Formalization of dynamic model of pneumatic drive with variable structure

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Abstract. The work is devoted to solving the actual technical problem of increasing the speed and accuracy of pneumatic servo drives. Pneumatic drives have a large number of advantages (high speed of the output link, environmental friendliness, low cost, etc.). But having a high compressibility of compressed air limits the possibility of realizing optimal trajectories of motion of control objects. The complexity in the organization of controlling the follow-up pneumatic drive is also introduced by a mathematical apparatus that takes into account the thermodynamic processes during the filling and emptying of the working cavities of a pneumatic cylinder. In connection with this, the goal of this work was the development of a mathematical model of a servomotor with a variable structure that takes into account the various structures of pneumatic valves with proportional control. The proposed mathematical model will make it possible to use the synergetic approach in controlling the pneumatic drive. This makes it possible to take into account not less important drive parameters such as energy efficiency, etc., with increasing speed and accuracy of the drive.

Introduction

Increasing the accuracy of positioning and speed of pneumatic actuators is undoubtedly associated with the improvement of existing control laws.

The use of classical control laws based on the use of typical regulators allows controlling overshoot, oscillation, time of the transient process, as well as established errors. However, the use of linear control laws does not provide a sufficiently effective and accurate control of the pneumatic drive, which is associated with the complexity of formalizing the thermodynamic processes and the nonlinear dynamics of the pneumatic system, the model of which is difficult to linearly approximate [1-5].

Now, the researches of many scientists, conducted in line with the improvement of the laws of management, are known. In [6], an adaptive quasi-optimal control law is pro-posed that improves the accuracy of positioning and the speed of the pneumatic drive. In [7-9], the pneumatic drive is considered as an adaptive system with variable structure, in which a sliding mode of transition from a motion correspond-ing to one linear structure is realized to a motion corresponding to another linear structure.

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In addition, methods have been developed to control the speed of the pneumatic drive using a fuzzy logic apparatus [10, 11].

Thus, research in the field of regulation of pneumatic systems is at the moment very relevant, using methods and approaches of both classical control theory and the apparatus of the modern control theory of nonlinear systems [12].

In this article, a detailed derivation of the mathematical model of the pneumatic drive and the method of controlling the pneumatic drive using the synergetic approach are considered.

Equation of motion of a mechanical part of the subsystem

In accordance with the design scheme presented in figure 1, we present a system of remote control, which relates the coordinate and speed of the piston in the cylinder, as well as the driving force, friction and load, in the following form:

$$\begin{cases} \dot{\ell} = \mathcal{V}; \\ M \cdot \dot{\mathcal{V}} + \mathcal{F}_{\text{тр}} = (S_1 P_1 - S_2 P_2 - P_a(S_1 - S_2) - N) \end{cases} \quad (1)$$

where ℓ – the piston displacement coordinate, $\mathcal{V}$ – moving speed of moving masses, M – the mass of the movable part of the piston and rod; $\mathcal{F}_{\text{тр}}$ – the sum of frictional forces; $S_1 S_2$ – the effective areas of the piston and rod cavities of the pneumatic cylinder; P_1 и P_2 – air pressures in the piston and rod cavities of the cylinder; P_a – atmospheric pressure acting on the end surface of the rod; N – force action of the load.

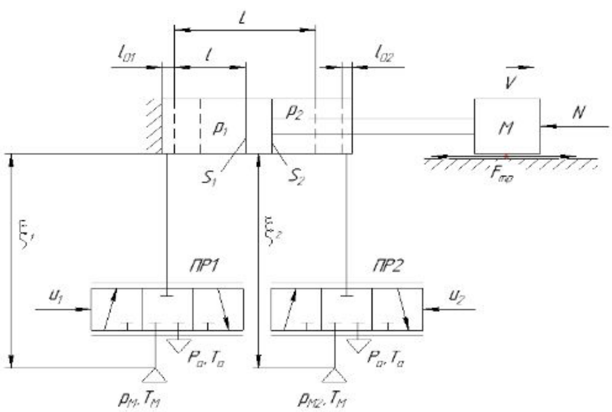

Fig. 1. The tracking scheme of the servantpneumatic drive.

The magnitude of the frictional force $\mathcal{F}_{\text{тр}}$consists of viscous friction $F_{\text{вяз}}$, dry friction $F_{\text{сух}}$ and frictional friction (sticking) $F_{\text{пок}}$: $\mathcal{F}_{\text{тр}} = F_{\text{вяз}} + F_{\text{сух}} + F_{\text{пок}}$.

Viscous friction manifests itself in the resistance of the medium to the relative shift of the gas layers with respect to each other and is determined by the formula:

$$F_{\text{вяз}} = k_{\text{Втр}} \cdot \mathcal{V} \quad (2)$$

where $k_{\text{Втр}}$ – the coefficient of viscous friction.

Dry friction can be expressed by the following dependence:

$$F_{\text{сух}} = \begin{cases} 0; \ at \quad \mathcal{V} = 0 \\ F_c \cdot sign(\mathcal{V}) \quad \text{in other cases} \end{cases} \quad (3)$$

The frictional force of rest (sticking) is a non-linear dependence of the straining force of the rod $F_{стр}$ and is expressed as:

$$F_{пок} = \begin{cases} 0; \text{ at } \mathcal{V} = 0; \\ F_{стр} \cdot sign(\mathcal{V}) \quad \text{in other cases} \end{cases} \tag{4}$$

When simulating the pneumatic system, it is important to know the numerical value of the frictional forces, which depend on different parameters, for example, such as the construction and material of the drive, geometric dimensions, and roughness. The methods for determining frictional forces are reduced to carrying out statistical experimental studies on special stands in order to obtain empirical data of frictional forces.

Equation of change of air pressure in the field of pumping pneumociliner

The change in pressure in the cylinder injection cavity is related to the airflow in the pneumatic distributor and is based on such fundamental laws of physics as the law on the state of an ideal gas and the laws of conservation of mass and energy [13].

According to the law of conservation of energy, the amount of energy supplied with gas $\dot{Q}_1$ is expended on changing the internal energy of the gas $\dot{U}_1$ and on performing external work by the gas inside the cavity $\dot{A}_1$:

$$\dot{Q}_1 = \dot{U}_1 + \dot{A}_1 \tag{5}$$

The amount of energy $\dot{Q}_1$ entering the cavity with a variable amount of mass of the gas is determined by the following relationship:

$$\dot{Q}_1 = q\dot{m} = i\dot{m} = C_p T_м \cdot \dot{m} \tag{6}$$

where q- is the specific thermal energy of the incoming gas, which is determined by its heat content as an enthalpy parameter i , related to the specific heat of the air at constant pressure C_p and the gas temperature in the main $T_м$.

Expressing the change in the mass of gas $\dot{m}$ enter-ing the cavity through the mass flow of gas $G_1$and substituting in (6), we obtain the amount of energy entering the injection zone:

$$\dot{Q}_1 = C_p T_м \cdot G_1 \tag{7}$$

The change in the internal energy of the gas can be written as:

$$\dot{U}_1 = C_V T_1 \dot{m}_1 \tag{8}$$

where C_V – the specific heat of the gas at a constant volume, T_1 – the temperature of the gas in the injection cavity, m_1- the mass of the gas in the injection cavity.

From the formula of the Mendeleev-Clapeyron law on the state of an ideal gas, let us express the mass of gas:

$$m_1 = \frac{P_1 W_1}{R T_1} \tag{9}$$

And substituting in (8) we get:

$$\dot{U}_1 = \frac{C_V}{R}\frac{d}{dt}(P_1 W_1) = \frac{C_V}{R} W_1 \dot{P}_1 + \frac{C_V}{R} P_1 \dot{W}_1 \tag{10}$$

The work done by the gas is expressed as:

$$\dot{A}_1 = P_1\dot{W}_1 \tag{11}$$

Substituting (11), (12), and (7) into the initial equation of the law of thermodynamics (5), we obtain:

$$C_pT_M \cdot G_1 = \frac{C_V}{R}W_1\dot{P}_1 + \frac{C_V}{R}P_1\dot{W}_1 + P_1\dot{W}_1 = \frac{C_V}{R}W_1\dot{P}_1 + P_1\dot{W}_1(\frac{C_V}{R} + 1) \tag{12}$$

Multiplying all the terms of the equation by R/C_V , taking into $k = C_p/C_V$ account what is the adiabatic exponent for air, and the gas constant $R = C_p - C_V$ is:

$$kRT_M \cdot G_1 = kP_1\dot{W}_1 + W_1\dot{P}_1 \tag{13}$$

Express the pressure in the injection cavity of the cylinder:

$$\dot{P}_1 = \frac{kRT_M \cdot G_1}{W_1} - \frac{kP_1\dot{W}_1}{W_1} \tag{14}$$

The volume of the cavity W_1 consists of a working - a variable volume of the cavity of the pneumatic cylinder W_{1P} and an initial, constant volume of the pneumatic drive W_{01} and can be expressed as follows:

$$W_1 = W_{1P} + W_{01} = S_1(\ell + \ell_{01}) \tag{15}$$

where S_1 - is the area of the piston; ℓ_{01}иℓ – the initial and actual coordinate of the piston position

Substituting (15) into (16), we obtain the equation of pressure variation in the cavity of the air cylinder filling:

$$\dot{P}_1 = \frac{kRT_M \cdot G_1}{S_1(\ell+\ell_{01})} - \frac{kP_1\mathcal{V}}{S_1(\ell+\ell_{01})} \tag{16}$$

Equation of air pressure changes in the exhaust penetration of pneumocylinder

Note that the equation connecting the change in pressure with the air flow in the exhaust cavity of the cylinder is derived taking into account the conditional assumptions adopted in the previous paragraph.

The amount of energy $\dot{Q}_2$, flowing with the gas from the exhaust cavity is directed at changing the internal energy of the gas in the cavity $\dot{U}_2$and on the performance of the drive

$$-\dot{Q}_2 = \dot{U}_2 + \dot{A}_2 \tag{17}$$

The amount of energy flowing with the gas from the exhaust cavity:

$$\dot{Q}_2 = C_pT_2\dot{m}_2 = C_pT_2 \cdot G_2 \tag{18}$$

The equation of change of internal energy:

$$\dot{U}_2 == \frac{C_V}{R}P_2\dot{W}_2 + \frac{C_V}{R}W_2\dot{P}_2 \tag{19}$$

In the exhaust cavity, work is associated with gas compression and is determined by the equation:

$$\dot{A}_2 = P_2\dot{W}_2 \tag{20}$$

Substituting the value of the equation for determining the amount of energy (18), the equation for changing the internal energy (19), and the equation of the gas operation in the exhaust cavity (20) into the initial equation (17), we obtain:

$$-kRT_2 \cdot G_2 = kP_2\dot{W}_2 + W_2\dot{P}_2 \tag{21}$$

The pressure change:

$$\dot{P}_2 = \frac{-kRT_2 \cdot G_2}{W_2} - \frac{kP_2\dot{W}_2}{W_2} \tag{22}$$

Volume of gas in the exhaust chamber:

$$W_2 = S_2 \cdot (L + \ell_{02} - \ell) \tag{23}$$

Substituting (23) into (22) after some transformations, we obtain the equation of pressure change in the drain cavity of the air cylinder:

$$\dot{P}_2 = \frac{-kRT_2 \cdot G_2}{S_2(L-\ell+\ell_{02})} + \frac{kP_2\mathcal{V}}{S_2(L-\ell+\ell_{02})} \tag{24}$$

The mass air flow $G_1 и G_2$ in formulas (16) and (24) is regulated by means of a pneumatic distributor, the derivation of which is presented in the next paragraph.

Equations of mass flow of air distributor air

The pneumatic distributor is the key link in the pneumatic actuator control system, since it allows controlling incoming and outgoing compressed air flows with high accuracy and speed.

Modeling the mass airflow of a pneumatic separator entering and leaving the air cylinder chamber is quite a challenge [14,15]. This is due to significant differences in pressure at the entrances and outlets of the openings of the pneumatic distributor, which lead to compressibility and turbulence of the air flow. Therefore, now there is no universal description of the mass air flow model.

In this paper, a model of mass airflow of a pneumatic transducer is used for filling and draining the chambers of a pneumatic cylinder developed by Donskoi A.S. [16], taking into account the isothermal process of gas flow, which takes place at constant values of temperature, pressure and volume of gas.

Calculation of the mass flow of air reduces to the following system of equations:

$$\begin{gathered} \boldsymbol{G_1} = \frac{\boldsymbol{f p_{\text{м}}}}{\sqrt{\boldsymbol{RT_{\text{м}}}}} \cdot \boldsymbol{\varphi(\sigma, \xi)}; \\ \boldsymbol{\varphi(\sigma, \xi)} = \sqrt{\frac{\boldsymbol{1-\sigma^2}}{\boldsymbol{\xi_1 - 2ln\sigma}}},\ \boldsymbol{at}\ \boldsymbol{\sigma_{\text{пр}}} < \boldsymbol{\sigma} < \boldsymbol{1}; \\ \boldsymbol{\varphi(\sigma, \xi)} = \boldsymbol{\sigma_{\text{пр}}},\quad \boldsymbol{at}\ \boldsymbol{\sigma_{\text{пр}}} \leq \boldsymbol{1}, \end{gathered} \tag{25}$$

where $\boldsymbol{f_1}$, $\boldsymbol{f_2}$ – the areas of the cross-sections of the openings of the pneumatic distributor; $\boldsymbol{R}$ - the universal gas con-stant; $\boldsymbol{T_{\text{м}}}$- gas temperature in the pipeline; $\boldsymbol{\xi_1}$, $\boldsymbol{\xi_2}$ – total coef-ficients of resistance of pneumatic equipment; $\boldsymbol{p_1}$, $\boldsymbol{p_2}$- the pressure in the rod less and rod chamber; $\boldsymbol{p_a}$- atmospheric pressure; $\boldsymbol{p_{\text{м}}}$– supply pressure of the pneumatic drive from the pipeline.

In this paper, we consider a pneumatic distributor with electromagnetic proportional control, which converts the input electrical signal - voltage u_i on the electromagnet into the cross-sectional area of the opening f_i of the air distributor:

$$f_i = \cdot\, u_i \cdot k$$

where k – voltage transfer ratio.

Generalized mathematical model of pneumatic system

The final view of the mathematical model of the pneumatic system, taking into account the absence of heat exchange with the environment and at constant compressed air parameters in the main line, will take the form of a system of nonlinear differential equations of the fourth order:

$$\begin{cases} \frac{dl}{dt} = V; \\ \frac{dV}{dt} = \frac{1}{M} \cdot \left(S_1 p_1 - S_2 p_2 - k_{\text{Втр}} \cdot (V) - F_{\text{сух}} - N - P_a (S_1 - S_2) \right); \\ \frac{dp_1}{dt} = \frac{k f_1 \sqrt{R T_{\text{М}}}}{S_1 (l + l_{01}) \sqrt{\xi}} \cdot \sqrt{p_{\text{М}}^2 - p_1^2} - \frac{k p_1 V}{(l + l_{01})}; \\ \frac{dp_2}{dt} = -\frac{k f_2 \sqrt{R T_{\text{M}}}}{S_2 (L - l + l_{02}) \sqrt{\xi_2}} \cdot \left(\frac{p_2}{p_a} \right)^{\frac{k-1}{2k}} \cdot \sqrt{p_2^2 - p_{\text{A}}^2} + \frac{k p_2 V}{(L - l + l_{02})}. \end{cases} \tag{26}$$

The equation of motion of the piston reflects the mechanics of the executive body of the pneumatic drive, and consists of constant quantities, the solution of which is to find the values of displacement, velocity and pressure in the cavities of the pneumatic cylinder at each moment of time.

The equation for changing the air pressure in rod less and rod cavities of a pneumatic cylinder is more complicated for analyzing the filling or discharge of a cavity.

When the rod of the pneumatic cylinder extends forward, the rod less cavity is filled with compressed air, and the air enters the atmosphere from the rod cavity. These processes correspond to the system of equations (26).

When the rod is pulled backwards, reverse processes of filling and discharge of the cavities take place, the rod area receives air under pressure, and from the rod less air, it flows into the atmosphere.

But such a model does not take into account the processes of the expiration of compressed air when the structure of the pneumatic valves is changed (Fig. 1) for realizing the optimal trajectories of the control object. For example, when moving to the right for intensive braking of the control object, the pneumatic distributor ПР1 can be switched to theright position to discharge air from the rod less cylinder cavity, and the pneumatic separator ПР2 switches to the left position to supply compressed air to the rod cavity. In addition, at one position of the distributor, two thermodynamic processes can be performed while filling and emptying the working cavity of the cylinder.

To formalize the processes of filling and draining from the cavities of the cylinder, it is advisable to introduce into the equation (26) an auxiliary coefficient A and B, indicating which filling or discharge process takes place in each cavity of the pneumatic drive with forward and reverse strokes:

$$
\begin{cases}
\frac{dl}{dt} = V; \\
\frac{dV}{dt} = \frac{1}{M} \cdot \left(S_1 x_3 - S_2 x_4 - k_{\text{Втр}} \cdot x_2 - F_y - P_a \cdot (S_1 - S_2) \right); \\
\frac{dp_1}{dt} = (1 - \mathrm{A}_1) \cdot [(1 - B_1) \cdot \frac{k|f_1|\sqrt{RT_{\text{М}}}}{S_1(l + l_{01})\sqrt{\xi_1}} \cdot \sqrt{|p_{\text{М}}^2 - p_1^2|} - \\
\quad -B_1 \cdot \frac{k|f_1|\sqrt{RT_{\text{М}}}}{S_1(l + l_{01})\sqrt{\xi_1}} \cdot \left(\frac{p_1}{p_{\text{М}}}\right)^{\frac{k-1}{2k}} \cdot \sqrt{|p_1^2 - p_{\text{М}}^2|}] + \\
\quad +\mathrm{A}_1 \cdot [-\frac{k|f_1|\sqrt{RT_{\text{М}}}}{S_1(l + l_{01})\sqrt{\xi_1}} \cdot \left(\frac{p_1}{p_{\text{М}}}\right)^{\frac{k-1}{2k}} \cdot \sqrt{|p_1^2 - p_{\text{А}}^2|}] - \frac{k \cdot p_1}{(l + l_{01})} \cdot V; \\
\frac{dp_2}{dt} = (1 - \mathrm{A}_2) \cdot [-\frac{k|f_2|\sqrt{RT_{\text{М}}}}{S_2(L - l + l_{02})\sqrt{\xi_2}} \cdot \left(\frac{p_2}{p_{\text{М}}}\right)^{\frac{k-1}{2k}} \cdot \sqrt{|p_2^2 - p_{\text{А}}^2|}] + \\
\quad +\mathrm{A}_2 \cdot [(1 - B_2) \cdot \frac{k|f_2|\sqrt{RT_{\text{М}}}}{S_2(L - l + l_{02})\sqrt{\xi_2}} \cdot \sqrt{|p_{\text{М}2}^2 - p_2^2|} - \\
\quad -B_2 \cdot \frac{k|f_2|\sqrt{RT_{\text{М}}}}{S_2(L - l + l_{02})\sqrt{\xi_2}} \cdot \left(\frac{p_2}{p_{\text{М}}}\right)^{\frac{k-1}{2k}} \cdot \sqrt{|p_2^2 - p_{\text{М}2}^2|}\Big] + \frac{k \cdot p_2}{(L - l + l_{02})} \cdot V \\
\mathrm{A}_1 = \begin{cases} 0, at\ u_1 \geq 0; \\ 1 \text{ in other cases} \end{cases}; \\
\mathrm{A}_2 = \begin{cases} 0, at\ u_2 \geq 0; \\ 1 \text{ in other cases} \end{cases}; \\
B_1 = \begin{cases} 0, at\ p_{\text{М}} \geq x_3; \\ 1 \text{ in other cases} \end{cases}; \\
B_2 = \begin{cases} 0, at\ p_{\text{М}2} \geq x_4; \\ 1 \text{ in other cases.} \end{cases}
\end{cases}
\tag{27}
$$

Analysis of the system of equations (26) allows us to conclude that the mathematical model of a pneumatic system is a model of a variable structure, in which the design of the equations varies, depending on the modeling of the process of filling or draining air from cavities, while the parameters of the system being simulated remain constant.

Conclusion

As a result, the developed mathematical model allows taking into account the change in the thermodynamic processes of gas flow during structural changes in the pneumatic drive. This makes it possible to apply a synergetic ap-proach to controlling the tracking pneumatic drive, taking into account the multi-tasking of organizing optimal trajectories of control objects for various purposes.

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