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Urban Metro Network Topology Evolution and Evaluation Modelling Based on Complex Network Theory: A Case Study of Guangzhou, China

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Abstract. As urban metro network is generally referred as a significant component of the modem urban transport system, the spatiotemporal evolution of spatial layout and topology structure of the network should be investigated and evaluated in order to promote urban transport services and optimize urban spatial pattern. This paper takes a case study of the city of Guangzhou, China, and applies the complex network theory and integrates geography information system (GIS) to explore and discuss the growth and topological structure characteristics of the Guangzhou metro network. Importantly, this paper focuses on accessing the formation process of the topology structure of the Guangzhou metro network from 1997 to 2016 on the basis of spatio-temporal sequence data analysis. This aims to provide scientific references for the future development and planning of urban metro network in China.

1 Introduction

With the rapid development of urban metro network for decades especially in China, the spatial structure of metro network evolves gradually from simple line crossing, grid, radial to complex patterns, the traditional quantitative evaluation of network topology is difficult to meet the complexity and dynamic measurement requirements by using network coverage and nodal connectivity [1].

Currently, the complex network theory has been increasingly deepening in the fields of ecology, economy and social network, obviously is also widely applied in transport network analysis and modelling. It not only provides an effective tool for the complexity analysis of transport network, but also is an important theoretical basis for the topology characteristics measurement. Presently, the complex network theory has gradually evolved into a new and wide range of scientific examples of network analysis and design. Particularly, rich achievements have been made by the applications of the complex network theory in the transport network analysis and modelling, including road network, highway network, railway network, aviation network and so on [1-7]. For example, Derrible and Kennedy (2010) comparatively analysed the statistical characteristics of the metro network topology evolution of thirty-two cities, and Zhang et al. (2013) discussed the complexity of the railway networks of thirty regions in the world. Both of studies aims to classify and construct an adaptive assessment system for the complex analysis of rail transit network in term of the state, form and structure of network [1,8]. Based on exploring the spatial distribution characteristics of urban travel demand, Cats (2016) demonstrated the spatial expansion mechanism of bus line network on its robustness, and further enriched the application of the complex network theory in the transport network analysis and modelling [9]. Currently, many Chinese scholars have also made rich achievements in urban transport network analysis and modelling based on complex network theory. These studies mainly aim to calculate the essential topological parameters such as average path length, clustering efficient, node degree distribution, node closeness or betweenness centrality, in order to explore the complexity of transport network, and to evaluate the network efficiency and performance, including reliability, robustness and invulnerability [10-13]. However, compared with the studies oriented to the evolution analysis of transport network based on time series data which have widely carried out in the western countries, the existing researches in China mainly focus on evaluating the transport network topology structure based on cross-section data analysis. Therefore, this paper focuses on exploring the formation process of the Guangzhou metro network from 1997 to 2016, especially discussing the complexity and evolution of network topology structure based on time series data analysis. This aims to provide scientific references for the future development and planning of urban metro network in China.

2 Methodology

In the L-space model, each metro station is defined as a node of network, and identified as s ($s \in S$, S is the set of nodes). If two metro stations are adjacent to one metro

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line, there is an edge which is represented by e (e∈E, E is the set of edges). Therefore, in this paper the metro network is defined as a directed planar map, and identified by G=(S, E). The number of nodes and edges in the network can be presented by ∣S∣ and ∣E∣, respectively.

2.1 Global network topology evaluation

The overall size of a network can be measured by its diameter, which is defined as:

$$d = MAX(d_{ij}) \quad i, j \in |S| \qquad)1)$$

d_{ij} is the shortest distance between node i and j, indicating the minimum number of edges go through two nodes in the network.

To evaluate the topology structure of transport network, some indexes usually are used, including β、μ、γ、α and q. Among these indexes, β is the ratio of the total number of edges of the network to the total number of nodes, i.e., β=∣E∣/∣S∣. When β<1, the network is a tree network, otherwise, a loop network. μ is the loop index, and calculated by μ=∣E∣-∣S∣+1, 1 is the number of subgraphs of a network, if all nodes are interconnected each other in the network, the number of subgraphs is 1. γ is defined as the ratio of the number of edges of the network to the maximum possible number of edges to reflect the networking level by the existing edges, as shown by Equation 2.

$$\gamma = \frac{|E|}{3(|S|-2)} \qquad (2)$$

α is identified as the ratio of the number of loops in the network to the maximum possible number of loops for reflecting the robustness of network, and defined as:

$$\alpha = \frac{|E|-|S|+1}{2|S|-5} \qquad (3)$$

q is the straightness index of network. Firstly, the sum of the shortest path length and geodesic distance between node i and all other nodes in the network should be calculated, respectively. Then, the ratio of the sum of the shortest path length to the sum of the geodesic distance can be attained. Finally, the average value of all nodes is obtained. The equation is as follows:

$$q = \frac{\sum_{i\in|S|}\sum_{i\neq j\in|S|} d_{ij}}{|S|(|S|-1)\sum_{i\in|S|}\sum_{i\neq j\in|S|}\tau_{ij}} \qquad)4)$$

τ_{ij} is the geodesic distance between node i and j. The value of q ranges from 0 to 1.0, which is used to access the straight-line travel efficiency through the network. The smaller the value, the lower the travel efficiency through the network is.

2.2 Complex network evaluation

In term of the complex network theory, the metrics for measuring the network topology statistical characteristics usually includes node degree, average path length, network efficiency, clustering coefficient, node betweenness and so on.

Node degree is usually defined as the total number of all connected edges of a node (see Equation 5). The larger the node degree, the more important node in the network is. When the ratio of the node degree of node i to the total number of edges which node i connects is calculated, the node degree distribution characteristic can be obtained. If the distribution characteristic presents a power law form, the network is a scale-free network.

$$k_i = \sum_{j\in|S|} e_{ij} \qquad (5)$$

The index of average path length describes the average value of the shortest distance between nodes i and j in the network, indicating the dispersion level of nodes, and can be identified by :

$$L = \frac{\sum_{i\neq j\in|S|} d_{ij}}{|S|(|S|+1)} \qquad (6)$$

Furthermore, the greater the average shortest path distance between any pair of nodes in the network, the lower the network efficiency is (see Equation 7).

$$F = \frac{\sum_{i\neq j\in|S|}\frac{1}{d_{ij}}}{|S|(|S|+1)} \qquad (7)$$

Clustering coefficient characterizes the average coupling degree of nodes in the network, and is defined as node i connecting to other N_i nodes through N_i edges. If all N_i nodes are connected to each other, the number of connected edges of all nodes is N_i (N_i-1)/2; if the number of actual connected edges of N_i nodes is E_i, and the ratio of n to (n-1)/2 is the clustering coefficient of node i. The equation is as follows:

$$c_i = \frac{2E_i}{N_i(N_i-1)} \qquad (8)$$

Node betweenness is defined as the ratio of the number of shortest paths passing through node i to the total number of shortest paths in a network (see Equation 9). The value of node betweenness ranges from 0 to 1. The greater the value of node betweenness, the more important the transfer node is. This implies that passengers are more inclined to choose their travel route through the node (station). This may result in the station is prone to arise a large cross-section passenger flow, and finally causing network congestion.

$$B_i = \sum_{k\neq j\in|S|}\frac{D_{kj}(i)}{D_{kj}} \qquad (9)$$

$D_{kj}(i)$ is the number of shortest path passing through node i, D_{kj} is the total number of shortest path between any pair of nodes in the network.

3 Results and analysis

3.1. Global topology structure evaluation

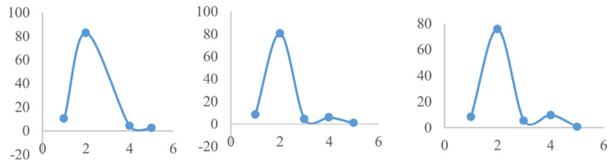
According to the methodology presented in this paper, the topology indexes of Guangzhou Metro Network (GMN) from 1997 and 2016 are listed in Table 1. As shown in Table 1, the diameter of GMN expanded from 4 km in 1997 to 35 km in 2010, and after 2010, the network diameter has been no significant increase. However, according to the loop index of μ, the number of loops in the network has been increasing after 2010, which indicates that the growth of GMN is directed to gradually strengthen the internal loop construction from 1997 to 2016. Besides, Table 1 shows that the value of μ and β was 0 and <1 before 2006, respectively. This implies that the metro lines did not form a network before 2006, and the topology structure is a tree structure. In 2006, the value of β and μ increased to 1.02 and 2, respectively. This shows the network topology structure began to evolve from a tree structure to a loop structure, indicating the networking was completed. With the topology structure of GMN evolving into a loop structure in 2006, the robustness index of α increases gradually with the loop index of μ, demonstrating the more developed the network, the higher the robustness is. In addition, the network straightness index of q is the only one of all indicators that illustrates a gradual decrease from 1997 to 2016. This shows that although the growing of GMN, the overall network efficiency has not been improved, but gradually decreased. The evolution of the index γ also confirm this scenario as the values of γ from 1997-2016 have been stable and low throughout the growth of network.

Table 1. Global topology index of Guangzhou Metro Network

Year	d	β	μ	γ	α	q
2016	35	1.09	15	0.36	0.05	0.00006
2015	33	1.09	14	0.37	0.05	0.00007
2013	33	1.09	13	0.36	0.05	0.00007
2010	35	1.05	7	0.35	0.03	0.00009
2009	32	1.05	5	0.35	0.03	0.00020
2007	32	1.01	2	0.35	0.01	0.00039
2006	22	1.02	2	0.35	0.02	0.00061
2003	19	0.96	0	0.34	0	0.00157
1999	15	0.93	0	0.35	0	0.00502
1997	4	0.80	0	0.44	0	0.05126

3.2 Complex topology analysis

Figure 1 illustrates the distribution characteristics of node degree of GMN in 2006, 2010 and 2016. It can be seen that the distribution characteristics are Poisson distribution, which indicates that GMN has evolved into been a stochastic network since 2006.



a.2006 b.2010 c. 2016

Figure 1. Node degree distribution of 2006, 2010, 2016.

The indicators to evaluate the statistical characteristics and evolution of GMN are listed in Table 2, involving with total mileage (M), number of stations (S), number of edges (E), average node degree (k^a), average path length (L), network efficiency (F), average clustering coefficient (c^a) and average node betweenness (B^a). As shown in Table 2, since 2003, each metro station (node) has connected to the other two stations (nodes) on average. Although the number of nodes and edges are rising, but the average node degree (k^a) increase is not remarkable, and basically maintained at about 2. This illustrates that the connectivity between metro lines in GMN is not strong.

Table 2. Complex topology index of Guangzhou metro network

Year	M	S	E	k^a	L	F	c^a	B^a
2016	278.25	153	167	2.18	12.53	0.13	0	0.16
2015	244.22	136	149	2.19	11.61	0.13	0	0.17
2013	237.87	133	145	2.18	11.72	0.13	0	0.18
2010	213.77	119	125	2.10	12.01	0.13	0	0.19
2009	146.18	80	84	2.10	10.62	0.16	0	0.23
2007	112.56	59	60	2.03	10.77	0.17	0	0.31
2006	71.68	47	48	2.04	8.03	0.21	0	0.22
2003	35.57	31	30	1.94	7.60	0.24	0	0.27
1999	18.19	16	15	1.88	5.82	0.36	0	0.42
1997	5.06	5	4	1.60	2.35	0.78	0	0.6

In addition, Table 2 reveals the evolution of network efficiency, which indicates a continued decline from 1997-2010 and then unchanged from 2013-2016 (F=0.13). This implies that the spatial extension of GMN in the early stage leads to the disadvantages of the increase of average path length and the aggravation of node dispersion. After 2013, as the network no longer expands outward in space, the average shortest path length between nodes remains stable, and keeps a smaller clustering coefficient, which verifies that GMN will approach a scale-free network with the growth of network. Average node betweenness reflects the function and influence of the corresponding nodes in the whole network, and is an important global geometric quantity. In fact, the station with the largest value of node betweenness indicates that it is the most important transfer station in the network. If the passenger flow of such a station is too large, the whole network should be congested. Table 2 further reveals that the average node betweenness of GMN has been declining steadily from 1997 to 2016. It illustrates that the distribution of node betweenness is gradually balanced, which reflects that the evacuation capacity of the network passenger flow is strengthened with the increase number of transfer stations.

Finally, according to the L-space model, the average clustering coefficient of GMN is 0, which does not mean that the stations in the network are independent of each other, but reveals that the intensity of connection between stations in the network needs to be strengthened. If the network topology structure is constructed according to the P-space model, the average clustering coefficient from 1997-2016 is shown in Table 3. In the P-space model, a metro station is defined as a node of the network, and if two nodes have a direct route by a metro line, there is an edge. Table 3 shows that the average node clustering coefficient of GMN decreases with the increase number of metro lines and stations, indicating that the shortcomings derived from the spatial extension of the network in the early stage have not been completely eliminated.

Table 3. Average clustering coefficients by P-space model

Year	**1997**	**1999**	**2003**	**2006**	**2007**
c^a	1	1	0.91	0.83	0.84
Year	**2009**	**2010**	**2013**	**2015**	**2016**
c^a	0.79	0.77	0.72	0.71	0.72

4 Conclusion

This paper takes a case study of Guangzhou Metro Network (GMN), and applies the complex network theory with the integration of GIS, in order to quantitatively access the growth and topology statistical characteristics of network. Importantly, this paper focuses on exploring the topology formation process of GMN from 1997-2016. Furthermore, the temporal and spatial evolution of the network complexity is discussed. The results derived from this study show: (1) the year of 2010 is a key point for the development of GMN, as before 2010, the development aimed to expand outward for increasing the service coverage, and after 2010, it engaged to encrypt the lines and stations inside the network, and strengthen the concentration between lines and stations in the main urban area; (2) the growth of GMN presents significant characteristics of a random network, particularly with the increasing of the lines and stations, the network approaches a scale-free network; (3) after rapid development of the Guangzhou metro network for decades, the growth of GMN is still at a developing stage. As a result, although the transfer station has a high level of aggregation in the network which actually strengthening the evacuation ability of passenger flow of the network, the anti-attack ability of network is relatively weak.

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