

Visualization Terrain Data Using Cubic Ball Triangular Patches

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Abstract. Scattered data technique is important to visualize the geometrical images of the surface data especially for terrain, earthquake, geochemical distribution, rainfall etc. The main objective of this study is to visualize the terrain data by using cubic Ball triangular patches. First step, the terrain data is triangulated by using Delaunay triangulation. Then partial derivative will be estimated at the data points. Sufficient condition for C^1 continuity will be derived for each triangle. Finally, a convex combination comprising three rational local scheme is used to construct the surface. The scheme is tested to visualize the terrain data collected at central region of Malaysia.

1 Introduction

Scattered data interpolation is an active research in many years. This method is useful when the user wants to visualize certain types of real data sets. For instance, rainfall distribution and geochemical data. Besides that, scattered data also can be used for shape preserving interpolation such as to preserves the positivity of surface data. Some early studied includes Said and Rahmat [5] consider cubic Ball triangular for scattered data interpolation. Goodman and Said [3] discussed the method to construct a C^1 surface by calculating three different inner Bezier ordinates for each triangle. The final surface is a convex combination from three local scheme. Piah et al. [4] has extended the work of [3] for positivity preserving. Their result is different that Chan and Ong [1] scheme. The main objective of this study is to improve the main stated result in Said and Rahmat [5] i.e. to derive the correct C^1 condition for scattered data interpolation.

2 Cubic Ball triangular patches

Said and Rahmat [5] has utilized the cubic Ball triangular for scattered data. They show that cubic Ball will produce C^1 surface for the data obtained from some true function. But from the results, they claim that, their scheme gives same result as in Goodman and Said [3] except with lesser computation time. In this study, we will prove that the scheme presented

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in Said and Rahmat [5] is not C^1 at each of the adjacent triangles. We propose new sufficient condition for C^1 continuity. We begin this section with some definition about cubic Ball triangular patch. Assume that, the barycentric coordinate u, v, w on triangle T with vertices V_1, V_2 and V_3 where $u + v + w = 1$ and $u, v, w \geq 0$. From basic geometry, any point $V(x, y) \in R^2$ inside the triangle, can be defined as

$$V = uV_1 + vV_2 + wV_3 \tag{1}$$

Figure 1 shows the distribution of the cubic Ball triangular bases on a triangular domain:

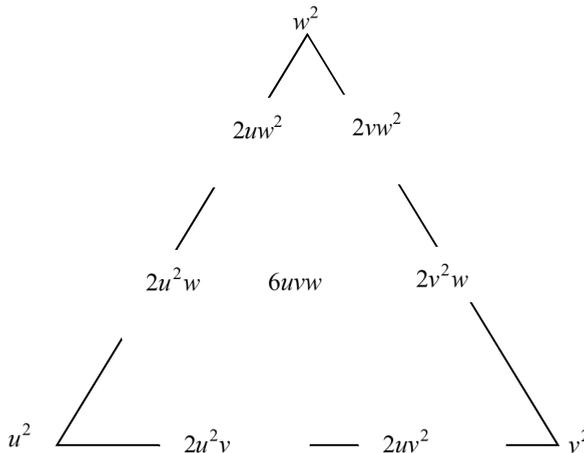


Fig. 1. Cubic Ball triangular bases.

Definition 1: The cubic Ball triangular patch with control points $b_{ijk}, i + j + k = 3$ is defined as

$$P(u, v, w) = u^2b_{300} + v^2b_{030} + w^2b_{003} + 2u^2vb_{210} + 2u^2wb_{201} + 2v^2ub_{120} + 2v^2wb_{021} + 2w^2ub_{102} + 2w^2vb_{021} + 6uvw b_{111} \tag{2}$$

where $u + v + w = 1$ and $b_{ijk} \in R^3, i + j + k = 3$.

3 Scattered data interpolation

The scheme comprising convex combination of three local schemes P_1, P_2 and P_3 and is defined as

$$P(u, v, w) = \frac{vwP_1 + uwP_2 + uvP_3}{vw_1 + uw + uv}, u + v + w = 1 \tag{3}$$

Local scheme P_i , $i=1,2,3$ is obtained by replacing b_{111} in (2) with b_{111}^i such that C^1 condition is satisfied only on the boundary e_i of triangle.

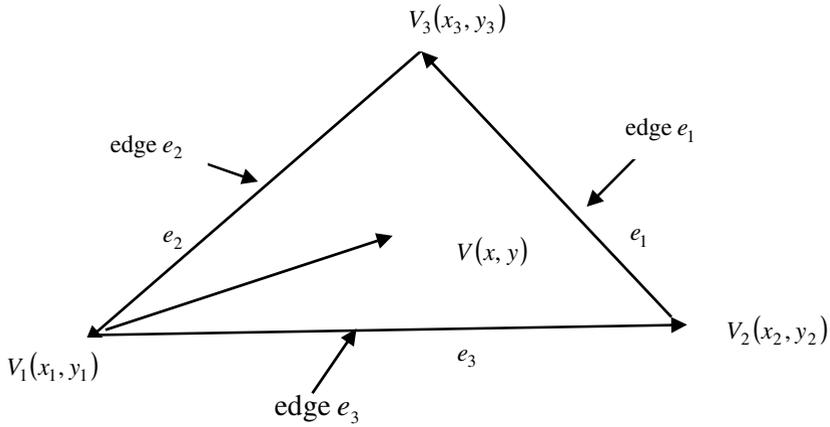


Fig. 2. Vertices and edges of the triangle.

Let the inward normal directions with the respective edges are given as n_1, n_2 and n_3 :

$$n_1 = -e_3 + \frac{e_3 \cdot e_1}{|e_1|^2}, n_{21} = -e_1 + \frac{e_1 \cdot e_2}{|e_2|^2} \text{ and } n_3 = -e_2 + \frac{e_2 \cdot e_3}{|e_3|^2}$$

where "." is a scalar product. Firstly we will find the Ball ordinate b_{111}^1 by considering the normal derivatives along $n_1 = (1, -1/2, -1/2)$ of local scheme P_1 i.e. on edge V_2V_3 such that $u = 0, v + w = 1$:

$$D_{n_3} P_1 = (1) \left(2b_{120}v^2 + 2b_{102}w^2 + 6vwb_{111}^1 \right) - \left(\frac{1}{2} \right) \left(2vb_{030} + 4vwb_{021} + 2b_{012}w^2 \right) - \left(\frac{1}{2} \right) \left(2wb_{003} + 2b_{021}v^2 + 4b_{012}vw \right)$$

which can be rearrange as

$$D_{n_3} P_1 = (2b_{120} - b_{021})v^2 + (6vwb_{111}^1 - 2b_{012} - 2b_{021})vw + (2b_{102} - b_{012})w^2 + b_{030}v - b_{0303}w$$

or

$$D_{n_3} P_1 = Av^2 + Bvw + Cw^2 + b_{030}v - b_{0303}w \tag{4}$$

For linearity,

$$D_{n_3} P_1 = Av^2 + Bvw + Cw^2 \tag{5}$$

where

$$\begin{aligned}
 A &= 2b_{120} - b_{021}, \\
 B &= 6vw b_{111}^1 - 2b_{012} - 2b_{021}, \\
 C &= 2b_{102} - b_{012}
 \end{aligned}$$

Theorem 1. The normal derivative in (5) must be linear on e_1 if and only if $B = A + C$.

Proof.

Substitute $B = A + C$ to (5) and $v + w = 1$ since $u = 0$ on edge e_1 , we obtain :

$$D_{n_3} P_1 = Av^2 + (A + C)vw + Cw^2 = Av(v + w) + Cw(v + w) = Av + Cw = A(1 - w) + Cw$$

which is linear.

From Theorem 1, the normal derivative will be linear on e_1 if

$$6vw b_{111}^1 - 2b_{012} - 2b_{021} = 2b_{120} - b_{021} + 2b_{102} - b_{012}$$

Thus

$$b_{111}^1 = \frac{1}{6}(2b_{120} + 2b_{102}) + \frac{1}{12}(2b_{021} + 2b_{012}) \tag{6}$$

Similarly, we can obtain the other Ball ordinates i.e. b_{111}^2 and b_{111}^3 for local schemes P_2 and P_3 , respectively.

Remark 2. In Said and Rahmat [5], they give the C^1 condition as

$$b_{111}^1 = \frac{1}{6}(2b_{120} + 2b_{102}) + \frac{1}{12}(2b_{021} - 2b_{012}) \tag{7}$$

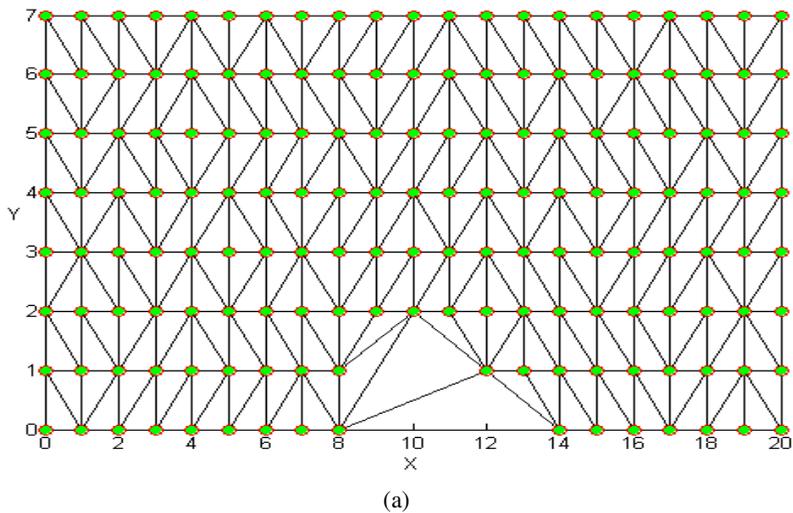
Based on **Remark 2** and our derivation for C^1 stated in (6), consequently we found that the scheme presented by Said and Rahmat [5] is not C^1 and the current study is an improvement to the result stated in Said and Rahmat [5]. We can see this fact in the following section.

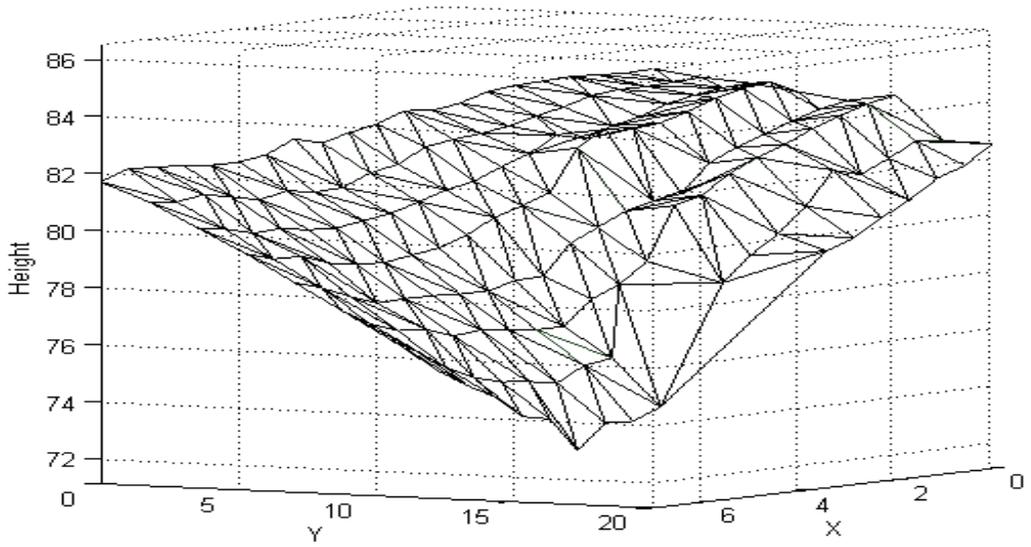
4 Results and Discussion

1. We test the proposed scattered data interpolation by using the new scheme based on cubic Ball triangular patches to visualize the digital elevation at Kalumpang Agricultural Station (3° 38' N, 101°34'E) in Selangor located about 90 km northeast of Kuala Lumpur, Malaysia as shown in Fig. 3. Meanwhile, Fig. 4 shows the results. Fig. 4(a) shows the Delaunay triangulation for all 165 data points. Fig. 4(b) shows the linear interpolant for the terrain data. Fig. 4(c) shows the example of surface reconstruction by using the proposed C^1 scheme. Clearly the surface is smooth with visually pleasing. Meanwhile in Fig. 4(d) shows the surface when the C^1 condition in Said and Rahmat [5] is used. Based on Fig. 4(d), we can see that, Said and Rahmat [5] scheme is not C^1 meanwhile the proposed scheme in this study is guaranteed to produce C^1 surface everywhere. This is the significance of the present study i.e. has improved the result in [5].

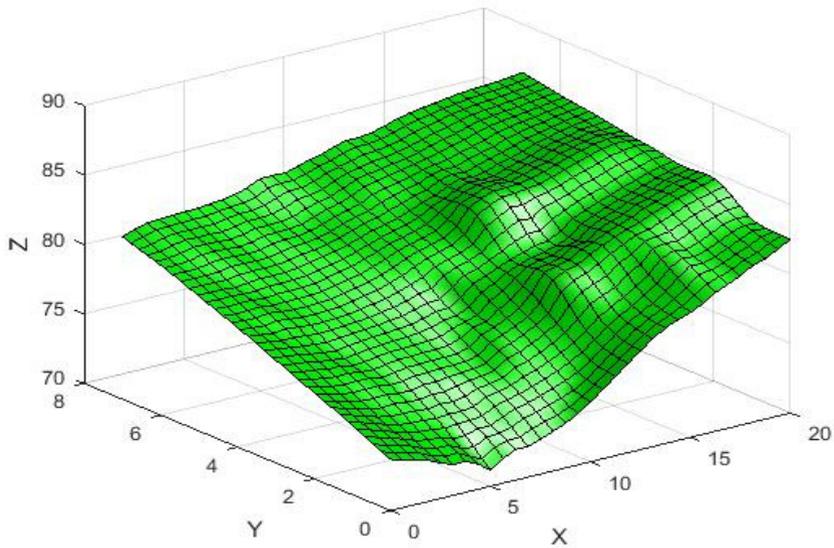


Fig. 3. Location of Kalumpang Agricultural Station in Malaysia.

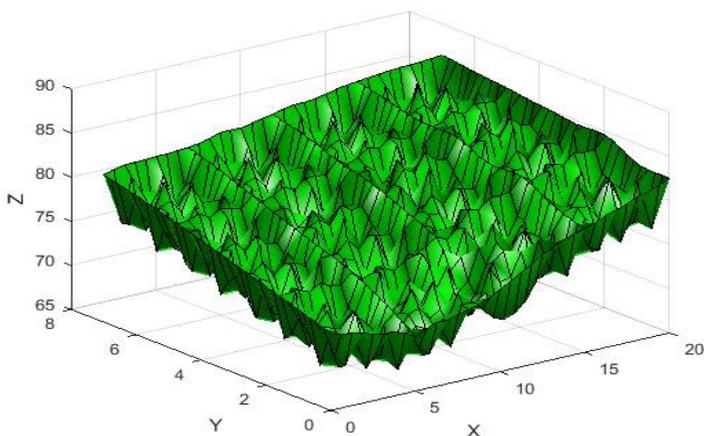




(b)



(c)



(d)

Fig. 4. Surface reconstruction for terrain data at Kalumpang Agricultural Station in Malaysia.

Conclusion

In this study, we derive a new C^1 condition for cubic Ball triangles. The new condition is correct compared with the condition derived in Said and Rahmat [5]. As an application, we test the scheme to visualize the terrain data. From the results, we can see that, the cubic Ball triangular can reconstruct the surface with good approximation properties such as nice surface shape as well as C^1 continuous along the boundaries. Future work will be applying the scheme for shape preserving interpolation and reconstruct the other types of surfaces, for instance, geo-chemical data and rainfall distribution as well as in medical imaging. An extension to the work of Foley and Optiz [3] is also possible.

Acknowledgement to Universiti Teknologi PETRONAS (UTP) for the financial support received in the form of research grant: **YUTP: 0153AA-H24** including the MATLAB Software. This work were carried out when the second author is visiting the first author from 27th August 2018 until 30th August 2018.

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