids solving the system of equations (6a), however, the results of these calculations will not be overstated as observed with equation (7).

4 Conclusions

The problem presented in this article is related to the optimization of heat treatment of the square steel profiles heated in the form of bundles. Due to the porous structure of this charge, its basic thermal property is effective thermal conductivity. This parameter quantifies the ability of the bundle to transmit heat as a result of the complex processes of conduction in steel and air, contact conduction, thermal radiation and free convection. The calculations of the k_{ef} coefficient were performed for two scenarios, which differ in the way of their modeling (exact and approximate) for radiation resistance. It has been shown that with the approximate method of determining resistance R_{rd} , the results of the k_{ef} calculations are overstated by an average of about 5%. In order to eliminate this discrepancy an updated version of the approximate equation was proposed, in which a corrective term was added. This term takes into account the linear impact of emissivity on the value of radiation resistance R_{rd} .

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