

# Calculation of the crack resistance of reinforced concrete elements with allowance for the levels of normal crack formation

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**Abstract.** The separate experimental investigations results and the major drawbacks of existing methods of calculating the reinforced concrete elements crack resistance are considered in detail, taking into account the process of multilevel formation and crack opening. Based on the basic provisions of the deformation and strength model of concrete and reinforced concrete resistance, an engineering method for calculating the formation and disclosure of normal cracks in the reinforced concrete elements and structures at the operational stages of their deformation is proposed. It is implemented on the system of generally accepted static, geometric and physical relations of a deformed solid mechanics. The simplicity of the proposed calculation method is due to the use of the nonlinear function of the average stresses of the reinforcement bond with concrete on the sections of the reinforced concrete element between adjacent cracks. The main advantages of the developed calculation method in comparison with others are outlined.

## 1 Introduction

The issues of crack resistance of reinforced concrete elements in the theory of concrete and reinforced concrete have always been and still remain one of the defining. It is connected, first of all, with the need to assess the actual stress-strain state of these elements in the operational stages. But describing this state is extremely difficult, since the length of the individual blocks, into which the reinforced concrete element is successively divided during the cracks formation, is variable and depends on many factors.

Since the process of crack formation and detection in reinforced concrete leads to violation of its continuity, the evaluation of the stress strain state of reinforced concrete elements tends to be associated with the basic provisions of the destruction mechanics. At the same time, the fact that the task of calculating distances between cracks in principle cannot have an exact analytical solution due to initial shrinkage micro-breaks of concrete continuity on the surface of its stretched zone is often neglected. Therefore, engineering methods for calculating the crack resistance of reinforced concrete elements, based on the general laws of changing the distances between cracks and reinforcement deformations and

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the resistance of the concrete displacement in the areas between cracks, will remain relevant in the future.

## 2 Analysis of recent studies and publications

A rather detailed classification and a thorough analysis of numerous investigations of the crack resistance of reinforced concrete elements are given in [1]. It should be noted that the vast majority of calculation methods based on the results of the above studies were developed for the stage of stabilized cracking of reinforced concrete elements. As for the methods of calculating the crack resistance of elements, taking into account the multilevel formation of cracks, then, despite the accumulation of experimental research results of [2-9], they are very limited and practically are only beginning to form. By their orientation, these techniques can be classified as based on:

- models of a “two-console” element in a section with a crack [4];
- solutions strictly within the framework of brittle fracture mechanics [9];
- models of stretched concrete resistance to reinforcement displacement in sections between adjacent cracks [7, 10, 11].

In general, a “two-console” element in a section with a crack is used to translate rather complicated dependencies of fracture mechanics into less complicated dependencies of a mechanics of deformed solid [4]. But the main disadvantage of this method is that the correct allocation of this “two-console” element is a rather complex task, which, in turn, should be carefully connected not only with the stress and strain state of the reinforced concrete element cross-section, but also with the laws of reinforcement bond with concrete.

Solutions proposed strictly within the fracture mechanics framework [9] are so complex, that today they can be implemented only software, in particular, using the finite element method. At the same time, the physical essence of the actual deformation process of reinforced concrete elements is often lost or lowered.

With regard to the calculation methods based on the models of the stretched concrete resistance to reinforcement displacement in the areas between adjacent cracks, then they can be divided into three groups.

In the methods of the first group, the concrete resistance to the reinforcement displacement is considered from the standpoint of the elastic-plastic material with strengthening [7] and is based on the use of a number of significant simplifications and empirical parameters and coefficients. Because of this, these techniques very closely reflect the processes of reinforced concrete elements crack resistance in the real conditions of their deformation.

The methods of the second group include the stepwise numerical integration of the differential equation of the composite transfer relative to the reinforcing rod [10] using a number of empirical parameters and coefficients. This is due to the direct integration impossibility of the spline function of the concrete and reinforcement mutual displacement over the entire range of reinforced concrete element deformation due to its lump approximation.

One of the simplest methods is the third group, by which the concrete resistance to reinforcement displacement is determined by the average stresses of reinforcement bond with concrete in the area between adjacent cracks [11]. However, there are a number of reservations about their methodological justification in the following issues:

- determination of the true curvature and stiffness of the reinforced concrete element in cross sections with cracks;
- evaluation of the element state according to the materials state in separate sections, on the one hand, and the average stresses of reinforcement bond with concrete in the area between adjacent cracks, on the other side;

- the linearity of the medium stresses of the reinforcement bond with concrete over the entire range of reinforced concrete element deformation.

### 3 Issues not solved within the common problem

The results of the above analysis allow to note: none of the above-mentioned techniques can be implemented in the engineering non-program version. The universal method for calculating the reinforced concrete elements strength, including those taking into account the levels of normal cracks formation, must be equally successful in both engineering and program execution.

### 4 Purpose and objectives of research

The purpose of the study is developing an engineering method for calculating the formation and disclosure of discrete cracks at any stage reinforced concrete elements of deformation. In this article two major tasks are solved:

- ensuring the universality of the method with the help of the basic provisions of the deformation and strength model of reinforced concrete resistance [1, 12];
- to ensure maximum simplicity of the method due to the use of nonlinear function of medium bond stresses of reinforcement with concrete on element sections between adjacent cracks.

### 5 Results and discussion

According to the deformation and force model [1], the appearance of normal first-level cracks in a reinforced concrete element is calculated according with two generally accepted equilibrium equations with boundary deformations of stretched concrete  $\varepsilon_{\text{ntu}}$ , which are fixed by the extreme criterion of its strength  $dN_{\text{ct}} / d\varepsilon_{\text{ct}} = 0$  or are accepted equal  $\varepsilon_{\text{ntu}} = 2f_{\text{ctk}} / E_{\text{no}}$ . In this case, the distance between cracks is calculated from the equilibrium condition of maximum efforts in stretched concrete  $N_{\text{ct,cr}} = f(\varepsilon_{\text{ntu}})$  and the efforts of active bond on the area between cracks  $N_{\text{bd,cr}}$ .

Strength of reinforced concrete active bond with stretched concrete, due to the known function of the average bond stresses  $\tau_{\text{bmi}}$ , can be determined in the cracks formation of the corresponding level by the formula

$$N_{\text{bd,cr}} = \int_{s_r} u \cdot \tau_b(z) \cdot dz \cong u \cdot \tau_{\text{bmi}} \cdot s_{\text{ri}}, \quad (1)$$

where  $u$  – cross-section perimeter of the reinforcing rod in diameter  $\varnothing_s$ , which has an area  $A_s$ , equal  $u = 4 \cdot A_s / \varnothing_s$ ;  $s_{\text{ri}}$  – the area of the reinforcing rod active bond with concrete (distance) between the cracks of the corresponding level;  $\tau_{\text{bmi}}$  – the value of the average stresses of reinforcement bond with concrete in the area between cracks of the corresponding level.

By the processing results of numerous experimental data [13-20] they are proposed to calculate by the following degree expression

$$\tau_{\text{bmi}} = \eta_1 \cdot \eta_2 \cdot f_{\text{ntk}} \cdot (\sigma_{\text{si}} / \sigma_y)^{1-1/\eta_s}, \quad (2)$$

where  $\eta_1$  – coefficient taking into account the profile of the reinforcing rod [1] by the Rehm's criterion [21];  $\eta_2$  – coefficient taking into account the influence of the rod diameter [22];  $f_{\text{rntk}}$  – characteristic value of concrete strength on stretching;  $\sigma_{\text{si}}$  – tension in the rod in the section with a crack of the corresponding level;  $\sigma_y$  – extreme stresses in rods ( $\sigma_y = f_y$ );  $1/\eta_s$  – bond intensity parameter (for periodic profile rods –  $\eta_s = \eta_1$ , for smooth profile rods –  $\eta_s = 6 \cdot \eta_1$ ).

Thus, taking into account all of the foregoing provisions, the distance between adjacent first-level cracks at the stretched rod gravity center, with stresses in it  $\sigma_{\text{si}} = \varepsilon_{\text{ctu}} \cdot E_s$ , can be determined by the following dependence

$$s_{r1} = \frac{\varnothing_s}{4 \cdot \eta_1 \cdot \eta_2 \cdot f_{\text{ctk}} \cdot (\varepsilon_{\text{ctu}} \cdot E_s / \sigma_y)^{1-1/\eta_s}} \cdot \frac{N_{\text{ct,cr}}}{A_s} \quad (3)$$

Since the average bond stresses in the cross sections between the cracks are different due to the direct connection with the size of the normal tensions in the reinforcement, then the distance between the normal cracks of the new and previous levels in the bending elements (Fig.1) will always be differentiated among themselves. They can be found in terms of expressions

$$s_{ri+1,j} = \frac{s_{ri,j} \cdot \tau_{bmi+1,j+1}}{\tau_{bmi+1,j} + \tau_{bmi+1,j+1}}; \quad s_{ri+1,j+1} = \frac{s_{ri,j} \cdot \tau_{bmi+1,j}}{\tau_{bmi+1,j} + \tau_{bmi+1,j+1}}, \quad (4)$$

where  $j$  – the crack number of the corresponding level of their formation –  $i$ .

If, however, neglected the difference in the rods stresses in the cross sections with adjacent cracks, then further calculations, including the cracks opening width, can be substantially simplified by taking  $s_{ri+1} = s_{ri} / 2$ .

The normal cracks opening calculation is expedient to perform in accordance with Thomas's hypothesis [23] from the positions of successive accumulation of reinforcement

and concrete mutual displacements  $w_k = 2 \int_0^{0.5s_r} \varepsilon_\tau(z) dz$  on active bond areas located on both

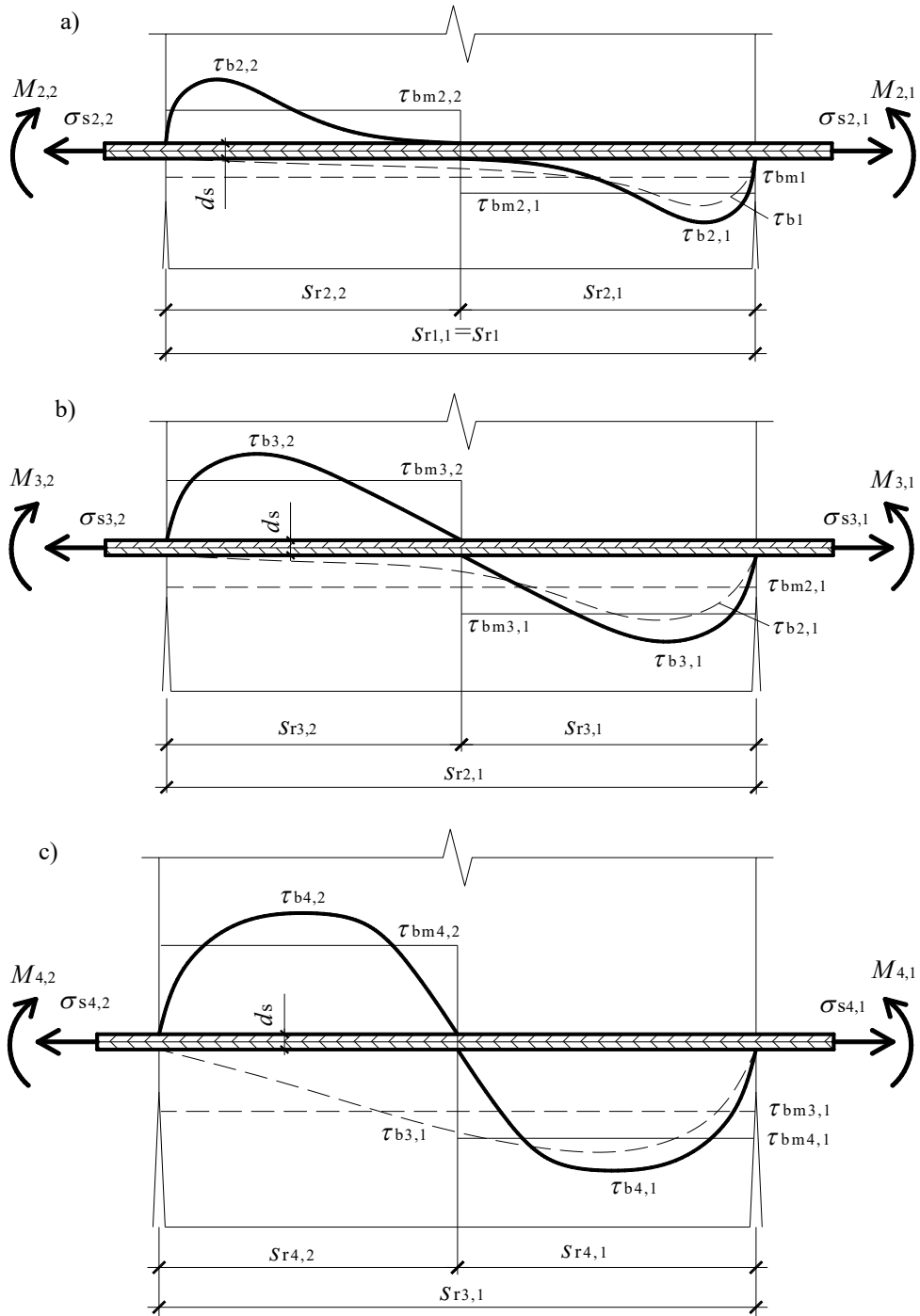
sides of the crack. And because the dependence of mutual reinforcement and concrete displacement  $\varepsilon_\tau(z) = \varepsilon_s(z) - \varepsilon_{\text{ct}}(z)$  is rather complex and can not be described by a single function, then the cracks opening width, taking into account the foregoing, is proposed to be calculated according to the following formula

$$w_k = s_{r1} \cdot \left( \sum_{i=1}^n \varepsilon_{\text{sm},i} - \varepsilon_{\text{ctm}} \right) - s_{ri,j} \cdot \left( \sum_{i=2}^n \varepsilon_{\text{sm},i} - \varepsilon_{\text{ctm}} \right) - \dots - s_{ri,j} \cdot \left( \sum_{i=n}^n \varepsilon_{\text{sm},i} - \varepsilon_{\text{ctm}} \right), \quad (5)$$

where  $\varepsilon_{\text{sm},i}$  – mean values of relative strains of stretched armature in the area between cracks of a certain level;  $\varepsilon_{\text{ctm}}$  – mean values of relative deformations of stretched concrete on the same site.

In general, the average reinforcement deformation in the area between cracks of a certain level should be determined directly from the diagram of the element state (by its curvature) in the averaged section of the block between the cracks  $\varepsilon_{\text{sm},i} = f(1/r)$ . As for

the average deformations of stretched concrete in the corresponding section, it is recommended to calculate them by the formula



**Fig. 1.** Scheme of tangential stresses of reinforcement bond with concrete after the formation of cracks of the first (a), second (b) and third (c) levels.

$$\varepsilon_{ctm} = 0.5 \cdot \varepsilon_{ctu} \cdot (d - x) / (h - x), \quad (6)$$

where  $d$  – working height of the element cross-section;  $h$  – section height of the element of a rectangular profile;  $x$  – compressed zone height in the cross section of the element.

Since in the deformation-power model [1, 12], the value of the average reinforcement deformations in the sections between cracks is controlled by the diagrams of the reinforced concrete elements state throughout the process of their deformation, this allows to link the normal cracks opening width with the determining parameters of the stress and strain state of these elements by a system of generally accepted static, geometric and physical correlations, or equations of mechanics of a deformed solid state:

$$\left. \begin{aligned} & \bullet \quad w_k = s_{r1} \cdot \left( \sum_{i=1}^n \varepsilon_{sm,i} - \varepsilon_{ctm} \right) - s_{ri,j} \cdot \left( \sum_{i=2}^n \varepsilon_{sm,i} - \varepsilon_{ctm} \right) - \dots - s_{ri,j} \cdot \left( \sum_{i=n}^n \varepsilon_{sm,i} - \varepsilon_{ctm} \right); \\ & \bullet \quad \text{static relations } \dot{I}_3 = f(\varepsilon_{\bar{n},3}, \varepsilon_{\bar{n}t,3}, \varepsilon_{s,3}), \quad N = f(\varepsilon_{\bar{n},3}, \varepsilon_{\bar{n}t,3}, \varepsilon_{s,3}); \\ & \bullet \quad \text{geometrical relations with the hypothesis of plane cross sections} \\ & 1/r_3 = f(\varepsilon_{\bar{n},3}, \varepsilon_{s,3}) \quad \text{or} \quad \varepsilon_{s,3} = f(1/r_3); \\ & \bullet \quad \text{physical relations: the elements state } 1/r_3 = f(\dot{I}_3) \quad \text{and materials state} \\ & \sigma_{\bar{n},3} = f(\varepsilon_{\bar{n},3}), \quad \sigma_{\bar{n}t,3} = f(\varepsilon_{\bar{n}t,3}), \quad \sigma_{s,3} = f(\varepsilon_{s,3}). \end{aligned} \right\} \quad (7)$$

## 6 Conclusions

Thus, on the basis of the above, it should be noted that the developed model of the step formation and the disclosure of normal cracks:

- complements the generalized model of reinforced concrete elements deformation;
- is universal, because its basis is as much as possible deprived of the empiricism influence and is acceptable for any reinforced concrete elements undergoing heterogeneous deformation;
- takes into account the qualitative and quantitative difference between the parameters of reinforcement bond with concrete and examines the distance between adjacent cracks as a value that varies discretely throughout the deformation process;
- reflects the dependence of the average tangential stresses of reinforcement with expanded concrete on a number of factors, including the type and profile of the reinforcement itself;
- directly correlates the reinforced concrete elements crack strength with the main parameters of their stress and strain state (curvilinear values  $1/r_i$ , bending moments values  $M_i$ , stresses in the reinforcement  $\sigma_{s,i}$ , its deformations  $\varepsilon_{s,i}$  etc.);
- allows control the process of the formation and disclosure of normal cracks in reinforced concrete elements at any stage of their deformation from the positions of successive accumulation of mutual displacements of the reinforcement relative to the concrete in the active bond areas.

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