Mathematical modeling of physical processes in the complex for testing of induction machines

Viktor Kharlamov^{1,*} and Denis Popov¹

¹Omsk State Transport University, 644046, Marx av., 35, Omsk, Russia

Abstract. The paper is devoted to the simulation of the test complex designed for energy-efficient load testing of induction machines by the method of mutual load with the exchange of electrical energy through the network. It is noted that for other similar test schemes, the mathematical model will have a slightly different form, but it will be identical in terms of asynchronous machines, network and frequency converter. The compiled mathematical model of the test complex allows studying the variable parameters of the system in all elements of the test scheme in static and dynamic modes of operation as well. The synthesized mathematical model can be used to determine the parameters of the equipment in the designed test complexes if the parameters of the test and load machines are known. The results of simulation of the test complex for the given parameters of the test and load induction machines are obtained.

1 Introduction

Nowadays, one of the priority areas for the development of industries related to electromechanical energy converters is the introduction of asynchronous converters in exchange for DC machines. This direction of technology development is supported both at the state and industry level and involves not only expanding the scope of use of induction machines but also introducing innovation systems for their diagnosis and monitoring.

One of the main factors in reducing costs in this process of modernization of production is energy saving. Therefore, the actual task is the development and implementation of innovative energy-efficient complexes designed to test asynchronous motors after repair, before commissioning, in accordance with existing requirements [1, 2] and according to a certain program [3].

In order to save electric energy in the process of testing DC motors, the back-to-back method has long been widely used [4, 5], which has not yet found wide application for testing asynchronous motors. However, for the implementation of this energy-efficient test method, a number of schemes [6-9] have now been developed that make it possible to create a complex for load testing of asynchronous motors.

Each of the listed technical solutions [6-9] has its advantages and disadvantages, which determine the expediency of their application in one case or another [9].

* Corresponding author: hvv-omgups@mail.ru

The introduction of these schemes into the production process requires studying the features of their work and developing techniques for determining the parameters of scheme elements with known parameters of motors that are being repaired at a specific plant.

To study the developed schemes, it is necessary to compile mathematical models of the physical processes that take place in them.

2 Materials and Methods

Let us consider the principle of mathematical modeling of the operation of these schemes by the example of one of them [6, 7]. Note that the operation of the remaining schemes can be modeled in the same way. Some common elements of mathematical models will be completely identical (network, asynchronous motors, frequency converter).

Studies [10, 11] consider a mathematical model of physical processes in the interaction of two induction machines, one of which operates in the motor mode and the other in the generator mode. This model can be extended to take into account losses in the frequency converter, for example, as suggested in [12-14].

Let's consider a mathematical model without taking into account the losses in the converter elements, since this refinement requires the presence of results of specific measurements with a specific type of induction machine. For simplicity of analysis, we divide the mathematical model under consideration into three parts.

The first part of the mathematical model describes the electromagnetic processes occurring in the first electric machine and has the following form:

$$\left\{ \begin{aligned} \frac{di_{1a}}{dt} &= \frac{1}{L_1\sigma_1} \left[u_{1a} - i_{1a}R_1 + k_1 \left[i'_{2a}R'_2 + \frac{p\omega}{\sqrt{3}}(L_{12}(i_{1b} - i_{1c}) + L_2(i'_{2b} - i'_{2c})) \right] \right]; \\ \frac{di_{1b}}{dt} &= \frac{1}{L_1\sigma_1} \left[u_{1b} - i_{1b}R_1 + k_1 \left[i'_{2b}R'_2 + \frac{p\omega}{\sqrt{3}}(L_{12}(i_{1c} - i_{1a}) + L_2(i'_{2c} - i'_{2a})) \right] \right]; \\ \frac{di_{1c}}{dt} &= \frac{1}{L_1\sigma_1} \left[u_{1c} - i_{1c}R_1 + k_1 \left[i'_{2c}R'_2 + \frac{p\omega}{\sqrt{3}}(L_{12}(i_{1a} - i_{1b}) + L_2(i'_{2a} - i'_{2b})) \right] \right]; \\ \frac{di'_{2a}}{dt} &= \frac{-1}{L_2\sigma_1} \left[k_1(u_{1a} - i_{1a}R_1) + \left[i'_{2a}R'_2 + \frac{p\omega}{\sqrt{3}}(L_{12}(i_{1b} - i_{1c}) + L_2(i'_{2b} - i'_{2c})) \right] \right]; \\ \frac{di'_{2b}}{dt} &= \frac{-1}{L_2\sigma_1} \left[k_1(u_{1b} - i_{1b}R_1) + \left[i'_{2b}R'_2 + \frac{p\omega}{\sqrt{3}}(L_{12}(i_{1c} - i_{1a}) + L_2(i'_{2c} - i'_{2a})) \right] \right]; \\ \frac{di'_{2c}}{dt} &= \frac{-1}{L_2\sigma_1} \left[k_1(u_{1c} - i_{1c}R_1) + \left[i'_{2c}R'_2 + \frac{p\omega}{\sqrt{3}}(L_{12}(i_{1a} - i_{1b}) + L_2(i'_{2a} - i'_{2b})) \right] \right]. \end{aligned} \right. \quad (1)$$

The second part of the mathematical model determines the total speed of rotation of mechanically coupled electric machines, connects electromagnetic moments, moments of resistance and moments of inertia of these machines, and has the following form:

$$\begin{aligned} \frac{d\omega}{dt} &= \frac{-p}{J\sqrt{3}} \{ L_{12}[i_{1a}(i'_{2b} - i'_{2c}) + i_{1b}(i'_{2c} - i'_{2a}) + i_{1c}(i'_{2a} - i'_{2b})] + \dots \\ &\dots + L_{21}[i_{2a}(i'_{2b} - i'_{2c}) + i_{2b}(i'_{2c} - i'_{2a}) + i_{2c}(i'_{2a} - i'_{2b})] \} - \frac{M_{loss}}{J} \text{sign}(\omega). \end{aligned} \quad (2)$$

The third part of the mathematical model describes the electromagnetic processes occurring in the second electric machine and has the following form:

$$\left\{ \begin{aligned} \frac{di_{21a}}{dt} &= \frac{1}{L_{21}\sigma_2} \left[u_{21a} - i_{21a}R_{11} + k_{2r} \left[i'_{22a}R_{22}' + \frac{p\omega}{\sqrt{3}} (L_{212}(i_{21b} - i_{21c}) + L_{22}(i'_{22b} - i'_{22c})) \right] \right]; \\ \frac{di_{21b}}{dt} &= \frac{1}{L_{21}\sigma_2} \left[u_{21b} - i_{21b}R_{11} + k_{2r} \left[i'_{22b}R_{22}' + \frac{p\omega}{\sqrt{3}} (L_{212}(i_{21c} - i_{21a}) + L_{22}(i'_{22c} - i'_{22a})) \right] \right]; \\ \frac{di_{21c}}{dt} &= \frac{1}{L_{21}\sigma_2} \left[u_{21c} - i_{21c}R_{11} + k_{2r} \left[i'_{22c}R_{22}' + \frac{p\omega}{\sqrt{3}} (L_{212}(i_{21a} - i_{21b}) + L_{22}(i'_{22a} - i'_{22b})) \right] \right]; \\ \frac{di'_{22a}}{dt} &= \frac{-1}{L_{22}\sigma_2} \left[k_{2s}(u_{21a} - i_{21a}R_{11}) + \left[i'_{22a}R_{22}' + \frac{p\omega}{\sqrt{3}} (L_{212}(i_{21b} - i_{21c}) + L_{22}(i'_{22b} - i'_{22c})) \right] \right]; \\ \frac{di'_{22b}}{dt} &= \frac{-1}{L_{22}\sigma_2} \left[k_{2s}(u_{21b} - i_{21b}R_{11}) + \left[i'_{22b}R_{22}' + \frac{p\omega}{\sqrt{3}} (L_{212}(i_{21c} - i_{21a}) + L_{22}(i'_{22c} - i'_{22a})) \right] \right]; \\ \frac{di'_{22c}}{dt} &= \frac{-1}{L_{22}\sigma_2} \left[k_{2s}(u_{21c} - i_{21c}R_{11}) + \left[i'_{22c}R_{22}' + \frac{p\omega}{\sqrt{3}} (L_{212}(i_{21a} - i_{21b}) + L_{22}(i'_{22a} - i'_{22b})) \right] \right]. \end{aligned} \right. \quad (3)$$

Equations (1) - (3) have the following parameters: the voltages supplied to the stator windings $u_{11a}, u_{11b}, u_{11c}, u_{21a}, u_{21b}, u_{21c}$, the active resistances of the stator winding R_{11} and R_{21} , the reduced resistance of the rotor windings R_{12}' and R_{22}' , the number of pole pairs p , inductance of stator and rotor windings $L_{11}, L_{12}, L_{21}, L_{22}$, mutual inductances L_{112} and L_{212} , the magnetic coupling ratios of the rotor and stator $k_{1r}, k_{1s}, k_{2r}, k_{2s}$, scattering coefficients σ_1 and σ_2 .

The following parameters are unknown in equations (1) - (3):

- stator currents of the first machine – $di_{11a}, di_{11b}, di_{11c}$;
- reduced rotor currents of the first machine – $di'_{12a}, di'_{12b}, di'_{12c}$;
- angular rotational speed of the rotor – ω ;
- stator currents of the second machine – $di_{21a}, di_{21b}, di_{21c}$;
- reduced rotor currents of the second machine – $di'_{22a}, di'_{22b}, di'_{22c}$;
- p – number of pairs of poles;
- M_{loss} – moment of losses;
- J – moment of inertia of induction motors.

However, the mathematical model consisting of equations (1) - (3) does not consider the processes occurring in the frequency converter (FC) and does not allow calculating the currents and power in its electrical circuits. To form a more complete mathematical model, we introduce the notation in accordance with the diagram given in fig. 1. The mathematical model of the voltage modeled by the frequency converter can be written in the form of the following expressions.

The voltage at the output of the rectifier, which is part of the power circuit of the FC:

$$u_V(t) = U_{lmax} \sin(2\pi ft + \pi/3 - \text{Int}(300t)\pi/3), \quad (4)$$

where U_{lmax} – peak value of the line voltage at the input of the FC;
 $\text{Int}(300t)$ – a function that separates the integer part of the number $(300t)$.

The sawtooth voltage generated in the control system of the FC:

$$u_s(t) = \begin{cases} U_{smax}(4f_s t - 4\text{Int}(f_s t) - 1), & \text{if } 0 \leq t - \text{Int}(f_s t)/f_s < 0,5/f_s; \\ -U_{smax}(4f_s t - 4\text{Int}(f_s t) - 3), & \text{if } 0,5/f_s \leq t - \text{Int}(f_s t)/f_s < 1/f_s, \end{cases} \quad (5)$$

where U_{smax} – peak value of the sawtooth voltage;
 f_s – sawtooth voltage frequency.

Fig. 1. Connection diagram of the test complex with the system asynchronous motor-asynchronous generator with the exchange of energy through the electric network.

The driving voltage in the control system that determines the frequency and the effective value of the alternating voltage generated at the output of the FC in three phases:

$$\begin{cases} u_{ctrl a}(t) = U_{ctrl max} \sin(2\pi f_{ctrl} t); \\ u_{ctrl b}(t) = U_{ctrl max} \sin(2\pi f_{ctrl} t - 2\pi/3); \\ u_{ctrl c}(t) = U_{ctrl max} \sin(2\pi f_{ctrl} t - 4\pi/3), \end{cases} \quad (6)$$

where $U_{ctrl max}$ – peak value of the driving voltage;

f_{ctrl} – driving voltage frequency.

The voltage generated at the output of the FC in phase “a”:

$$u_{1a}(t) = \begin{cases} \frac{2}{3} u_V(t) \text{sign}(u_{ctrl a}(t) - u_s(t)), & \text{if } \text{sign}(u_{ctrl b}(t) - u_s(t)) = \dots \\ \dots = \text{sign}(u_{ctrl c}(t) - u_s(t)) \neq \text{sign}(u_{ctrl a}(t) - u_s(t)); \\ \frac{1}{3} u_V(t) \text{sign}(u_{ctrl a}(t) - u_s(t)), & \text{if } \text{sign}(u_{ctrl b}(t) - u_s(t)) \neq \text{sign}(u_{ctrl c}(t) - u_s(t)); \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

Similarly, stresses are formed in phases “b” and “c”.

Expressions for currents in transistors of the frequency converter can be written in the form of the following equations with logical functions:

$$i_{VT1} = \begin{cases} i_{l1a}, & \text{if } (u_{l1a} > 0) \cap (i_{l1a} > 0); \\ 0, & \text{otherwise;} \end{cases} \quad (8)$$

$$i_{VT2} = \begin{cases} i_{l1b}, & \text{if } (u_{l1b} > 0) \cap (i_{l1b} > 0); \\ 0, & \text{otherwise;} \end{cases} \quad (9)$$

$$i_{VT3} = \begin{cases} i_{l1c}, & \text{if } (u_{l1c} > 0) \cap (i_{l1c} > 0); \\ 0, & \text{otherwise;} \end{cases} \quad (10)$$

$$i_{VT4} = \begin{cases} i_{l1a}, & \text{if } (u_{l1a} < 0) \cap (i_{l1a} < 0); \\ 0, & \text{otherwise;} \end{cases} \quad (11)$$

$$i_{VT5} = \begin{cases} i_{l1b}, & \text{if } (u_{l1b} < 0) \cap (i_{l1b} < 0); \\ 0, & \text{otherwise;} \end{cases} \quad (12)$$

$$i_{VT6} = \begin{cases} i_{l1c}, & \text{if } (u_{l1c} < 0) \cap (i_{l1c} < 0); \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

For currents in the valves, the following expressions are valid:

$$i_{VD1} = \begin{cases} i_{l1a}, & \text{if } (u_{l1a} > 0) \cap (i_{l1a} < 0); \\ 0, & \text{otherwise;} \end{cases} \quad (14)$$

$$i_{VD2} = \begin{cases} i_{l1b}, & \text{if } (u_{l1b} > 0) \cap (i_{l1b} < 0); \\ 0, & \text{otherwise;} \end{cases} \quad (15)$$

$$i_{VD3} = \begin{cases} i_{l1c}, & \text{if } (u_{l1c} > 0) \cap (i_{l1c} < 0); \\ 0, & \text{otherwise;} \end{cases} \quad (16)$$

$$i_{VD4} = \begin{cases} i_{l1a}, & \text{if } (u_{l1a} < 0) \cap (i_{l1a} > 0); \\ 0, & \text{otherwise;} \end{cases} \quad (17)$$

$$i_{VD5} = \begin{cases} i_{l1b}, & \text{if } (u_{l1b} < 0) \cap (i_{l1b} > 0); \\ 0, & \text{otherwise;} \end{cases} \quad (18)$$

$$i_{VD6} = \begin{cases} i_{l1c}, & \text{if } (u_{l1c} < 0) \cap (i_{l1c} > 0); \\ 0, & \text{otherwise.} \end{cases} \quad (19)$$

The current in the DC link of the FC can be found in the form of the following sum:

$$i_{DC} = i_{VT1} + i_{VT2} + i_{VT3} + i_{VD1} + i_{VD2} + i_{VD3}. \quad (20)$$

The voltage across the capacitor u_C is determined from the following system of equations:

$$\begin{cases} u_C = u_V, & \text{if } i_{DC} \geq 0; \\ du_C = -\frac{i_{DC} dt}{C}, & \text{if } i_{DC} < 0. \end{cases} \quad (21)$$

The current flowing through the capacitor circuit:

$$i_C = C \frac{du_c}{dt}. \quad (22)$$

The current flowing through the rectifier:

$$i_V = i_{DC} + i_C. \quad (23)$$

A complete mathematical model that allows determining the currents and voltages in all elements of the scheme of the test complex with the asynchronous motor - asynchronous generator system with energy exchange through the network consists of equations (1) - (23).

3 Results

The resulting mathematical model can be formalized in the form of a set of constants, functions and program modules in the Mathcad program, which allows obtaining time-based graphs of behaviour of the variables of interest for given constant parameters.

The calculation in the Mathcad program for induction machines of 5A type with a rated power of 0.37 kW and the number of pole pairs equal to four and a frequency converter with a modulation frequency of 4 kHz [15] and a frequency of the output voltage of 60 Hz allowed obtaining the following results shown in fig.2- 7.

Fig. 2. The phase current of the first induction machine (M1) operating in motor mode.

Fig. 3. The phase current of the second induction machine (M1) operating in generator mode.

Fig. 4. The phase voltage at the frequency converter output.

Fig. 5. The voltage on the capacitor.

Fig. 6. The current passing through the capacitor.

Fig. 7. The current output of the rectifier of the frequency converter.

4 Discussion

The obtained mathematical model allows studying physical processes in the considered scheme of tests of induction machines both in static and dynamic modes of operation.

Studying of static operating modes with application of the obtained mathematical model can be carried out as follows. When solving a system of equations consisting of expressions (1) - (3) under zero initial conditions, its unknowns (currents and rotational speed of the rotor of induction machines) will have variable values for a certain time interval, which are subsequently fixed unchanged. The change in the calculation of the initial conditions in the range from zero to nominal values, as a rule, does not lead to a change in the final result.

The study of dynamic operating modes with application of the obtained mathematical model can be carried out by setting the frequency of the voltage at the output of the frequency converter as a function of time $f_U = f(t)$, which can be, for example, piecewise linear. In this case, it will be necessary to make calculation at two time intervals. First, perform a calculation to the steady state mode of interest, and then set the dynamic mode of interest.

5 Conclusions

The obtained mathematical model of the electrical complex for testing induction machines with the exchange of electrical energy through the network can be used to solve the following engineering problems:

- determination of necessary equipment parameters with known parameters of the tested motors in the process of designing new test complexes according to the considered scheme;
- modeling of transient processes in electrical circuits of the test scheme with the purpose of virtual verification of possible algorithms for setting the load to the tested motor.

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