

Evaluation of fatigue life of polycrystalline structural alloys

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Abstract. A mathematical model describing the processes of elastoplastic deformation and damage accumulation under low-cycle loading has been developed, based on the viewpoint of mechanics of damaged media (MDM). The MDM model consists of three interrelated parts: defining relations describing elastoplastic behavior of the materials, taking into account its dependence on the failure process; evolutionary equations describing the kinetics of damage accumulation; strength criteria of the damaged material. In order to assess the reliability and scope of applicability of the defining relations of mechanics of damaged media, the processes of plastic deformation and damage accumulation in variety of structural steels in low-cycle tests have been numerically analyzed, and numerical results obtained have been compared with the data of full-scale experiments. It is shown that the presented model of mechanics of damaged media adequately describes, both qualitatively and quantitatively, with accuracy, necessary for practical calculations, the main effects of the processes of plastic deformation and damage accumulation in structural alloys under block-type non-stationary non-symmetrical low-cycle loading.

1 Introduction

Cyclic properties of structural materials are of great importance for reliable evaluation of the strength and service life of elements and components of load-bearing structures under alternating combined thermo-mechanical effects. Calculation of the resource of structural elements based on the finite element analysis of inelastic deformations in hazardous zones of structural elements requires the formulation of the defining thermoplastic relations taking into account the real cyclic properties of materials [1].

Equations of state constructed on the basis of monotonous loading processes and not taking into account the peculiarities of cyclic deformation under proportional and disproportional loadings can lead to large errors in determining the basic parameters of the stress-strain state, which are then used to evaluate the resource characteristics of the material. The formulation of reliable defining thermoplastic relations for these processes requires, first of all, experimental studies of the effects of cyclic behavior of structural materials under proportional and disproportional loadings [2].

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The classical methods of predicting the fatigue life of materials using semi-empirical formulas (rules) based on a stabilized analysis of the deformation process and relating parameters of plastic hysteresis loops with the number of cycles prior to failure require a large amount of experimental deformation and are valid only for a narrow class of loading regimes within the available basic experimental data [3].

In recent years, to solve such problems, a new research area is being successfully developed – the mechanics of damaged medium (MDM) [4, 5].

2 Defining relations of MDM

The MDM model, developed in [4, 5], consists of three interrelated parts:

- relations defining elastoplastic behavior of materials, accounting for the effect of the failure process;
- evolutionary equations describing the kinetics of damage accumulation;
- strength criteria of the damaged material.

2.1 Defining relations of plasticity

Defining relations of plasticity [2] are based on the following main principals:

- components of stress tensors e_{ij} and their velocities \dot{e}_{ij} include elastic deformations e_{ij}^e , \dot{e}_{ij}^e and plastic deformations e_{ij}^p , \dot{e}_{ij}^p ;
- initial yield surface for different temperatures is described using a Mises form of the yield surface. Evolution of change in yield surface is described by the change in its radius C_p and its centre ρ_{ij} ;
- change in body volume is elastic;
- initially isotropic media are being considered. Only anisotropy due to plasticity processes is taken into account.

To describe the effects of monotonous and cyclic deformation, the yield surface is introduced in the form [2]:

$$F_s = S_{ij}S_{ij} - C_p^2 = 0, \quad S_{ij} = \sigma'_{ij} - \rho_{ij}. \quad (1)$$

To describe complex cyclic deformation modes in the stress space, a cyclic ‘memory’ surface is introduced:

$$F_p = \rho_{ij}\rho_{ij} - \rho_{\max}^2 = 0, \quad (2)$$

where ρ_{\max} is maximum module in the loading history ρ_{ij} .

Evolutionary equation for the yield surface radius has the form [2]:

$$\dot{C}_p = [q_\chi H(F_\rho) + a(Q_s - C_p)G(F_\rho)]\dot{\chi} + q_3\dot{T} \quad (3)$$

$$C_p = C_p^0 + \int_0^t \dot{C}_p dt, \quad \dot{\chi} = \left(\frac{2}{3} \dot{e}_{ij}^p \dot{e}_{ij}^p \right)^{1/2}, \quad \chi_m = \int_0^t \dot{\chi} H(F_\rho) dt, \quad \chi = \int_0^t \dot{\chi} dt. \quad (4)$$

$$q_\chi = \frac{q_2 A \psi_1 + (1-A)q_1}{A \psi_1 + (1-A)}, \quad Q_s = \frac{Q_2 A \psi_2 + (1-A)Q_1}{A \psi_2 + (1-A)}, \quad 0 \leq \psi_i \leq 1, \quad i=1,2.$$

$$A = 1 - \cos^2 \theta, \quad \cos \theta = n_{ij}^e n_{ij}^s, \quad n_{ij}^e = \frac{\dot{e}_{ij}'}{(\dot{e}_{ij}^e \dot{e}_{ij}^e)^{1/2}}, \quad n_{ij}^s = \frac{S_{ij}}{(S_{ij} S_{ij})^{1/2}}, \quad (5)$$

$$H(F_\rho) = \begin{cases} 1, F_\rho = 0 \wedge \rho_{ij} \dot{\rho}_{ij} > 0 \\ 0, F_\rho < 0 \vee \rho_{ij} \dot{\rho}_{ij} \leq 0 \end{cases}, \quad \Gamma(F_\rho) = 1 - H(F_\rho).$$

where q_1, q_2, q_3 are modules of isotropic hardening, Q_1 and Q_2 are modules of cyclic isotropic hardening, a is constant, that determines the speed of the process of stabilization of the shape of the hysteresis loop of cyclic deformation of the material, Q_s is stationary value of the yield surface radius for the given ρ_{\max} and T , C_p^0 are initial values of the yield surface radius. We postulate that the evolution of an internal variable ρ_{ij} is assumed in the form:

$$\dot{\rho}_{ij} = f(\chi_m) [g_1 \dot{\epsilon}_{ij}^p - g_2 \rho_{ij} \dot{\chi}] + g_T \rho_{ij} \langle \dot{T} \rangle + \dot{\rho}_{ij}^*, \quad \rho_{ij} = \int_0^t \dot{\rho}_{ij} dt, \quad f(\chi_m) = 1 + k_1 (1 - e^{-k_2 \chi_m}), \quad (6)$$

$$\dot{\rho}_{ij}^* = g_3 \dot{\epsilon}_{ij}^p H(F_\rho) - g_4 \rho_{ij} \dot{\chi} \Gamma(F_\rho) \langle \cos \beta \rangle, \quad (7)$$

$$\langle \cos \beta \rangle = \frac{\dot{\rho}_{ij} \rho_{ij}}{(\dot{\rho}_{ij} \dot{\rho}_{ij})^{1/2} (\rho_{ij} \rho_{ij})^{1/2}}, \quad (8)$$

where $g_1, g_2, g_3, g_4, g_T, k_1$ and k_2 are experimentally determined material parameters. For asymmetric rigid and soft cyclic loading due to $\dot{\rho}_{ij}^*$ the equation (6) describes the processes of landing and "ratcheting" of the cyclic plastic hysteresis loop. At $g_T = g_3 = g_4 = k_1 = 0$ from (9) we obtain a special case of equation (6) - the Armstrong-Frederic-Kadashevich equation.

$$\dot{\rho}_{ij} = g_1 \dot{\epsilon}_{ij}^p - g_2 \rho_{ij} \dot{\chi}. \quad (9)$$

To characterize the behavior of the "memory" surface, it is necessary to formulate an evolution equation for ρ_{\max} :

$$\dot{\rho}_{\max} = \frac{(\rho_{ij} \dot{\rho}_{ij}) H(F_\rho)}{(\rho_{mn} \rho_{mn})^{1/2}} - g_2 \rho_{\max} \dot{\chi} - g_T \rho_{\max} \langle \dot{T} \rangle. \quad (10)$$

At the stage of development of the scattered by volume damages, the effect of damage degree on the physico-mechanical properties of material is observed. This effect can be accounted by the introducing the effective stresses [4]:

$$\tilde{\sigma}'_{ij} = F_1(\omega) \sigma'_{ij} = \frac{G}{\tilde{G}} \sigma'_{ij} = \frac{\sigma'_{ij}}{(1-\omega) \left[1 - \frac{(6K+12G)\omega}{(9K+8G)} \right]}, \quad (11)$$

$$\tilde{\sigma} = F_2(\omega) \sigma = \frac{K}{\tilde{K}} \sigma = \frac{\sigma}{4G(1-\omega)/(4G+3K\omega)}, \quad (12)$$

where \tilde{G}, \tilde{K} are effective elastic moduli determined by the McKenzie formulas [4].

The effective variable $\tilde{\rho}_{ij}$ is defined similarly:

$$\tilde{\rho}_{ij} = F_1(\omega) \rho_{ij}. \quad (13)$$

2.2. Evolutionary equations of fatigue damage accumulation

Let us postulate that the speed of damage accumulation under low cycle fatigue (LCF) is determined by an evolution equation of the form [4, 5]:

$$\dot{\omega} = f_1(\beta) f_2(\omega) f_3(W) f_4(\dot{W}), \quad (14)$$

where functions f_i , $i=1..4$ account for volumetric character of the stress state ($f_1(\beta)$), level of damage accumulation ($f_2(\omega)$), accumulated relative energy of damage resulted in macrocrack formation ($f_3(W)$) and rate of change in energy of damage ($f_4(\dot{W})$).

In equation (14):

$$f_1(\beta) = \exp(\beta), \quad f_2(\omega) = \begin{cases} 0, & W \leq W_a \\ \omega^{1/3} (1-\omega)^{2/3} \wedge W > W_a \wedge \omega \leq 1/3 \\ \frac{\sqrt[3]{16}}{9} \omega^{-1/3} (1-\omega)^{-2/3} \wedge W > W_a \wedge \omega > 1/3 \end{cases} \quad (15)$$

$$f_3(W) = \frac{W - W_a}{W_f}, \quad f_4(\dot{W}) = \dot{W} / W_f. \quad (16)$$

where β is parameter of the volumetric character of the stress state ($\beta = \sigma/\sigma_u$), W_a is damage energy for the formation of scattered fatigue damage under low cycle fatigue (LCF); and W_f is energy of microcrack formation.

2.3. Criterion of strength of damaged material

The condition when the damage degree ω reaches its critical value is taken as a criterion of the completion of the phase of development of scattered microcracks:

$$\omega = \omega_f \leq 1. \quad (17)$$

3 The investigation results

Below are the results of experimental and theoretical studies of samples of 12X18H9 stainless steel under rigid non-stationary asymmetric cyclic loading consisting of two blocks:

- on the first block the sample is compressed until $e_{11} = 0,01$ deformation, and then stretches to $e_{11} = 0,05$ deformation;

- on the second block, asymmetric rigid cyclic loading is realized with a range of deformations $\Delta e_{11} = e_{11}^{(+)} - e_{11}^{(-)} = 0,01$ up to $N_f = 850$ destruction. Tables 1-3 presents the main physical and mechanical characteristics and material parameters of the MDM model used in the calculations for 12X18H9 steel.

Table 1. Physical-mechanical characteristics and parameters of MDM model of 12X18H9 stainless steel.

K	G	C_p^o , MPa	g_1 , MPa	g_2	g_3 , MPa	g_4	k_1 , MPa	k_2	a	W_a	W_f
165277	76282	190	24090	286	800	2	10000	0,2	5	0	800

Table 2. Value of cyclic hardening modulus $Q_s(\rho_{max})$ (MPa) 12X18H9 stainless steel.

Q_s , МПа	190	205	210	215	220	225	225
P_{max} , МПа	0	20	40	60	80	100	120

Table 3. Value of monotonous hardening modulus q_χ (MPa) of 12X18H9 stainless steel.

q_χ , МПа	-5000	-4471	-4188	-3859	-2460	-182	888	1531	1274	913	913	913
χ	0	0,002	0,004	0,006	0,008	0,01	0,015	0,02	0,03	0,04	0,05	0,06

Fig. 1 depicts deformation process of 12X18H9 stainless steel on the second loading block (the 500th cycle).

Fig. 2 shows the change in average cycle stress $\sigma_{11}^{(m)}$ under cyclic loading on the second block. It can be seen that a particular case of the thermoplastic model using defining relation for ρ_{ij} in the form (9) implements the hysteresis loop immediately on the first loading cycle (dotted curve in Fig. 2), i.e. this variant of defining relations of plasticity can not describe the process of fitting of the cyclic hysteresis loop.

The model with the use of defining relation for ρ_{ij} in the form (6) qualitatively and quantitatively describes the fitting process.

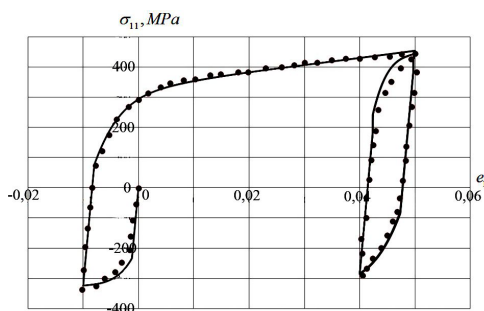


Fig. 1

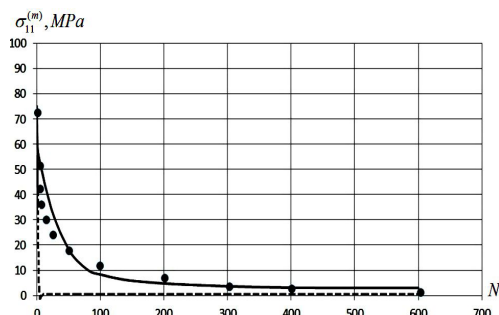


Fig. 2

Fig. 3 shows the fatigue curve of stainless steel 12X18H9 with rigid symmetrical cyclic loading. Here the solid line shows the experimental curve, and the markers show the results of calculations using defining relations of the MDM. In Fig. 3 round marks show the calculated data of thermoplasticity model using the defining relations for ρ_{ij} in the form (6), and square markers show the data using the ratio for ρ_{ij} in the form (9). It can be seen that the calculations of fatigue life on both models almost coincide.

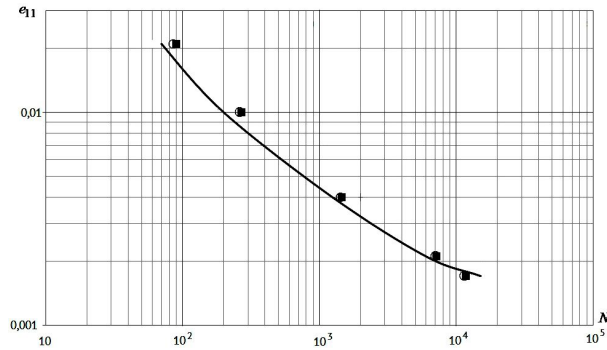


Fig. 3. the fatigue curve of stainless steel 12X18H9 with rigid symmetrical cyclic loading.

4 Conclusion

To describe the fatigue life of polycrystalline structural alloys under block asymmetric low-cycle loading, the reliability of defining relations of the MDM has been assessed by comparing the results of numerical experiments with experimental data on plastic deformation and damage accumulation in 12X18H9 steel under block unsteady asymmetric low-cycle loading, which corroborated the adequacy of the modeling and determining material parameters .

The work is financially supported by the Federal Targeted Program for Research and Development in Priority Areas of Development of the Russian Scientific and Technological Complex for 2014-2020 under the contract No. 14.578.21.0246 (unique identifier RFMEFI57817X0246).

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