Modeling of capillary-sorption process steam condensation in the system of microcapillaries of grain shells

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Abstract. The aim of this study is to simulate the capillary-sorption process of steam condensation in the system of microcapillaries of grain shells [1]. Modeling of the processes of intensive moisture transfer during drying of a number of materials was considered in [2,3]. In order to significantly reduce the time of moistening, moisture transfer by effusion is considered in the form of steam produced at low pressure (vacuum) with its further guaranteed condensation in micro-cavities (microcapillaries) of a porous shell of grain [4]. Study of intensity of moisture transport in accordance with the changes in concentrations of vapour along the length of the capillary in a vacuum proved the possibility of intensifying the process of moistening the grain before grinding. The obtained mathematical model will allow to calculate the values of the theoretical weight gain of moisture in microcapillaries depending on the parameters and the time of vacuum-sorption moisture.

1 Introduction

The processes of heat and mass transfer under vacuum conditions through a porous medium in materials, which include porous shells of grain, are accompanied by a change in the mechanism of mass transfer from molecular diffusion to effusive, in the form of steam [1, 2]. Its further condensation in pores (microcapillaries), commensurate with the size of its molecules, requires taking into account the weight of moisture in them, due to sorption phenomena. Thus, the problem of creating a model of such a process of high-intensity heat and mass transfer is relevant.

2 The timing of vacuum and sorption moistening

Capillary condensation of water vapor occurs only in small capillaries with an effective radius $\tau < 10^{-7} \text{ m}$ and even in cases where the walls of capillaries hydrophobic [1]. It follows that in vacuum and sorption moistening, the gain of moisture in the porous shells of the grains is due to the condensation of moisture in microcapillaries [5].

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To simulate this process, we will present skin capillaries as cylindrical tubes, the length of each of which is significantly greater than its diameter $2r_0$, where r_0 is the radius of a single capillary.

Let's assume that at the time the process of steam transfer to the tube and its deposition on the walls begins. The process of steam condensation can be considered uniform, because, firstly, the diffusion of vapors (as it was found out above) occurs very quickly, (the air from the capillaries is removed during vacuuming), and secondly, the number of condensed vapor molecules depends on the initial temperature difference of the wall, water vapor and surface tension forces that are the same along the entire length of the capillary.

Due to the condensation of steam on the walls, the radius of the part of the tube not occupied by the steam will decrease. Changes in this radius will be denoted by $r(\tau)$ and find out how it depends on time.

During the time $\Delta\tau$ on the walls of the tube, the mass of moisture Δm is condensed, for which the expression is true:

$$\Delta m = \Delta V \cdot \rho_b, \quad (1)$$

where ΔV – the increment of the volume of water layer, m³; ρ_b – water density, kg / m³.

The increment of the volume of moisture ΔV is equal to:

$$\Delta V = 2\pi r \cdot \Delta r \cdot L, \quad (2)$$

where Δr – is the distance between the ends of the tube (shell thickness); L – tube length; $L = \xi \cdot l$, where l – is the distance between the ends of the tube (shell thickness); ξ – the tortuosity of the capillaries (taken equal to 2 m/m with a shell thickness of 1.5 mm [1]).

Mass Δm according to the law Fika [4]:

$$\Delta m = -D \frac{d\rho_n}{dx} \cdot S(\tau) \cdot d\tau, \quad (3)$$

where $S(\tau)$ is the cross-sectional area of the tube at the time $\Delta\tau$ ($S(\tau) = \pi[r(\tau)]^2$); ρ_n – is the vapor density.

Substituting expressions (2) and (3) in (1), we obtain:

$$-D \frac{d\rho_n}{dx} \cdot \pi r^2 \cdot \Delta\tau = 2\pi r \cdot \Delta r \cdot L \cdot \rho_b \quad \text{or} \quad -D \frac{d\rho_n}{dx} \cdot r \cdot \Delta\tau = 2L \cdot \rho_b \cdot \Delta r. \quad (4)$$

Replacing the increment with differentials, we obtain a differential equation with separable variables:

$$-D \frac{d\rho_n}{dx} \cdot d\tau = \frac{dr}{r}. \quad (5)$$

By integrating the expression (5), we obtain:

$$\ln(r) = -\frac{D \frac{d\rho_n}{dx}}{2L \cdot \rho_b} \cdot \tau + C_1, \quad (6)$$

$$\text{or} \quad r(\tau) = C_1 \cdot e^{-\frac{D \rho'_n}{2L \cdot \rho_b} \cdot \tau}. \quad (7)$$

At $\tau = 0$ the radius is r_o , therefore $C_1 = r_o$, $\rho'_n = \rho'_n(0)$ – the gradient of vapor concentration at the inlet of the capillary, then

$$r(\tau) = r_o \cdot e^{-\frac{D\rho'_n(0)}{2L \cdot \rho_B} \cdot \tau} \quad (8)$$

To determine the volume of condensed moisture in the capillary tube, use the expression:

$$\begin{aligned} V_{\text{вп}} &= V_{\text{тр.нач}} - V_{\text{тр.кон}} = \pi r_o^2 L - \pi r_o^2 \cdot \left(e^{-\frac{D\rho'_n(0)}{2L \cdot \rho_B} \cdot \tau} \right)^2 \cdot L = \\ &= \pi r_o^2 L \cdot \left(1 - e^{-\frac{D\rho'_n(0)}{2L \cdot \rho_B} \cdot \tau} \right)^2 \end{aligned} \quad (9)$$

To calculate the volume of moisture according to the formula (6) it is necessary to know the values of D and $\rho'_n(0)$, which depend on steam pressure and temperature. Setting their average values, we can theoretically determine the volume of condensed moisture. The average gradient of the vapor concentration along the length of the tube is equal to:

$$\frac{d\rho_n}{dx} = \frac{\rho_n}{x} \quad (10)$$

The dependences between the mean gradient of vapor concentration and the gradient of concentration at the inlet of the capillary, we establish by solving the diffusion equation with absorption [5]:

$$D \frac{d^2 \rho_n}{dx^2} - K_1 = 0 \quad (11)$$

where K_1 – mass of steam condensed per unit of time per unit of tube length.

Integrating the above equation twice, we obtain:

$$\rho_n = \frac{K_1}{2D} \cdot x^2 + C_1 x + C_2 \quad (12)$$

To determine the values C_1 and C_2 take advantage of the initial conditions:

$$\rho_n(0) = \rho_{n_o},$$

Where ρ_{n_o} – the density of the steam at the inlet of the capillary.

$$\left. \frac{d\rho_n}{dx} \right|_{x=L} = 0 \quad \text{– from the impermeability condition.}$$

Therefore,

$$C_2 = \rho_{n_o}, \quad C_1 = -\frac{K_1}{D} \cdot L$$

Substituting the values found in the equation (11), we obtain:

$$\rho_n = \frac{K_1}{2D} \cdot x^2 - \frac{K_1}{2D} \cdot x \cdot L + \rho_{n_0} \quad (13)$$

Hence

$$\frac{d\rho_n}{dx} = \frac{K_1}{D} \cdot x - \frac{K_1}{D} \cdot L$$

Then under $x=0$:

$$\left. \frac{d\rho_n}{dx} \right|_{x=0} = -\frac{K_1}{D} \cdot L$$

Find the mean gradient of the pair on the segment $(0; L)$ using the formula:

$$\left(\frac{d\rho_n}{dx} \right)_{cp} = \frac{1}{L} \cdot \int_0^L \frac{d\rho_n}{dx} \cdot dx, \quad (14)$$

from which have:

$$\left(\frac{d\rho_n}{dx} \right)_{cp} = \frac{1}{L} \cdot \left(\frac{K_1}{D} \cdot \frac{x^2}{2} - \frac{K_1}{D} \cdot L \cdot x \right) \Big|_0^L = \frac{1}{L} \cdot \left(\frac{K_1}{D} \cdot \frac{L^2}{2} - \frac{K_1}{D} \cdot L \cdot L \right) = -\frac{K_1 L}{2D} \quad (15)$$

The found value of the mean gradient of the vapor concentration on the segment $(0; L)$ is twice less than the gradient at $x=0$.

Gradient of vapor concentration at the capillary inlet:

$$\rho'_n(0) = \frac{d\rho_n}{dx}(0) = \frac{2\rho_n}{L} \quad (16)$$

Substituting expression (16) in equation (9), we obtain:

$$V_{\text{вп}} = \pi r_o^2 L \cdot \left(1 - e^{-\frac{2D\rho_{n,\tau}}{L^2 \cdot \rho_n}} \right) \quad (17)$$

Equation (17) is an expression that can mathematically determine the amount of moisture condensed in a microcapillary, and equation 18 defines the mass of condensed moisture:

$$m_c = \pi \cdot r_o^2 L \cdot \rho_n \cdot \left(1 - e^{-\frac{2D\rho_{n,\tau}}{L^2 \cdot \rho_n}} \right), \quad (18)$$

The average vapor density can be calculated using the Clapeyron – Mendelev equation for rarefied gases:

$$p \cdot V = \frac{m}{\mu} RT \quad (19)$$

Where the vapor density is:

$$\rho_n = \frac{m}{V} = \frac{p\mu}{RT}, \quad (20)$$

where μ – molar mass of water vapor equal to 18 kg / KMOL; R – universal gas constant equal to $8,31 \cdot 10^3$ j/(mol K).

The results of the calculation of the theoretical weight gain of moisture are given in table 1, taking into account the fact that the moisture of the capillaries [6] accounts for 12% of the total grain volume.

Table 1. Values of the theoretical weight gain of moisture in microcapillaries depending on the parameters and time of vacuum sorption moisture

Main parameters vacuum and sorption moistening	The wetting time, s			
	120	240	360	480
The average pressure, PA	25·103	30·103	40·103	50·103
Mean temperature, ° C	55	57	58	59
Average vapor density, kg / m ³	16·10-2	19·10-2	26·10-2	36·10-2
Average speed of thermal motion molecules, m/ s	6,23·102	6,24·102	6,24·102	6,26·102
Mean free path length molecules, m	0,4·10-6	0,37·10-6	0,28·10-6	0,22·10-6
Diffusion coefficient, m2/ s	0,83·10-4	0,76·10-4	0,58·10-4	0,45·10-4
The amount of moisture of the capillary, m3	3,51·10-14	5,25·10-14	5,84·10-14	6,09·10-14
Theoretical weight gain of microcapillary moisture, %	6,9	10,3	11,5	12,0

As can be seen from the above data, with the increase in moisture time, the intensity of steam condensation due to capillary-sorption phenomena falls.

3 Results and discussion

With the effusive mechanism of moisture penetration-moisture transfer in the form of steam occurs almost instantly, and its condensation time is about 2 minutes. The resulting model does not take into account the theoretical weight gain of moisture due to the heat exchange between the processed grain and steam [6].

4 Summary

- 1) The concentration of steam in microcapillaries is from 2 to 6 minutes, further increase in the exposure time in the steam environment in a vacuum does not have a significant effect on the weight gain of moisture in them.
- 2) To intensify the process of moisturizing the grain before grinding, it is possible to use well-known devices for hygrothermal treatment of tannery materials [7,8].

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