Review of methods for solving inverse problems in identifying permanent magnets of executive elements in dynamic devices

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Abstract. Permanent magnets due to their overall-saving and energy-saving properties are increasingly used in the executive elements in dynamic systems of various fields of technology (electric motors, electromagnetic drives, electromechanical devices – relays, contactors, etc.). At the same time, devices with permanent magnets have a disadvantage – under the effect of overheating, mechanical shocks and current surges in the windings, their demagnetization occurs, which can lead to disruption of operation and failure of devices. In this regard, there is a need for identification of permanent magnets, that is, assessment of their condition by investigating the distribution of the magnetization by the volume of the permanent magnets by using the solution of inverse problems. The article provides the overview of methods for solving such problems. The analysis of publications has shown that two approaches to the solution of inverse problems can be distinguished: – the linearization of some functional and the multiple solution of the direct problem of calculating the magnetic field; – solution of the ill-posed problem and determination of the pseudo-solution stable to small perturbations by regularization methods. Both approaches have disadvantages. For example, the application of mesh-based methods in field calculations leads to the need to solve systems of equations of large dimension, parallel calculations are not used, etc. Therefore, the improvement of methods for solving inverse problems in the identification permanent magnets of executive elements in dynamic devices is an actual problem.

1 Introduction

Permanent magnets due to their overall-saving and energy-saving properties are increasingly used in the executive elements in dynamic systems of various fields of technology (electric motors, electromagnetic drives, electromechanical devices – relays, contactors, etc.).

At the same time, devices with permanent magnets have a disadvantage – under the effect of overheating, mechanical shocks and current surges in the windings, their demagnetization occurs, which can lead to disruption of operation and failure of devices.

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In this regard, there is a need for identification of permanent magnets, that is, assessment of their condition by investigating the distribution of the magnetization by the volume of the permanent magnets by using the solution of inverse problems [1, 2]. This is due to the fact that the magnetization cannot be measured directly. However, it can be estimated indirectly by measuring the values of magnetic induction in accessible places. For example, it is possible to measure induction in the air gap of the device or in the space surrounding the magnet, and then, solving the inverse problem, determine the magnetization. The tasks of identification are divided into diagnostic, macro-modeling and flaw detection problems. It is noted that inverse problems are also used for optimal design: for structural and parametric synthesis of objects. The inverse problems of identification and synthesis differ in the form of additional information. Additional information in identification is a collection of measured values of a certain value, for example, magnetic induction. Additional information in synthesis problems is the values of mainly integral characteristics, for example, the force of attraction of the armature to the core of the electromagnet. Synthesis algorithms, unlike identification, may contain a procedure for optimizing any characteristic of an object of investigation, for example, minimizing the mass of the device.

Thus, a critical analysis of methods for solving inverse problems and finding ways to improve them is an actual problem. The following is a review of methods for solving inverse problems in identifying PM of executive elements in dynamic systems.

2 Inverse problems of electrical engineering

As is known, inverse problems refer to ill-posed ones in the classical sense. The nonuniqueness and instability of their solutions are possible. In the article only conditionally correct problems are considered. It is proved that the problems considered in this article have a unique solution, and the stability of the solution is ensured by narrowing the class of unknown functions, that is, by passing to conditionally correct problems (correct according by A. N. Tikhonov) [3-5].

The main part of the algorithms for solving inverse problems has the following stages [2]: 1. Selection of the initial values of the unknown quantities; 2. The solution of the direct problem; 3. Calculation of the values of functionals subject to minimization; 4. Verification of the completion of the conditions for the completion of calculations; 5. If at least one of the conditions is not fulfilled, new values of the unknown quantities are calculated by minimizing the functionals and returning to step 2. When the conditions are satisfied, the problem is solved.

At present, a large number of numerical methods for modeling the magnetic field are developed: methods for reducing field calculation problems to the calculation of equivalent replacement schemes (magnetic circuits) [6], the finite difference method [7], the finite element method [8], the boundary element method [9], the boundary integral equation method [10], the method of spatial integral equations [11], as well as a combination of these methods [12]. The above methods are mesh-based and therefore lead to systems of algebraic equations of large dimension. Mesh-free methods are more economical, but are practically not used at present. For example, this is the method of fundamental solutions and its modifications [13,14] and also approaches, combining mesh-based and mesh-free methods [15, 16].

Parallel calculations are proposed to be used to reduce the calculation time of three-dimensional magnetic fields [17].

Deterministic methods are used to minimize functionals (see section 5 of the main part of the algorithm for solving inverse problems): gradient descent with constant step, steepest gradient descent and coordinate descent [18], and evolutionary and stochastic methods [19].

Recently, combined methods is developed, using the evolutionary (stochastic) method to find the area where the global minimum is located, and the deterministic method to quickly approach the exact value [20].

The solution of the inverse problem of determining the PM magnetization in the design of energy-saving devices is considered in paper [21], but the method can also be used to solve the PM identification problem. The algorithm for solving the inverse problem is an iterative process, at each step of which the direct problem of analyzing the magnetic field by the finite element method is solved and the functional is minimized by the method of gradient descent. The disadvantage of the work is that PM is considered to be magnetized uniformly.

The task of identifying of the PM in the electric machine is considered in paper [22]. The inverse problem was solved by the methods of gradient descent and boundary elements. PM are assumed to be magnetized uniformly. Each pole of the electrical machine contains two magnets.

The method for determining the PM demagnetization curve by the natural-model method based on the iterative approximation of the PM demagnetization curve is proposed in work [23]. The stationary magnetic field is calculated at each step of the algorithm by integrating the magnetization over the volume of the magnet.

The reconstruction of the magnetization state in the PM was considered in work [24]. The arising inverse magnetostatic problem is solved with the help of the conjugate approach based on the Fredkin-Köhler method for solving the direct problem [25] and the regularization method by Tikhonov. The optimal value of the regularization parameter is chosen from the L-curve [26]. Due to the combined FEM-BEM method for calculating the field in combination with the matrix compression methods, the proposed algorithm is effective for identifying objects with open borders.

The method for determining the magnetization of a PM from the known value of the magnetic induction at some point of the surrounding PM space is proposed in work [27]. The PM is assumed to be magnetically homogeneous, sources of an external magnetic field and ferromagnetic bodies are absent. This allows us to estimate the state of a PM without resorting to solving a system of algebraic equations.

The paper [28] also considers a solitary PM, which can be heterogeneous. The PM is divided into small elements, each of which is considered magnetically homogeneous. Magnetic induction is measured in the surrounding PM space. Then the connection between the magnetization values of the elements and the values of the measured magnetic induction is established. The resulting system of algebraic equations is poorly conditioned and is solved by the regularization method of A.N. Tikhonov [3]. The regularization parameter is given in the form of a decreasing sequence. In the analyzed article, the second approach to the solution of the inverse problem of PM identification is actually proposed: the the formulation and solution of an ill-posed problem and the determination by means of the regularization method of a pseudo-solution that is stable to small perturbations of experimental data.

This approach is shown in paper [29], where an electromagnetic system is considered, composed of PM and magnetically soft carbide ferromagnets instead of a solitary PM. The characteristics of the carbide ferromagnets are unambiguous and known. The reaction field of a ferromagnet is taken into account by adding to the model an integral equation of the second kind relative to the densities of the scalar field sources placed on the ferromagnets of the framework.

The paper [30] is devoted to the diagnosis of demagnetization of PM in synchronous electromotor. The main methods of diagnostics are indicated in the article, as well as ways to increase resistance to demagnetization of PM by constructive changes in electric motors. A method based on the analytical model and the gradient descent method is also proposed, which makes it possible to determine the region of local demagnetization of the PM in the motors.

A review of papers devoted to the method for determining the magnetization of soft magnetic materials, which can be used to identify the PM, is given below.

The problem of reconstructing the ship's magnetization from the measured values of the magnetic field is solved in paper [31, 32]. The mathematical model is obtained in the form of an integral equation of the first kind with respect to a vector function, namely, the magnetization of the elements of the hull of the ship. Each element of the discretization is considered to be magnetized uniformly. Since the problem is ill-posed, a regularizing algorithm based on minimizing the A. N. Tikhonov functional with the choice of the regularization parameter in accordance with the generalized discrepancy principle and using the conjugate gradient method is used. One-dimensional, two-dimensional, three-dimensional problems of calculating fields are considered. The algorithm of calculation in multiprocessor systems is developed. However, even the use of parallel computations does not guarantee a quick solution of the problem. The calculation time is 29 hours for 22500 sampling elements in the three-dimensional formulation of the problem (67.500 unknown components of the magnetization vector), using the supercomputer complex of the Moscow State University.

The problem of magnetostatic flaw detection was solved in paper [33]: from the known pattern of the field, the deviations of the magnetic susceptibility from the given values in the volume of the object under study are determined. The inverse problem is formulated in the form of a system of three-dimensional integral equations of the first and second kind with respect to magnetization. Regularizing constraints are introduced. It is established that the choice of initial data affects the result. The solution has the best stability and sufficient accuracy when using field values in certain parts of the object.

The inverse magnetostatic problem is reduced to a system of integral equations for the magnetization and the intensity inside the ferromagnetic body in [34]. An analytical solution is obtained on the basis of such a model for a ball with a model magnetic permeability in a constant external field. In this example, the problem of the uniqueness of the solution of the inverse problem is investigated. The analytical model is used to identify demagnetization defects in synchronous machines with PM [35].

Parallel calculations for solving the three-dimensional structural inverse magnetometry problem are used in paper [36].

The use of the natural-model approach combining field tests and modeling of magnetic fields is a promising direction in the identification of ferromagnetic materials [37, 38].

An effective solution of the coupled experimental-numerical inverse problem for the identification of the properties of magnetic materials is given in papers [39, 40]. The proposed methodology allows a priori to estimate the result error, which takes into account the measurement noise and uncertainty of the parameters of the mathematical model, using the method of the lower boundary of Cramer-Rao. The stochastic Bayesian approach is used to reduce the error when using a coarse model. This approach is used to identify magnetic material within large-scale industrial installations, but can be used for small-scale objects, for example, for the reconstruction of magnetic nanoparticles.

3 Conclusions

It follows from the above review that two approaches to solving the inverse problem of identifying the parameters of the PM are applied: multiple solutions of the direct problem of magnetic field calculation by one of the numerical methods and minimization of the functional corresponding to the problem by the gradient descent method; formulation and solution of the ill-posed problem and determination of the pseudo-solution that is stable to small perturbations, using the regularization method. The first approach is more universal and is used both in identification and in the optimal design of new devices. The second approach is used more often, but only when identifying the parameters. It is revealed that the solution of inverse problems of identification by mesh-based methods is extremely resource-consuming, therefore parallel computing is used. The deterministic and stochastic approaches to the estimation of the inaccuracy of solving inverse identification problems are applied. The problem of the arrangement of the sensors for measuring the magnetic field remains unsolved. Insufficient attention is paid to the application of the hierarchy of mathematical models. All this points to the need to further improve the methods for solving the inverse problems of identifying the permanent magnets of the executive elements of dynamical systems.

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