

Optimization of Tooth Root Profile Using Bezier Curve with G2 Continuity to Reduce Bending Stress of Asymmetric Spur Gear Tooth

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Abstract. This research describes simple and innovative approach to reduce bending stress at tooth root of asymmetric spur gear tooth which is desire for improve high load carrying capacity. In gear design at root of tooth circular-filleted is widely used. Blending of the involute profile of tooth and circular fillet creates discontinuity at root of tooth causes stress concentration occurs. In order to minimize stress concentration, geometric continuity of order 2 at the blending of gear tooth plays very important role. Bezier curve is used with geometric continuity of order 2 at tooth root of asymmetric spur gear to reduce bending stress.

1 Introduction

Gears are very important part in all of machinery used in current times. Gear design is an extremely difficult art. High load-carrying capacity, less expensive, quieter running, lighter weight, and longer life are the primary aim of present industry for making gears. Current focus of gear design in industry is on failure occurs at critical section/root of tooth. One major cause of gear failure is fracture at root of the gear tooth due to bending fatigue [1]. The bending strength is influenced by: load location at HPSTC, load value, number teeth on the gear, fillet radius at root of tooth etc.

Failure at root of tooth can be reduced by optimizing geometrical shape of standard symmetric gear tooth [1-2]. In order to achieve this goal, pressure angle on drive side profile (as compare to coast side) can be increases within constraint [3-4]. Due to different pressure angle on both sides of gear tooth gear becomes asymmetric gear. In asymmetric spur gear bending stress at tooth root is reduced as compare to standard symmetric spur gear [5-6].

For further reduction in bending stress, it is necessary to optimize root profile of asymmetric spur gear tooth. Blending of the involute profile of tooth and circular fillet creates discontinuity at root of tooth causes stress concentration occurs [7-8]. In order to minimize stress concentration, geometric continuity of order 2 at the blending of gear tooth plays very important role [9-10]. In this paper a bezier curve is used with geometric continuity of order 2 at tooth root to reduce bending stress of asymmetric spur gear tooth.

2 Mathematical model of gear tooth

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Drive side pressure angle increases, base circle radius for drive side decreases compare to coast side. Hence long involute profile is available on drive side from tip (point A) to base circle radius (point C).

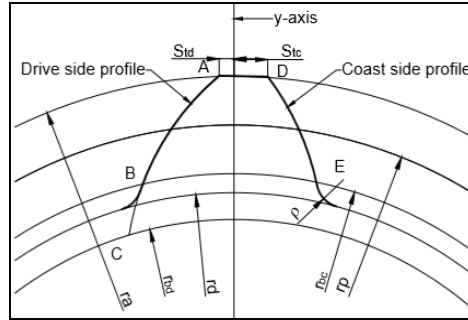


Figure 1. Asymmetric involute spur gear tooth

Whereas on coast side available profile is from tip (point D) to base circle radius (point E) as shown in figure 1.

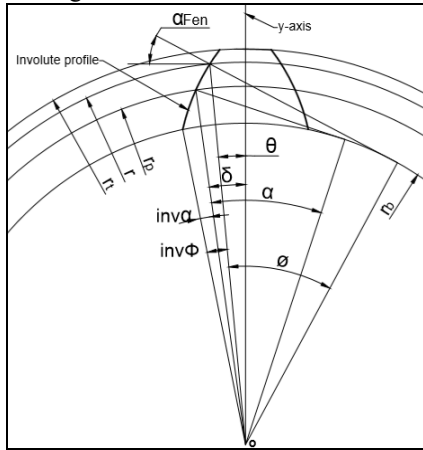


Figure 2(a). Representation of angle θ

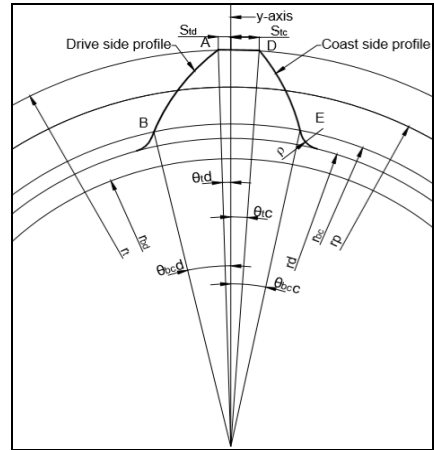


Figure 2(b). Mathematical representation of involute profile/gear tooth

Consider centre of gear as origin and vertical line passes through origin is a y axis. As shown in figure 2(a), θ is an angle at radius r w.r.t y-axis. Angle θ is a variable, which is required to develop involute profile of asymmetric spur gear tooth. Angle θ and radius r define location of points on the involute profile which is given by:

$$\theta = \frac{\pi}{2 \cdot z} + \tan \alpha - \alpha - \left(\tan \left(\cos^{-1} \left(\frac{r_p}{r} \cdot \cos \alpha \right) \right) - \left(\cos^{-1} \left(\frac{r_p}{r} \cdot \cos \alpha \right) \right) \right) \tag{1}$$

To develop drive and coast side profile, θ is varied between $\theta_{bc,d} \leq \theta \leq \theta_d$ and $\theta_{bc,c} \leq \theta \leq \theta_c$ respectively as shown in figure 2(b).

Involute profile coordinate (x_i, y_i) of an arbitrary point i for drive and coast side given by:

$$x_i = r_i \cdot \cos \theta_i$$

$$y_i = r_i \cdot \sin \theta_i$$

$$\theta_i = \frac{\pi}{2 \cdot z} + \tan \alpha - \alpha - \left(\tan \left(\cos^{-1} \left(\frac{r_p}{r_i} \cdot \cos \alpha \right) \right) - \left(\cos^{-1} \left(\frac{r_p}{r_i} \cdot \cos \alpha \right) \right) \right) \tag{2}$$

Where design parameter θ_i is varied

$$\theta_{bc,d} \leq \theta_i \leq \theta_{i,d} \text{ and } \alpha = \alpha_d \text{ for drive side profile}$$

$$-\theta_{bc,c} \leq \theta_i \leq -\theta_{i,c} \text{ and } \alpha = \alpha_c \text{ for coast side profile}$$

3 Optimization of tooth root profile

3.1 G2 Continuity

G2 continuity means geometric continuity of order 2. It represents position of end point, equal tangent and equal curvature continuity between two geometric curves [10]. It is necessary to reduce stress concentration at tooth root to reduce bending stress.

3.2 Bezier curve

Bezier curve mimics the shape of its control polygon and always passes through its ends points [11].

Bezier curve is defined by

$$P(t) = \sum_{i=0}^n B_i J_{n,i}(t), \quad 0 \leq t \leq 1 \tag{3}$$

$$J_{n,i}(t) = \binom{n}{i} \cdot t^i (1-t)^{n-i}, \quad \binom{n}{i} = \frac{n!}{i!(n-i)!}$$

3.3 Optimize Bezier curve fillet with G2 continuity at the blending /critical section

In gear design at root of tooth circular-filletted is used [12] as shown in figure 3(a). Due to this at root, stress concentration occurs. To reduce stress concentration at the common end point of involute tooth profile and circular fillet, Bezier curve (at root of tooth) with G2 continuity has been developed as shown in figure 3(b).

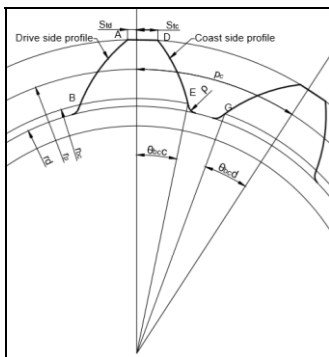


Figure 3(a). Circular fillet at gear tooth root

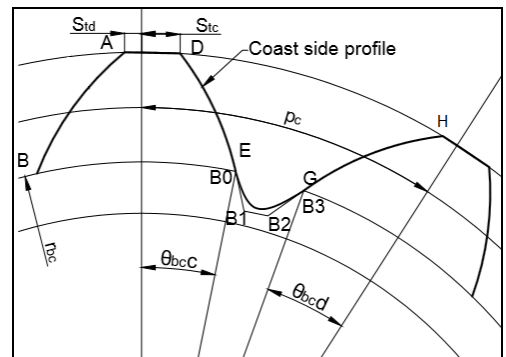


Figure 3(b). Developed fillet at tooth root with G2 continuity

If the radius of curvature is same at common end point, curves are known as curvature continuous. Tangent and curvature continuous is possible only when both first and second derivations of the equations are equal at common end point [10-11]. First and second derivations of bezier curve at end points are [11].

$$P'(0) = n(B_1 - B_0) \tag{4}$$

$$P'(1) = n(B_n - B_{n-1}) \quad (5)$$

$$P''(0) = n(n-1)(B_0 - 2B_1 + B_2) \quad (6)$$

$$P''(1) = n(n-1)(B_n - 2B_{n-1} + B_{n-2}) \quad (7)$$

First and second order derivative of the curve DE and GH at end points gives value of $P'(0)$, $P''(0)$, $P'(1)$ and $P''(1)$ respectively. B_0 and B_3 are end points of curve DE and GH respectively. B_0 and B_3 are known points. Based on equation 4 to 7 a code has been developed to obtain required points B1 and B2 at root of tooth.

4 Geometric model of asymmetric spur gear tooth

Following parameters and material used for geometric model and analysis. It is presented in table 1 & 2 [13].

Table 1. Design parameters [13]

Design Parameters	Value	Unit
Pressure angle, Drive / Coast side	$38^\circ / 20^\circ$	Degree
Total tooth on pinion and gear	25 and 25	-
Module	4	mm
Power	18	KW
Rotation	1600	RPM

Table 2. Material- (Used for ANSYS Simulation)

Parameters	Value	Unit
Young Modulus	2×10^{11}	N/m^2
Poisson ratio	0.266	-
Density	7860	Kg/m^3
Yield strength	2.5×10^8	N/m^2

As per above parameter and developed mathematical model, involute profile of asymmetric spur gear tooth is generated shown in figure 4(a) and (b).

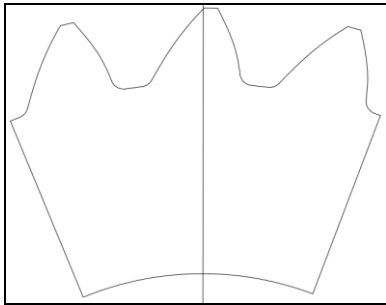


Figure 4(a). Asymmetric involute spur gear ($\alpha_d=38^\circ$, $\alpha_c=20^\circ$) with normal fillet ($\rho=1.2$ mm) at root of tooth

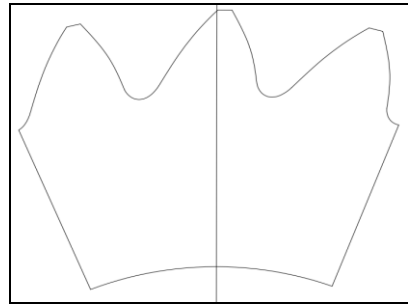


Figure 4(b). Asymmetric involute spur gear ($\alpha_d=38^\circ$, $\alpha_c=20^\circ$) with G2 continuity at root of tooth

5 Analysis of tooth in ANSYS software

Finite element analyses have been carried out for developed tooth shown in fig. 5(a) and (b). In this simulation tooth is considered as cantilever beam and loaded on drive side. The material used for simulation is presented in table 2. For analysis, following assumptions are made: 1) There is no misalignment in gear transmission, 2) Load is static, 3) Load is uniformly distributed and 4) Gear tooth material is homogenous.

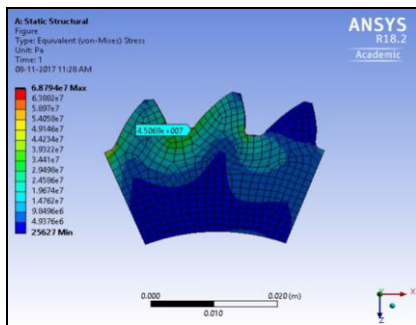


Figure 5(a). ANSYS result for asymmetric involute spur gear ($\alpha_d=38^\circ$, $\alpha_c=20^\circ$) with normal fillet ($\rho=1.2$ mm) at root of tooth

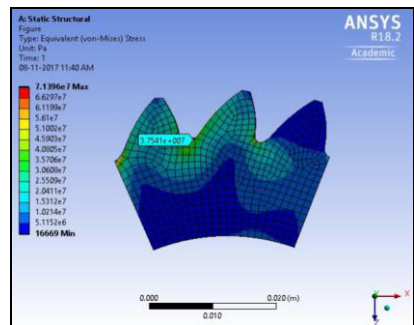


Figure 5(b). Asymmetric involute spur gear ($\alpha_d=38^\circ$, $\alpha_c=20^\circ$) with G2 continuity at root of tooth.

6 Results

Comparison has been made between normal fillet asymmetric spur gear tooth with modified gear tooth. FEA simulation show that Von mises stress at root of symmetric gear ($\alpha_d = 38^\circ$ and $\alpha_c = 20^\circ$) with normal fillet is 45.06 Mpa. Von mises stress at root of tooth of asymmetric gear ($\alpha_d = 38^\circ$ and $\alpha_c = 20^\circ$) with G2 continuity is 37.54 Mpa. Von mises stress is reduced by 16.68% compared with normal fillet asymmetric spur gear. A result has been presented in table 3.

Table 3. Result table

Parameters	Value	Unit
Bending stress (asymmetric gear /20 ⁰ -38 ⁰) with normal fillet radius	45.06	Mpa
Bending stress (asymmetric gear/20 ⁰ -38 ⁰) with G2 continuity	37.54	Mpa

7 Conclusions

Root profile is optimized to reduce bending stress of asymmetric involute spur gear tooth. Von mises stress of optimized root profile is analyzed and compared with normal fillet profile of asymmetric gear. Von mises stress is reduced by 16.68 % by optimization of root profile. Stress concentration of modified asymmetric spur gear has been significantly reduced.

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