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Kinematics Analysis of a 4PRR-P Hybrid Machining Mechanism

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Abstract. In this paper, a new type of 4PRR-P hybrid machining mechanism is presented. Based on the screw theory, the degree of freedom of the mechanism is verified. We solve the inverse position model of the the mechanism. The workspace of the mechanism is solved by the boundary search algorithm. The reasonable ranges of key scale parameters are obtained by investigating the influence of the parameters of the mechansm on its workspace. The above kinematic analyses and solving results can provide corresponding references for subsequent mechanism dynamics analysis and structural design.

1 Introduction

Compared with the spatial parallel mechanism, the planar parallel mechanism has the advantages of simple structure, simple kinematics and dynamics model and low manufacturing cost[1].However, the planar parallel mechanism has some shortcomings, such as small workspace, low flexibility of moving platform and singularity of mechanism, which restrict its application in practical engineering. In view of the shortcomings of planar parallel mechanism, redundant drive or hybrid configuration can be added to reduce the singularity of planar parallel mechanism, increase the workspace of mechanism and improve the flexibility of moving platform.

Zhang Dongsheng[2] constructed a new five-degree-of-freedom serial-parallel hybrid robot based on two moving-rotation three-degree-of-freedom planar parallel mechanisms, and planar parallel mechanism $(2PRR)^2$+R with two different modes of motion redundancy and drive redundancy. (P stands for moving pair, R stands for rotating pair) for kinematics analysis; Wu Cuncun[3] proposes a new four-degree-of-freedom parallel mechanism that can realize three movements and one rotation - 2PPPaR (Pa refers to generalized moving pair) parallel mechanism, the parallel The mechanism can be widely used in high-speed and high-precision sorting, packaging and palletizing operations; Chen Chun[4] performs kinematics and dynamic modal characteristics analysis and simulation on planar 2-PRR parallel mechanism; Wang Xinhui[5] uses isomorphic mapping and differential motion Based on the principle of learning, the optimal configuration and design sensitivity of 3-PRR full-flexible parallel mechanism are solved.

Literature research shows that few literatures systematically study the Hybrid Machining Mechanism Based on 2R1T (T represents the degree of freedom of movement) planar parallel mechanism. Therefore, this paper takes the new 4PRR-P hybrid machining mechanism proposed by the author's team as the research object, verifies the degree of freedom and properties of the hybrid mechanism based on screw theory, derives the forward and inverse solutions of the mechanism position in analytical form, which is convenient for subsequent motion planning and control, and determines the motion/scale parameters of the singular shape. The influence of key dimension parameters on workspace volume was investigated, and the reasonable range of dimension parameters was optimized. The above analysis and calculation results show that the kinematics model of the mechanism is correct and reasonable, and the corresponding results of kinematics performance and scale optimization can provide necessary reference for the follow-up study.

2 Configuration of 4PRR-P hybrid mechanism

2.1 Mechanism configuration

The 4PRR-P hybrid mechanism presented in this paper is shown in Fig. 1. It is noted that the center of the moving slider is A_j（j=1~4）,A_j（j=1~4）, the two guides of the four moving sliders are fixed platforms, and the P pair where A_j is located is the driving pair. A moving platform is composed of four rotating pairs'centers B_j（j=1~4）, four parallel branch chains 4PRR, and a moving pair is connected in series with the moving platform. The moving slider's centroid is A_5, and the A_5 guide is perpendicular to the plane where the 4PRR of the parallel mechanism is located.

We set the fixed platform coordinate system o-xyz: the origin o is the midpoint of the line at the lowest end of the two driving guides, the y-axis is connected to the right at the lowest end of the two driving guides, the z-axis is parallel to the driving guide, and the x-axis is determined by the right-hand rule. We set the moving platform coordinate system o'-$x'y'z'$: the origin o' is the center of the rectangular $B_1B_2B_3B_4$, the y' axis crosses

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the *o'* point forward from B_1 to B_4, the *z'* axis crosses the *o'* point forward from B_2 to B_1, and the *x'* axis is determined by the right hand rule.

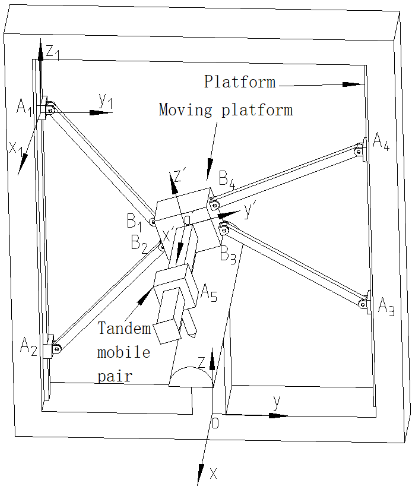


Figure 1. Schematic diagram of 4PRR-P hybrid mechanism

The scale parameters of the 4PRR-P hybrid mechanism are shown in Table 1.

Table 1. Scale parameters of hybrid mechanism

Symbol	Definition
a	The distance between the center of rotation of $B1$ and $B4$ (or $B2$, $B3$)
b	Vertical distance between *yoz* planar two driving paths
d	The distance between the center of rotation of $B1$ and $B2$ (or $B3$, $B4$)
q_i	Driving displacement of mobile sub Ai (i=1, 2, 3, 4, 5)
L	$AjBj$ rod length ($j=1, 2, 3, 4$)
α_j	Angle between $AjBj$ and Z to drive path ($0\le\alpha_j\le\pi/2$)
L_1	Travel of series mobile pair

2.2 Analysis of degree of freedom

As shown in Fig. 2, we set the branch coordinate system o_1-$x_1y_1z_1$: the origin o_1 takes the drive slider centroid A_1, the y_1 axis is perpendicular to the z drive guide and is to the right, the z_1 axis is parallel to the z axis, and the x_1 axis is forward through the right hand. The rules are determined. The one-way arrow in the figure indicates the restraining force vector, and the double-headed arrow indicates the binding couple. In the coordinate system o_1-$x_1y_1z_1$, the motion spiral[6] corresponding to each single-degree-of-freedom motion pair in the branch A_1B_1 is

$$\begin{cases} \$_{11} = (0 \quad 0 \quad 0 \quad ; \quad 0 \quad 0 \quad 1) \\ \$_{12} = (1 \quad 0 \quad 0 \quad ; \quad 0 \quad 0 \quad 0) \\ \$_{13} = (1 \quad 0 \quad 0 \quad ; \quad 0 \quad L\cos\alpha_1 \quad L\sin\alpha_1) \end{cases} \qquad (1)$$

Where, $\$_{11}$ represents the motion spiral driving the moving pair at A_1, $\$_{12}$ represents the motion spiral of the rotating pair at A_1, and $\$_{13}$ represents the motion spiral of the rotating pair at B_1. According to the principle that the reciprocal product is 0, the inverse of the equation (1) is obtained.

$$\begin{cases} \$_{11}^r = (1 \quad 0 \quad 0 \quad ; \quad 0 \quad 0 \quad 0) \\ \$_{12}^r = (0 \quad 0 \quad 0 \quad ; \quad 0 \quad 1 \quad 0) \\ \$_{13}^r = (0 \quad 0 \quad 0 \quad ; \quad 0 \quad 0 \quad 1) \end{cases} \qquad (2)$$

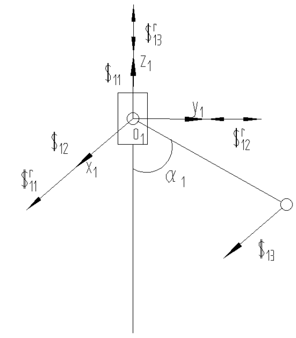


Figure 2. Movement of the branch $O_1(A_1B_1)$, constrained spiral

Similarly, the reverse spirals of the other three branches can be obtained, and the reverse spirals of the four branches A_iB_i are the same. Therefore, the constrained spiral system $\$^r$ of the 4PRR parallel mechanism is

$$\begin{cases} \$_1^r = (1 \quad 0 \quad 0 \quad ; \quad 0 \quad 0 \quad 0) \\ \$_2^r = (0 \quad 0 \quad 0 \quad ; \quad 0 \quad 1 \quad 0) \\ \$_3^r = (0 \quad 0 \quad 0 \quad ; \quad 0 \quad 0 \quad 1) \end{cases} \qquad (3)$$

For the inverse of the equation (4), the motion spiral system of the parallel platform is $\$^m$.

$$\begin{cases} \$_1^m = (1 \quad 0 \quad 0 \quad ; \quad 0 \quad 0 \quad 0) \\ \$_2^m = (0 \quad 0 \quad 0 \quad ; \quad 0 \quad 1 \quad 0) \\ \$_3^m = (0 \quad 0 \quad 0 \quad ; \quad 0 \quad 0 \quad 1) \end{cases} \qquad (4)$$

In the formula (4), the motion spiral $\$_1^m$ represents the degree of freedom of the moving platform about the x-axis, and the motion spirals $\$_2^m$ and $\$_3^m$ respectively represent the freedom of movement of the moving platform along the y and z-axis directions, and at the same time, the series branches in the moving platform have along the x. A degree of freedom of movement in the direction of the axis. Therefore, the 4PRR-P hybrid mechanism has 3 movements 1 rotation and 4 degrees of freedom.

3 Position inverse solution modeling

The positional inverse analysis is the output pose (y, z, θ) of the known mechanism, and the drive displacement (q_1, q_2, q_3, q_4) of the solution mechanism.

In the fixed coordinate system, the position vector *oo'* of the motion platform centroid *o'* is denoted as (x,y,z).

Since the 4PRR parallel mechanism has a degree of freedom of rotation about the x-axis (rotation angle is θ), the attitude conversion matrix $\boldsymbol{R}_{BE}$ of the parallel mechanism from the fixed coordinate system *o-xyz* to the moving coordinate system *o'-x'y'z'* is

$$\boldsymbol{R}_{BE} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \quad (5)$$

The position analysis vector relationship proposed in this paper is

$\overrightarrow{\boldsymbol{OB}_i} = \overrightarrow{\boldsymbol{OO'}} + \boldsymbol{R}_{BE} \cdot \overrightarrow{\boldsymbol{O'B}_i}$ ， among them， i=1,2,3,4 （6）

The z coordinate of the A_j point in the fixed coordinate system o-xyz is the drive displacement q_j, ie

$$\begin{cases} q_1 = z - a/2\cdot\sin\theta + d/2\cdot\cos\theta \\ +\sqrt{L^2-(y-a/2\cdot\cos\theta - d/2\cdot\sin\theta + b/2)^2} \\ q_2 = z - a/2\cdot\sin\theta - d/2\cdot\cos\theta \\ -\sqrt{L^2-(y-a/2\cdot\cos\theta + d/2\cdot\sin\theta + b/2)^2} \\ q_3 = z + a/2\cdot\sin\theta - d/2\cdot\cos\theta \\ -\sqrt{L^2-(b/2-y-a/2\cdot\cos\theta - d/2\cdot\sin\theta)^2} \\ q_4 = z + a/2\cdot\sin\theta + d/2\cdot\cos\theta \\ +\sqrt{L^2-(b/2-y-a/2\cdot\cos\theta + d/2\cdot\sin\theta)^2} \end{cases} \quad (7)$$

4 Workspace analyses

4.1 Workspace solving

The working space of the 4PRR-P hybrid mechanism is based on the inverse solution of the position of the 4PRR parallel mechanism. Considering the constraint of the length of the rod, the angle of the sub-rotation of the motion and the interference between the poles, the computer is used to search the motion space of the centroid of the moving platform of the mechanism. owned.

The constraints for solving the workspace are as follows:

$$\begin{cases} 0 \le x \le L_1 \\ -a/2+b/2-L \le y \le a/2+L-b/2 \\ d/2 \le z \le L_2 - d/2 \\ 0 \le q_j \le L_2 \\ \arccos[(L_2-d)/2L] \le \alpha \le 90^\circ \\ 0^\circ \le \theta \le \arccos[a/(b-2L\sin\alpha_{\min})] \end{cases} \quad (8)$$

Where j=1， 2， 3， 4； $\sin\alpha_{\min} = \sqrt{1-[(L_2-d)/2L]^2}$

The boundary search method usually uses the kinematic inverse solution, and considers the scale parameters of the mechanism and the limitation of the motion displacement, and searches for the set of boundary points of the workspace[7]. We give the angle and step size of θ, let θ be from $-\pi/2 \sim \pi/2$, and substitute each point in the closed space composed of $y_{\max}$, $y_{\min}$, $z_{\max}$, and $z_{\min}$ into the inverse kinematics of the mechanism, and determines whether the point exists is in the workspace. After all the searches are completed, you can get the posture workspace of the moving platform.

The scale parameters of a set of 4PRR-P hybrid mechanisms are given as follows: L=1m, L_1=1.5m, L_2=2.3m, a=0.42m, b=1.8m, d=0.22m. Workspace can be obtained by substituting the instance values,as shown in Figure 3.

When the attitude angle of the moving platform is θ=1.53 rad, the two-dimensional working space of the 4PRR-P hybrid mechanism is as shown in Fig. 4. From the direction view of the workspace, there is no void area inside the workspace of the mechanism.

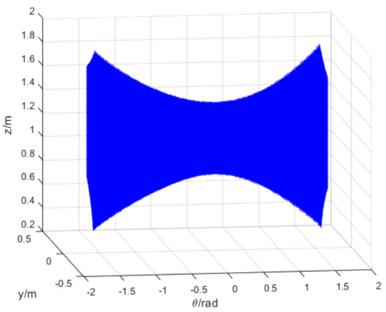


Figure 3. 4PRR mechanism 3D workspace

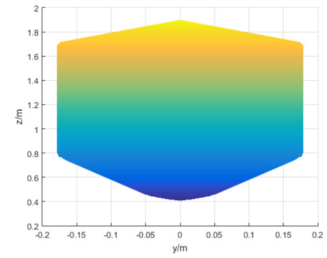


Figure 4. Workspace *yoz* plane when θ=1.53

4.2 Influence of scale parameters on workspace

As shown in Table 1, the L, b, a, and d scale parameters are important scale parameters. The following four factors are studied for the influence of the working space of the moving platform.

(1) The influence of the length L of the rod on the working space of the moving platform

For the 4PRR-P hybrid mechanism example, the influence of the rod length L on the *yoz* plane working space generated by the centroid o' of the moving platform is analyzed by the univariate method. As shown in Fig. 5(a), considering the parameter limitation of the mechanism, the design range of the rod length L here is $L_{\max}$ = 1.38 m and $L_{\min}$ = 0.83 m. It can be seen that in the range of values, the influence of the length L of the rod on the *yoz* plane working space of the moving platform is first increased and then decreased. When the rod length L is about 1.05, the maximum working space is obtained.

(2) The influence of parameter b on the working space of the moving platform

When the moving platform rotation angle θ is fixed, the reachable position working space of the 4PRR-P hybrid mechanism can be regarded as a space formed by stretching the working space of the *yoz* plane in the x direction.

The scale parameters of a set of 4PRR-P hybrid mechanisms are given as follows: L=1m, L_1=1.5m, L_2=2.3m, a=0.42m, b=1.8m, d=0.22m. Substituting the instance values to get the workspace, as shown in Figure 3.

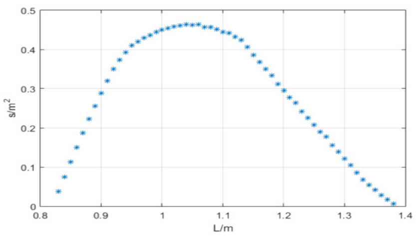
When the attitude angle of the moving platform is θ=1.53 rad, the two-dimensional working space of the 4PRR-P hybrid mechanism is as shown in Fig. 5. From the direction view of the workspace, there is no void area inside the workspace of the mechanism.

As shown in Fig. 5(b), the design range of b here is b_{max} = 2.2 m and b_{min} = 0.42 m. It can be seen that within the range of values, the influence of parameter b on the working space of the moving platform is first increased and then decreased. When the parameter b is about 1.55, the maximum working space is available.

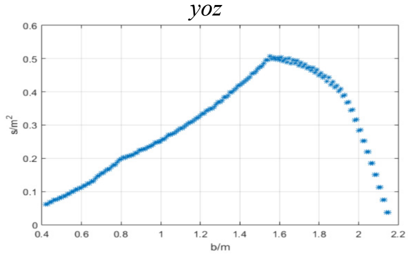
(3) Influence of parameter a on the working space of the moving platform

As shown in Fig. 5(c), the design range of a here is a_{max} = 1.8 m, and a_{min} > 0 m. It can be seen that within the range of values, the influence of parameter a on the *yoz* plane working space of the moving platform is first increased and then decreased. When the parameter a is about 0.6, the maximum working space is available.

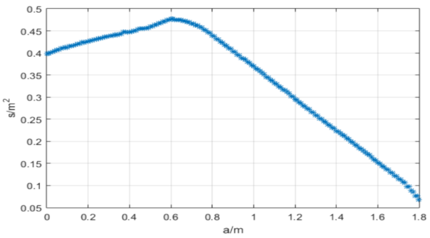
(4) Influence of parameter d on the working space of the moving platform



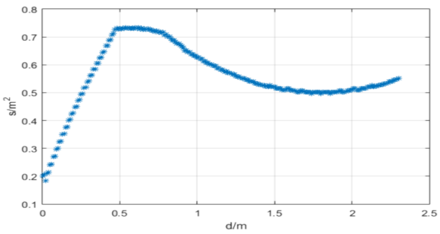
(a) L influence on the cross-sectional area of the workspace *yoz*



(b) b influence on the cross-sectional area of the workspace *yoz*



(c) a influence on the cross-sectional area of the workspace *yoz*



(d) d influence on the cross-sectional area of the workspace *yoz*

Fig 5. Effect of scale parameters on the cross-sectional area of the moving platform *yoz*

As shown in Fig. 5(d), considering the parameter limitation of the mechanism, the design range of d is d_{max}=2.3m and d_{min}=0m. It can be seen that in the range of values, the influence of the parameter d on the working space of the *yoz* plane of the moving platform is first increased, then decreased, and then increased. When the parameter d is about 0.5, the maximum working space is available.

5 Conclusions

1) Using the spiral theory, the degree of freedom analysis of the 4PRR-P hybrid mechanism proves that the 4PRR-P hybrid mechanism has four degrees of freedom in the x, y, and z directions and in the x direction.

2) We solve the workspace of the dispatching platform by the boundary search method and solve the influence of important parameters on the workspace of the moving platform: within the range of values, the influence of the length L of the rod on the *yoz* plane working space of the moving platform is first increased and then decreased; the parameter b is the working space of the moving platform. The influence trend is first increased and then decreased; the influence of parameter a on the *yoz* plane working space of the moving platform is first increased and then decreased; the influence of parameter d on the working space of the moving platform *yoz* plane is first increased and then decreased. Small and then increase.

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