

Heat transfer coefficient determination using the FEM with time-dependent Trefftz-type basis functions in subcooled flow boiling in a minichannel

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Abstract. Results concerning flow boiling heat transfer in a vertical minichannel of 1.7 mm depth were shown. The channel was asymmetrically heated by a thin foil. Its surface temperature was recorded continuously in points by thermocouples. Measurements were carried out in 0.01 s intervals. The objective of the numerical calculations was to determine the heat transfer coefficient on the heated foil–fluid contact surface in the minichannel from the Robin boundary condition. Both the foil and fluid temperatures were the result of solving the nonstationary two-dimensional problem in the foil and flowing fluid. The problem was solved by using the FEM combined with Trefftz-type basis functions. The values of the time-dependent local heat transfer coefficient were presented and discussed.

1 Introduction

Miniaturization of devices is being progressively applied in cooling technologies designed to prevent exceeding operating temperatures. A compact heat exchanger responds to the issues of transfer in confined spaces in which conventional size channels cannot be used. Extensive efforts to recognize boiling phenomena in minichannels include theoretical analyses based on experimental measurements. The results can be applied to the construction of compact heat exchangers and using them in heating, cooling and thermoregulation applications.

In order to estimate the intensity of heat transfer accompanying flow boiling in a mini heat exchanger device, the main experimental data are needed for the heat transfer coefficient identification: the heated wall temperature, the temperature gradient and the temperature of the fluid flowing along a minichannel. These quantities were obtained by solving the inverse problem [1] in the heated wall and in the flowing fluid. The method proposed by Trefftz [2] can be used to solve the inverse problems. This approach is based on an approximation of the unknown solution by a linear combination of functions that exactly satisfy the differential equation.

Details of the method based on the Trefftz functions can be found in [3]–[12]. In this work, to solve the non-stationary two-dimensional problem in the flowing fluid the time-dependent Trefftz functions for the Fourier–Kirchhoff equation were determined. These functions were used to construct the nonstationary FEM basis functions.

2 Experiment

The test section with two parallel minichannels (Fig. 1) is the main element of flow loop realized in the experimental stand, which view is shown in Fig. 2.

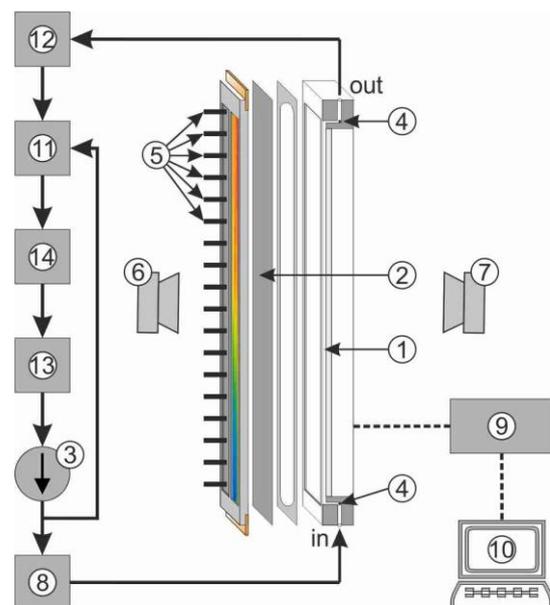


Fig. 1. The main elements of the experimental stand: 1 - a minichannel, 2 - a heated foil, 3 - a gear pump; 4,5 - thermocouples, 6 - an infrared camera, 7 - a high speed camera, 8 - a Coriolis mass flow meter, 9 - data acquisition stations, 10 - a pc computer, 11 - a compensating tank/ a pressure regulator, 12 - a heat exchanger, 13 - a filter, 14 - a deaerator.

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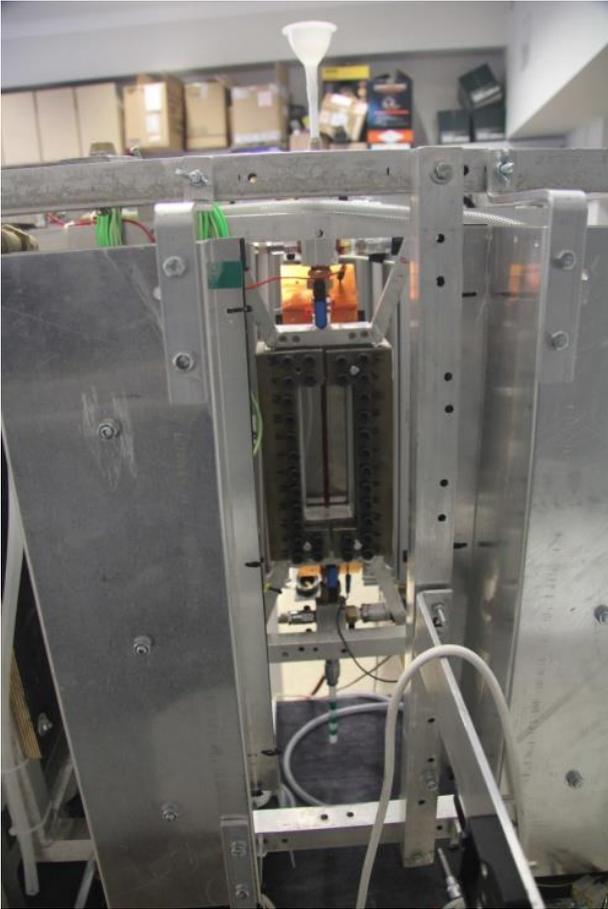


Fig. 2. The view of the experimental stand.

The components of the test section are two 26.5 mm wide, 180 mm long and 1.7 mm deep minichannels (1). A 0.1 mm thick foil (2), made of Haynes-230 alloy is heated while FC-72 Fluorinert flowing along the minichannels thanks to the gear pump (3). In the selected minichannel temperature of the outer side of the foil is measured with 18 points by K-type thermocouples (5). Two other thermocouples control fluid temperature at the inlet and outlet of the minichannels (4). Foil temperature of the second minichannel is monitored by an infrared camera (6). The opposite surface of the heated foil is observed through the glass using high-speed camera (7), simultaneously. A mass flow meter (8) is used to control the fluid flow in the flow loop. Pressure at the minichannel inlet and outlet, current supplied to the foil and voltage drop across a foil were also monitored continuously.

The supply and control system contains: an inverter welder, a shunt, an ammeter and a voltmeter. The data acquisition system consists of two data acquisition stations (9) and a computer (10) with a appropriate software.

During an experiment series, there was a gradual increase in the electric power supplied to the foil. All measurements were carried out in 0.01 s intervals.

3 Analysis and modelling

The objective of the numerical calculations, refer to an experiment, was to determine the heat transfer coefficient during FC-72 flow boiling in a minichannel from the Robin boundary condition:

$$\alpha(x,t) = \frac{-\lambda_f \frac{\partial T_f(x, \delta_f, t)}{\partial y}}{T_f(x, \delta_f, t) - T_f(x, \delta_f + \delta_t, t)} \quad (1)$$

where λ_f – the thermal conductivity coefficient of the foil, δ_f – the foil thickness, T_f – the foil temperature, T_f – the fluid temperature, δ_t – the thickness of the thermal boundary layer, x – the spatial variable referred to the flow direction, y – the spatial variable referred to the direction perpendicular to the flow direction (referring to the thickness of the heated foil).

Both the foil and fluid temperatures were the result of solving the inverse nonstationary two-dimensional problem in two neighboring domains: the heated foil and flowing fluid. It was assumed that nonstationary temperature distribution in the heated foil $(x, y) \in \Omega_f$ for $t > 0$ was described by the heat equation

$$\nabla^2 T_f - \frac{\rho_f c_{p-f}}{\lambda_f} \frac{\partial T_f}{\partial t} = -\frac{I \cdot \Delta U}{A \cdot \delta_f \cdot \lambda_f} \quad (2)$$

whereas in the thermal boundary layer of fluid $((x, y) \in \Omega_T)$ for $t > 0$ by the Fourier–Kirchhoff equation

$$\nabla^2 T_f - w_x(y) \frac{\partial T_f}{\partial x} - \frac{\rho_f c_{p-f}}{\lambda_f} \frac{\partial T_f}{\partial t} = 0 \quad (3)$$

The boundary conditions have the form:

$$T_f(x, y, 0) = T_{0-f}(x, y) \quad (4)$$

$$T_f(x, y, 0) = T_{0-f}(x, y) \quad (5)$$

$$T_f(x_p, 0, t_k) = T_p(t_k) \text{ for } p = 1, 2, \dots, P, k = 1, 2, \dots, K \quad (6)$$

$$\frac{\partial T_f}{\partial x}(0, y, t) = 0 \quad (7)$$

$$\frac{\partial T_f}{\partial x}(L, y, t) = 0 \quad (8)$$

$$\lambda_f \frac{\partial T_f}{\partial y}(x, 0, t) = q_{\text{loss}}(t) \quad (9)$$

$$T_f(x, \delta_f, t) = T_f(x, \delta_f, t) \quad (10)$$

$$T_f(0, y, t) = T_f^{\text{in}}(t) \quad (11)$$

$$T_f(L, y, t) = T_f^{\text{out}}(t) \quad (12)$$

where $\Omega_F = \{(x, y) \in R^2 : 0 < x < L, 0 < y < \delta_F\}$,
 $\Omega_T = \{(x, y) \in R^2 : 0 < x < L, \delta_F < y < \delta_F + \delta_T\}$, L – the
 minichannel length, I – the electrical current, ΔU – the
 voltage drop, A – the surface area of the heated foil,
 P – the number of measurements, K – the number of
 time intervals, T_p – the foil temperature measured by
 thermocouples at the boundary $y=0$, q_{loss} – the heat
 loss to the surroundings [13], w_x – component of vector
 fluid velocity parallel to the minichannel heated surface
 [14], ρ_F , ρ_f – the density of the foil and fluid,
 respectively, c_{p-F} , c_{p-f} – specific heat of the foil and
 fluid, respectively, λ_f , δ_f , δ_T – defined as for Eq. (1).
 The boundary conditions are shown in Fig. 3.

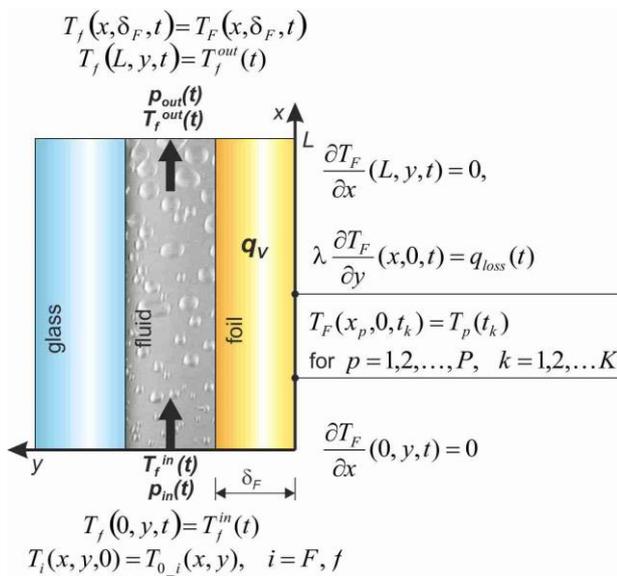


Fig. 3. Boundary conditions.

The solution method of Eqs. (2-3) presented in this
 article is based on an approach described in [15]
 concerning to the one-dimensional heat equation.

The problem described in Eqs. (3), (4), (10), (11) and
 (12) in the thermal boundary layer of fluid Ω_T was
 solved in the time subintervals by means of the FEM
 with time-dependent basis functions. This base functions
 are a linear combinations of Trefftz functions, strictly
 satisfying the governing equation, and have the
 following properties in nodes:

$$f_{jk}(x_i, y_i, t_i) = \delta_{ki}, \quad i=1,2,\dots,lw \quad (13)$$

where δ_{ki} – the Kronecker delta, j – the element number,
 k – the basis function number in the j -th element,
 lw – number of nodes in the j -th element.

The Trefftz functions are coefficients at partial
 derivatives in expansion of the differential equation
 solution in the Taylor series, in which the derivative
 $\frac{\partial^2 T_f}{\partial y^2}$ is replaced by the formula resulting from the
 Fourier–Kirchhoff equation:

$$\frac{\partial^2 T_f}{\partial y^2} = w_x(y) \frac{\partial T_f}{\partial x} + \frac{\rho_f c_{p-f}}{\lambda_f} \frac{\partial T_f}{\partial t} - \frac{\partial^2 T_f}{\partial x^2} \quad (14)$$

The fluid temperature in each time – space element
 Ω_T^j is approximated by a linear combination of time-
 dependent Trefftz-type basis functions $f_{jk}(x, y, t)$:

$$\tilde{T}_f^j(x, y, t) = \sum_{k=1}^{lw} T_f^n f_{jk}(x, y, t) \quad (15)$$

where n is the node number in the entire domain Ω_T ,
 T_f^n denotes the approximate values of the fluid
 temperature in nodes j, k , lw are defined as for Eq. (13).

The unknown temperature values at nodes T_f^n were
 calculated by minimizing the functional, similar to this
 described in [16]. It represents the mean square error of
 the approximate solution on the boundary, in the initial
 time and along common edges of neighboring elements.

The set of Eqs. (2) and (5-9) was solved like in [16].

4 Results and discussion

The study shows the results obtained for the subcooled
 boiling region and the most important concern was to the
 heat transfer coefficient identification. The coefficient
 was found by solving the inverse heat conduction
 problem by using the FEM with time-dependent Trefftz-
 type basis functions. The calculations were performed
 using two minutes time-dependent temperature
 measurements but only selected results - from 1 s to
 100 s - were presented. This data were selected to
 present the results obtained for the subcooled boiling
 region.

The relationship between the heat transfer coefficient
 and the distance from the minichannel inlet, were
 determined for three times: $t_{10} = 10$ s, $t_{50} = 50$ s and
 $t_{80} = 80$ s and shown in Fig. 4a. Figure 4b presents the
 heat transfer coefficient vs. time calculated on the basis
 of temperature measurements from six selected
 thermocouples (T2, T5, T8, T11, T14 and T17) with the
 increasing heat flux being supplied to the foil. Results
 obtained for the points where listed above thermocouples
 were selected as representative for the subcooled boiling
 region data.

When analysing the dependence transfer coefficient
 vs. the distance from the minichannel inlet shown in
 Fig. 4a, it can be noticed that the highest values of the
 heat transfer coefficient up to 1.6 kW/(m²K) are
 achieved near the outlet of the minichannel and the
 lowest are obtained at its inlet. It is worth mentioning
 that presented results concerns first part of experiment
 when the lowest heat flux was supplied to the heated
 foil.

The results in Fig. 4b are presented as the heat
 transfer coefficient vs. time. The local heat transfer
 coefficients determined on the basis of the foil
 temperature measured at six selected points measured
 between 1 s and 100 s were shown in this graph. It was
 observed that local heat transfer coefficient values

increased with increasing distance from the minichannel and achieved the highest values up to 1.6 kW/(m²K), at the minichannel outlet and the lowest were near the minichannel inlet approx. 0.2 kW/(m²K).

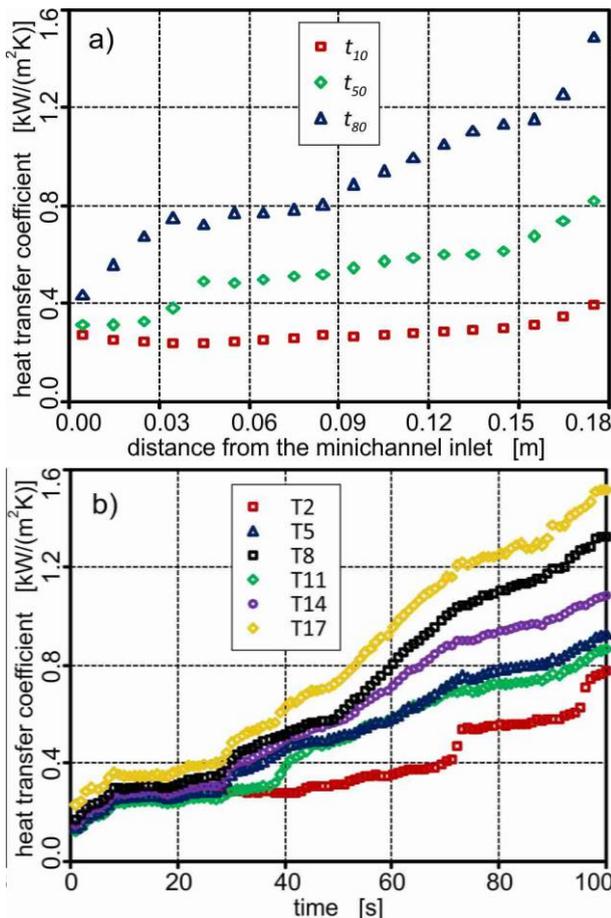


Fig. 4. Heat transfer coefficient vs.:
 a) the distance from the minichannel inlet for selected time intervals : $t_{10} = 10$ s, $t_{50} = 50$ s and $t_{80} = 80$ s,
 b) time calculated on the basis of temperature measurements from selected thermocouples: T2, T5, T8, T11, T14 and T17 ; experimental parameters: heat flux density from 1.8 kW/m² to 46 kW/m², average inlet pressure 122 kPa, average mass flow rate 0.012 kg/s.

5 Conclusions

The method of solving the time-dependent inverse heat transfer problem in flowing fluid using the FEM with time-dependent Trefftz-type basis functions were discussed in this paper. The objective of the numerical calculations, refer to an experiment, was to determine the heat transfer coefficient during FC-72 flow in an asymmetrically heated minichannel. Only results concerned subcooled boiling region was discussed. The heated foil surface temperature was recorded continuously in 18 points by thermocouples. Other experimental parameters necessary for calculations were also monitored in 0.01 s intervals. The heat transfer coefficient, on the heated foil–fluid in the minichannel, was obtained from the Robin boundary condition. It was assumed that the nonstationary temperature distribution

in the flowing fluid was described by the Fourier–Kirchhoff equation. The unknown temperature values at nodes were computed by minimizing the functional which describes the mean square error of the approximate solution on the boundary, in the initial time and along common edges of adjacent elements. The auxiliary nonstationary temperature distribution in the heated foil, described by the heat equation, was obtained by Trefftz method. The values of the time-dependent local heat transfer coefficient were discussed. It was confirmed that under the subcooled boiling, local heat transfer coefficients achieved rather values up to 1.6 kW/(m²K), at the minichannel outlet. The lowest values of the coefficient were obtained at the minichannel inlet.

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