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Spectrum estimation of electromagnetic signal based on AR model parametric spectrum estimation

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Abstract. For the low accuracy of classical spectrum estimation, the AR model method in modern parameter estimation is proposed to analyze the spectrum of electromagnetic signal. The basic principle of AR model is introduced and the Burg algorithm is simulated by Matlab. The electromagnetic signal in the real environment is simulated by the classical FFT method and the AR model method. Comparing the results show that the AR model is more accurate and reliable in the detection of electromagnetic signals, so modern parameter spectral estimation can be used as a method for spectrum analysis of electromagnetic signals.

1 Introduction

Traditional spectral estimation method makes DTFT or DFT on N observed sampling data to estimate the frequency spectrum. The signal data outside the observation interval is treated as zero, which is obviously inconsistent with the actual, resulting in the disadvantage of poor frequency resolution. Therefore, someone proposed a parametric spectrum estimation method, which is usually based on the prior knowledge of the process, to establish a model that approximates to the real process, and then use the observation data or correlation function to estimate the hypothetical model parameters, finally identify or estimate the spectrum. The most commonly used models are the ARMA model, the AR model, and the MA model, which are widely used because they can improve the signal resolution with smooth curve[1]. In general, the AR model is an all-pole model, which is easy to reflect spectral peaks and has sharp peaks but no deep valleys. Moreover, the model's parameter estimation algorithm is a linear system of equations with a small amount of calculation and relatively easy calculation. The MA model is an all-zero model, which is easy to reflect spectrum valleys and has deep valleys but no sharp peaks; ARMA can also reflect spectral peaks and spectral valleys. The main purpose of spectral estimation of electromagnetic signals is to observe the power distribution state of electromagnetic signals. Therefore, AR model is suitable for signal power spectrum estimation and detection[2].

2 Basic princeple of AR model parameter spectrum estimation

The linear prediction autoregressive model method (abbreviated as AR model method) considers that the observation data satisfy the autoregressive model and can predict the data outside the observation interval according to the data in the observation interval, and the signal data outside the observation interval is no longer treated as zero, which is equivalent extending the length of the data, then the frequency resolution capability increases naturally[2].

The AR model method is a spectrum estimation method based on a signal model. The output of the parametric model is a linear combination of the immediate input and the previous p outputs, so this model is called an autoregressive model, and denoted as AR(p), where p is the order of the AR model. The transfer function of the AR model contains only poles, does not contain zeros, so the AR model is also called the all-pole model. Assume that the observed data is obtained from a zero-mean white noise sequence with a mean squared error exciting an all-pole linear time-invariant discrete-time system. Represented by difference equation as:

$$x(n) = -\sum_{k=1}^{p} a_k x(n-k) + w(n) \tag{1}$$

Where, the constant p is the order of the parametric model, p is the parameter of p-order AR model. This model is denoted as AR(p) and its system transfer function is

$$H(z) = \frac{X(z)}{W(z)} = \frac{1}{1+\sum_{k=1}^{p} a_k z^{-k}} \tag{2}$$

In power spectrum estimation, if the observed data is a stationary random process, then the input of the system can also be considered stable[2]. Therefore, according to the response theory of the linear system to the stationary

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random signal, the power spectrum of the observed data is:

$$p\left(e^{j\omega}\right)=\left|H\left(e^{j\omega}\right)\right|^2\sigma_\omega^2=\frac{\sigma_\omega^2}{\left|1+\sum_{k=1}^{p}a_k e^{-j\omega k}\right|^2} \tag{3}$$

The main property of the AR model is that the output autocorrelation function has a recursive nature. Suppose the difference equation of the p-order AR model is as shown in Equation (1)[3].

Multiplying to both sides of equation (1) and taking the mathematical expectation, we can obtain the relationship between the AR(p) model parameters and the autocorrelation function of the observed data:

$$E[x(n)x(n-i)]=E[w(n)x(n-i)] \\ -E[\sum_{k=1}^{p}a_k x(n-k)x(n-i)] \tag{4}$$

Finishing

$$R_y(i)=-\sum_{k=1}^{p}a_k R_y(i-k) \tag{5}$$

This is the Yule-Walker equation. Expand the equation above and write it as a matrix:

$$\begin{bmatrix} R_0 & R_{-1} & \cdots & R_{1-p} \\ R_1 & R_0 & \cdots & R_{2-p} \\ \vdots & \vdots & \vdots & \vdots \\ R_{p-1} & R_{p-2} & \cdots & R_0 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix}=-\begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_p \end{bmatrix} \tag{6}$$

The coefficient matrix of the equations is an autocorrelation matrix, since the autocorrelation function is an even function $R_k=R_{-k}$, Therefore, this matrix is symmetric, and the diagonal elements are all equal. This feature is called a Toeplitz type matrix in linear algebra. The Yule-Walker equation shows that as long as the signal's autocorrelation function is known, each coefficient can be solved [3].

The recursive properties of the AR model autocorrelation function indicate that it can be extended to autocorrelation. This is why the AR model requires less data for spectral estimation. It is found that the Yule-Walker equations are closely related to the predictor designed by the minimum error criterion. The output of the p-order one-step predictive filter is the prediction of the current output x_n by the first p outputs.

$$\hat{x}_n=-\sum_{k=1}^{p}h_k x_{n-k} \tag{7}$$

Make a Z transformation for (7):

$$\hat{\mathrm{X}}(z)=\sum_{k=1}^{p}h_k \mathrm{X}(z)z^{-k} \tag{8}$$

$$H(z)=\frac{\hat{\mathrm{X}}(z)}{\mathrm{X}(z)}=\sum_{k=1}^{p}h_k z^{-k} \tag{9}$$

The orthogonal principle states that the filter coefficients are chosen such that the estimation error is orthogonal to all observations, so the prediction error $e_n=x_n-\hat{x}_n$ should be orthogonal to all inputs (i = 1 ~ p). which is

$$E\left[\left(x_n-\sum_{k=1}^{p}h_k x_{n-k}\right)x_{n-k}\right]=0 \quad k=1\sim p \tag{10}$$

Finished :

$$R_i=\sum_{k=1}^{p}h_k R_{i-k} \quad i=1\sim p \tag{11}$$

Comparing (11) with the Yule-Walker equation, it can be seen that the two equations are completely identical except that the coefficients are represented by h_k and a_k respectively and a negative sign. Therefore, the p-order one-step prediction filter has the coefficients of the impulse response h_k which are the coefficients of the denominator of the p-order AR model a_k . Fundamentally, the AR model is actually a model estimated by the principle of minimum mean square error.

AR model estimates the parameters of the signal model through a limited number of N data samples, then obtains the power spectrum of the signal based on the calculated parameters. The nature of the AR model can be considered as a reasonable extension of the N data through the model. Extension breaks the limit of only the known N points data, which improves the quality of the spectrum estimation [4].

3 AR model spectrum estimation of electromagnetic signals

When estimating the power spectrum based on the AR model, the parameters of the AR model must be extracted first. At present, the extraction algorithms of these parameters mainly include autocorrelation Levinson-Durbin algorithm, Burg algorithm and covariance/Marple algorithm. Among them, Burg algorithm has higher resolution, less complicated calculation, and has the best overall performance, it is often used for power spectrum estimation[5].

3.1. Spectral Estimation of Simulated Signal

FFT and Burg algorithms were used to estimate the spectrum of the signals and compare the results. Signal sequence is x(n)=cos(2π*200t)+cos(2π*205t) +cos(2π*210t)+0.1*r(t). Where r(t) is a random noise sequence. The sampling rate is 1kHz, the number of sampling points is 128, and the FFT length is 1024. The result is shown in Figure 1.

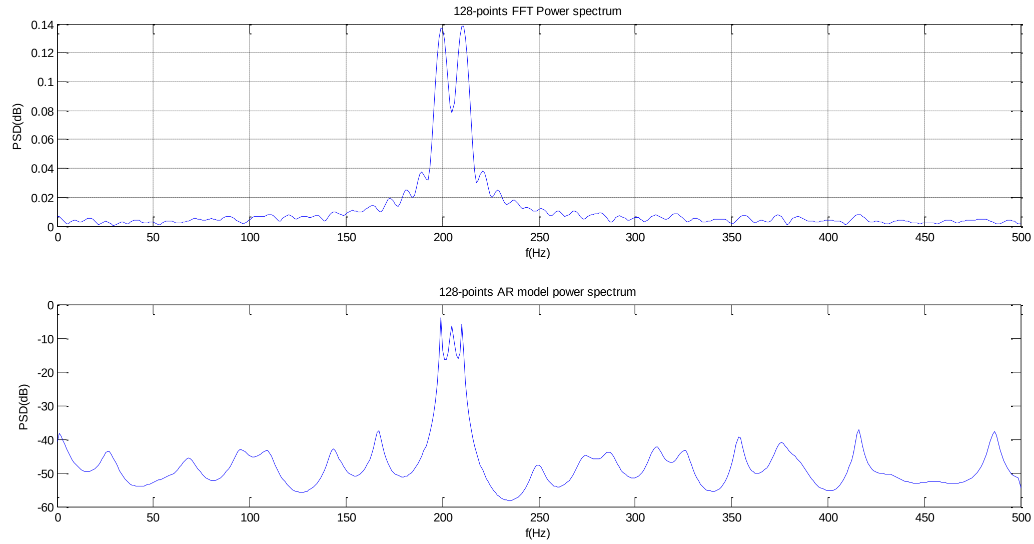

Figure.1 Comparison of FFT and AR model spectral estimation of simulated signa

From Figure 1, we can see that the traditional FFT method can't distinguish two frequency signals at 200Hz, 205Hz and 210Hz. in the case of a small number of samples, while the Burg algorithm can accurately distinguish, and the spectral peak is narrow and sharp, with high resolution.

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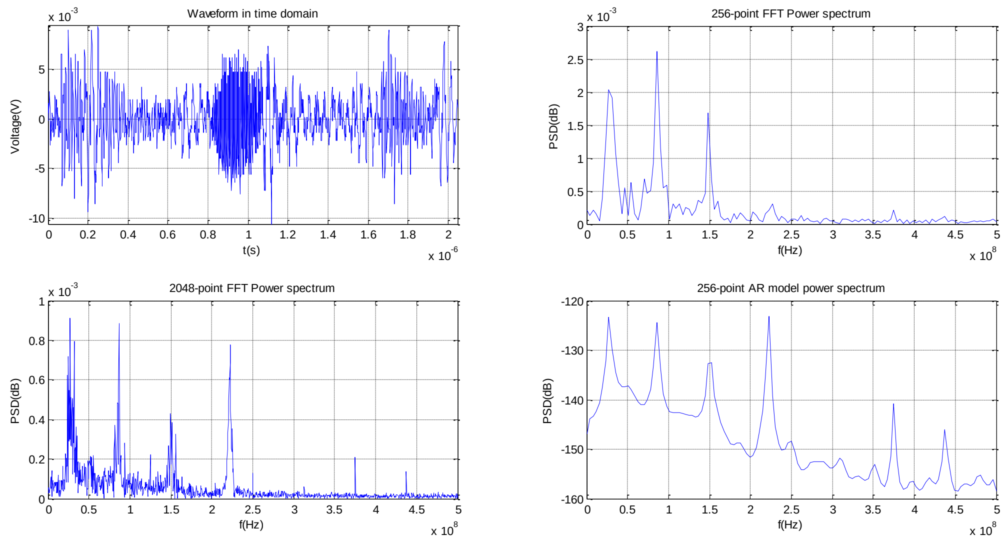
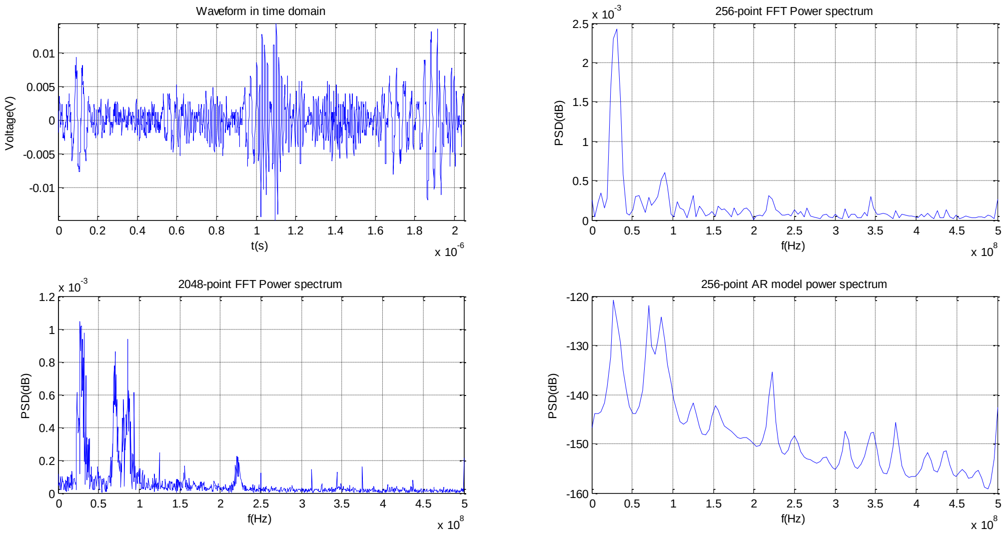
3.2 Spectral Estimation of real Electromagnetic Signal

In the simulation, the sample data was an electromagnetic wave signal acquired in a real environment acquired by an ADC with a sampling rate of 1 GHz. In MATLAB, the spectrum of the FFT and the Burg algorithm AR model parametric spectrum estimate with different lengths were separately calculated. FIG. 2 shows the time-domain waveforms and spectral analysis results of the two sets of collected electromagnetic wave signals.



(a) The first set of data



(b) The second set of data

Figure.2 Comparison of FFT and AR model spectral estimation of electromagnetic wave signals

The simulation results show that the classical FFT method could not accurately calculate the power spectrum of the electromagnetic signal in the case of a few sample data. In the first set of results, the 256-point FFT spectrum was compared with the 2048-point FFT result, the 256-point FFT couldn't resolve the peak at 220 kHz, and the power spectrum density of the first two peaks was not same as the 2048-point FFT. The 256-point AR model parametric spectrum estimation can use a small amount of data for accurate spectrum estimation. The second set of data in Figure (b) shows that the 256-point FFT results cannot distinguish between two spectral peaks between 50 k-100 kHz and 220 kHz, where the AR model parametric spectrum estimation results were consistent with 2048-point FFT results. The simulation results show that the AR model parameter spectrum estimation has higher computational accuracy than the classical FFT method and is suitable for the case with less sample data.

4 Conclusion

Compared with the classical spectrum estimation method, the modern parametric spectrum estimation has the advantage of high accuracy. In the dissertation, spectrum analysis of electromagnetic signals in space is performed by using the traditional FFT and AR model parametric spectrum estimation respectively. It can be concluded that the parametric estimation of AR model is more accurate than the FFT result when the number of signal samples is small. Therefore, this method is suitable for spectrum analysis of electromagnetic signals. Equations should be centred and numbered with the number on the right-hand side.

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