

Wind Velocity Data Interpolation Using Rational Cubic Spline

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Abstract. Wind velocity data is always having positive value and the minimum value approximately close to zero. The standard cubic spline interpolation (not-a-knot and natural) as well as cubic Hermite polynomial may be produces interpolating curve with negative values on some sub-intervals. To cater this problem, a new rational cubic spline with three parameters is constructed. This rational spline will be used to preserve the positivity of the wind velocity data. Numerical results shows that the proposed scheme work very well and give visually pleasing interpolating curve on the given domain.

1 Introduction

Meteorological data parameters are important especially when the scientist dealing with some problems such as dengue transmission at certain location, the amount of rainfall received in a month as well as the earthquake events or history etc. Some common meteorological data are rainfall, temperature (maximum, minimum and average), humidity, wind velocity or speed as well as wind direction. Wu et al. [8] studied the multiquadric interpolation to approximate the wind speed data. Their method is only capable to approximate the wind velocity where the interpolations to all data points are lost. Karim [4] discussed the monotonic interpolating cure by using rational cubic spline of Karim and Kong [3]. Sarfraz et al. [6] discussed the positivity preserving interpolation by using rational cubic spline with two parameters. The main drawback of [6] is that, on some intervals, the positivity of the data is not preserved as proved by Qin et al. [7]. Hussain et al. [2] scheme also does not guarantee to produces positive interpolating curve everywhere.

In this study, a new rational cubic spline in the form of cubic numerator and quadratic denominator is proposed. This rational spline has three parameters for shape modification. In order to produce positive rational interpolant, the sufficient condition for the positivity of the rational spline interpolant is derived on one parameter meanwhile the other two parameters can be used to refine the shape of positive interpolating curve.

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2 Rational cubic spline interpolant

The construction of new rational cubic spline is discussed below. Given data points $\{(x_i, f_i), i = 0, 1, \dots, n\}$ which satisfy $x_0 < x_1 < \dots < x_n$ and the first derivatives $d_i, i = 0, 1, \dots, n$. Let the step size $h_i = x_{i+1} - x_i$, the gradient is $\Delta_i = (f_{i+1} - f_i)/h_i$ and $\theta = (x - x_i)/h_i$ where $0 \leq \theta \leq 1$. For each sub-intervals $x \in [x_i, x_{i+1}], i = 0, 1, \dots, n - 1$, the rational cubic spline with three parameters α_i, β_i , and γ_i , for $i = 0, 1, \dots, n - 1$ is defined by

$$S(x) = S_i(x) = \frac{P_i(\theta)}{Q_i(\theta)}, \tag{1}$$

where

$$P_i(\theta) = (1 - \theta)^2 U_i + (1 - \theta)^2 \theta V_i + (1 - \theta) \theta^2 W_i + \theta^2 Z_i$$

$$Q_i(\theta) = \alpha_i (1 - \theta)^2 + 2\gamma_i (1 - \theta)\theta + \beta_i \theta^2$$

In this study, we construct C^1 rational cubic spline that satisfies the following conditions:

$$\begin{aligned} S(x_i) &= f_i, & S^{(1)}(x_i) &= d_i \\ S(x_{i+1}) &= f_{i+1}, & S^{(1)}(x_{i+1}) &= d_{i+1} \end{aligned} \tag{2}$$

By using (2) and the rational cubic spline in (1), the unknowns, U_i, V_i, W_i and Z_i for $i = 0, 1, \dots, n - 1$ are:

$$\begin{aligned} U_i &= \alpha_i f_i, & V_i &= 2\gamma_i f_i + \alpha_i h_i d_i, \\ W_i &= \gamma_i f_{i+1} - \beta_i h_i d_{i+1}, & Z_i &= \beta_i f_{i+1} \end{aligned}$$

With this value, the rational cubic spline defined in (1) can be written as

$$S(x) = \frac{(1 - \theta)^2 \alpha_i f_i + (1 - \theta)^2 \theta (2\gamma_i f_i + \alpha_i h_i d_i) + (1 - \theta) \theta^2 (\gamma_i f_{i+1} - \beta_i h_i d_{i+1}) + \theta^2 \beta_i f_{i+1}}{\alpha_i (1 - \theta)^2 + 2\gamma_i (1 - \theta)\theta + \beta_i \theta^2} \tag{3}$$

When $\alpha_i = \beta_i = \gamma_i = 1$, the rational cubic spline in (3) is reduce to the cubic Ball polynomial.

3 Positivity preserving data interpolation

The rational cubic spline defined by (3) can be used to interpolate the wind velocity data (or any other positive data) by supplying any positive value to the shape parameters α_i, β_i , and γ_i , for $i = 0, 1, \dots, n - 1$. Table 1 shows the wind velocity data obtained in Wu et al. [8]. Fig. 1 shows some examples of interpolating curve by varying α_i, β_i , and γ_i .

Table 1. A wind data [8]

i	0	1	2	3	4	5	6
x_i	0	0.25	0.5	1	1.2	1.8	2
f_i	2	0.8	0.5	0.1	1	0.5	1

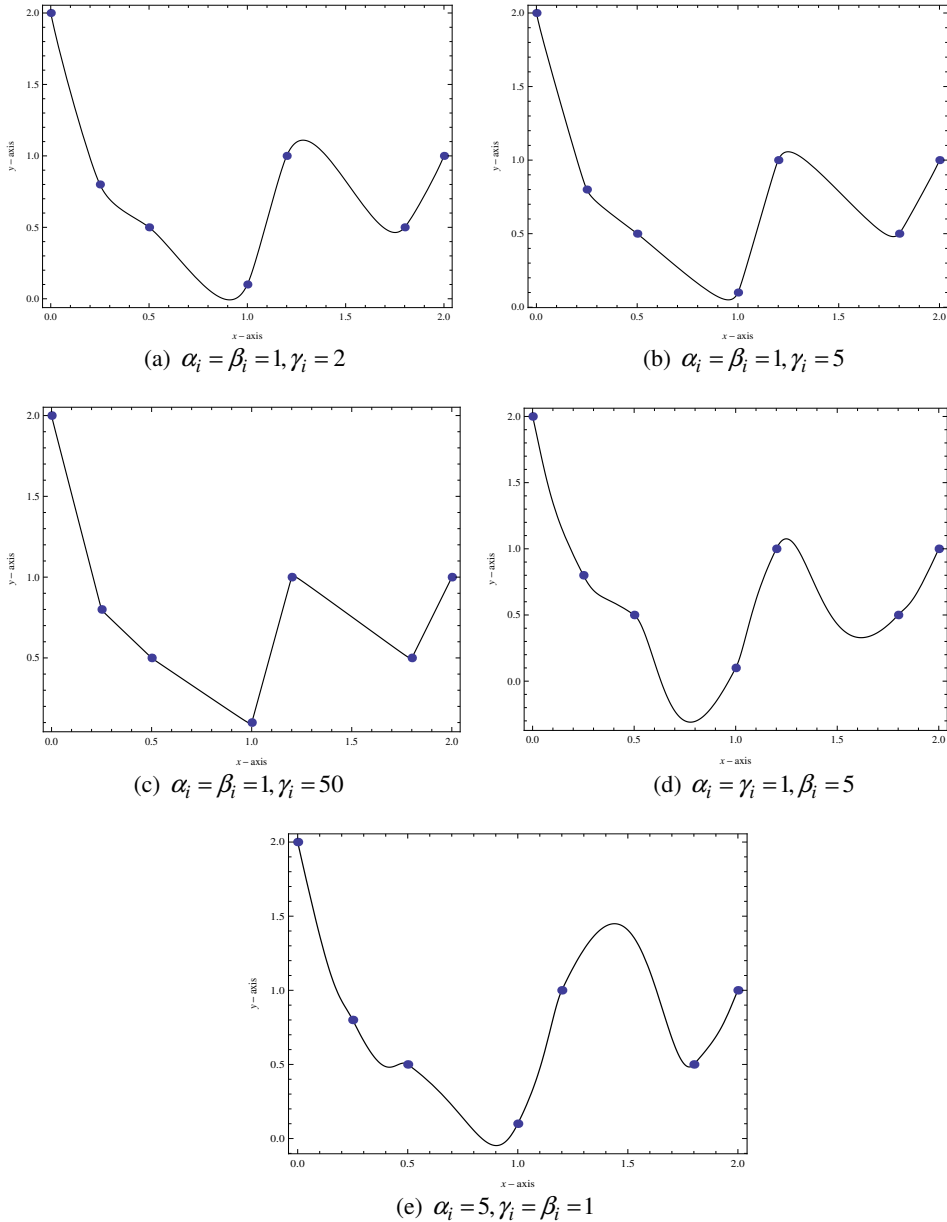


Fig. 1. Various interpolating curves

It is not easy to find suitable value of each parameter that will ensure that the resulting interpolating curve is positive everywhere. The automated choice of parameter $\gamma_i > 0, i = 0, 1, \dots, n - 1$ is derived that will ensure the positive rational interpolant is produced. Since we are using rational spline, then data must be strictly positive in order to avoid the dividing by zero in the mathematical derivation later:

$$f_i > 0, \quad i = 0, 1, 2, \dots, n \tag{4}$$

Condition (4) is the same as

$$S_i(x) > 0, \quad i = 0,1,2,\dots,n-1 \tag{5}$$

When $\alpha_i, \beta_i > 0$, and $\gamma_i \geq 0$ for $i = 0,1,\dots,n-1$, the quadratic polynomial $Q_i(\theta)$ is always positive such that $Q_i(\theta) > 0$. Thus $S_i(x) > 0$, if and only if the cubic polynomial $P_i(\theta)$ [7] is positive i.e. $P_i(\theta) > 0$ for all $i = 0,1,2,\dots,n-1$. Theorem 1 state the sufficient condition for the positivity of the rational cubic interpolant defined by (3).

Theorem 1. For strictly positive data, the rational cubic spline interpolant $S(x)$ in (3) will be positive everywhere if in each sub-interval $[x_i, x_{i+1}]$, $i = 0,1,\dots,n-1$, the parameter $\gamma_i > 0$, $i = 0,1,\dots,n-1$ satisfies:

$$\gamma_i > \text{Max} \left\{ 0, -\alpha_i \left(\frac{h_i d_i}{2f_i} + \frac{1}{2} \right), \beta_i \left(\frac{h_i d_{i+1}}{2f_{i+1}} - 1 \right) - \frac{1}{2} \right\}, \quad \alpha_i, \beta_i > 0. \tag{6}$$

Proof.

Firstly, the first derivative of $P_i(\theta)$ is equal to

$$P_i'(\theta) = \frac{-2\alpha_i f_i (1-\theta) + V_i ((1-\theta)^2 - 2\theta(1-\theta)) + W_i (-\theta^2 + 2\theta(1-\theta)) + 2\beta_i f_{i+1} \theta}{h_i}$$

The sufficient conditions for positivity of the rational cubic spline are given below:

$$P_i'(0) > \frac{-3P_i(0)}{h_i} \Rightarrow \frac{-2\alpha_i f_i + 2\gamma_i f_i + \alpha_i h_i d_i}{h_i} > \frac{-3\alpha_i f_i}{h_i} \tag{7}$$

and

$$P_i'(1) < \frac{3P_i(1)}{h_i} \Rightarrow \frac{2\beta_i f_{i+1} - 2\gamma_i f_{i+1} + \beta_i h_i d_{i+1}}{h_i} < \frac{3\beta_i f_{i+1}}{h_i} \tag{8}$$

Both conditions (7) and (8) can be further simplified to obtain the following:

$$\gamma_i > -\alpha_i \left(\frac{h_i d_i}{2f_i} + \frac{1}{2} \right) \tag{9}$$

and similarly

$$\gamma_i > \beta_i \left(\frac{h_i d_{i+1}}{2f_{i+1}} - \frac{1}{2} \right) \tag{10}$$

Combining Conditions (9) and (10), the rational cubic spline is positive i.e. $S_i(x) > 0$, $i = 0,1,2,\dots,n-1$ when the parameter $\gamma_i > 0$, $i = 0,1,\dots,n-1$ satisfies

$$\gamma_i > \text{Max} \left\{ 0, -\alpha_i \left(\frac{h_i d_i}{2f_i} + \frac{1}{2} \right), \beta_i \left(\frac{h_i d_{i+1}}{2f_{i+1}} - 1 \right) - \frac{1}{2} \right\}$$

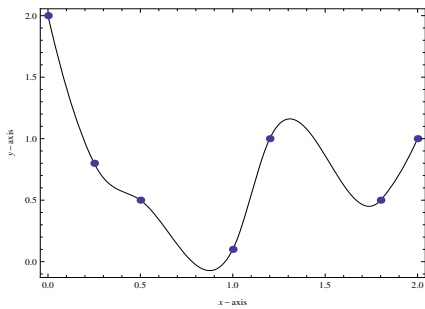
Furthermore, this condition can be written as:

$$\gamma_i = \chi_i + \text{Max} \left\{ 0, -\alpha_i \left(\frac{h_i d_i}{2f_i} + \frac{1}{2} \right), \beta_i \left(\frac{h_i d_{i+1}}{2f_{i+1}} - 1 \right) - \frac{1}{2} \right\}, \quad i = 0,1,\dots,n-1 \tag{11}$$

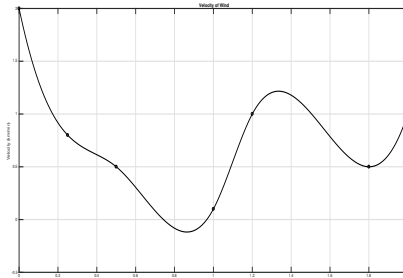
such that $0 < \chi_i \leq 0.50$.

4 Numerical results

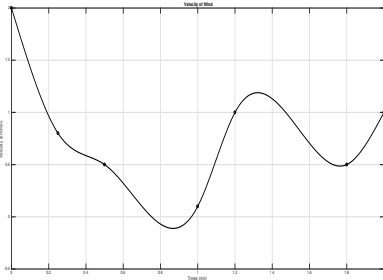
To test the positivity preserving by using the proposed rational cubic spline, we use the wind velocity data given in Table 1. Fig. 2 shows the examples of various interpolating curves produces by using cubic spline interpolations and the proposed scheme. Fig. 2(a) shows the interpolating curve when $\alpha_i = \beta_i = \gamma_i = 1$ for all $i = 0,1,\dots,6$. This is cubic Ball polynomial. Figs. 2(b) and 2(c) show the interpolating curve by using cubic spline interpolation with not-a-knot and natural end conditions respectively. Clearly both cubic splines fail to preserves the positivity of the wind velocity data. Fig. 2(d) shows the example of interpolating curve by using PCHIP [1] that well documented in MATLAB software PCHIP can preserves the positivity of wind velocity, but the interpolating curve is not visually pleasing since on some intervals, the interpolating curves are very tight as well as very sharp. To avoid this, we implement the proposed scheme with positivity shape preserving. Figs. 2(f) until 2(h) show the example of positive interpolating curve by using the proposed scheme with various value of the free parameters α_i and β_i .



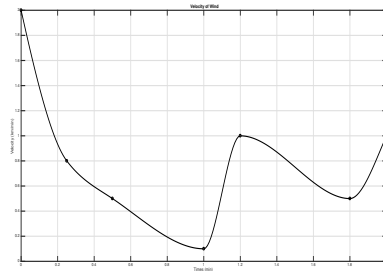
(a) The proposed scheme with $\alpha_i = \beta_i = \gamma_i = 1$



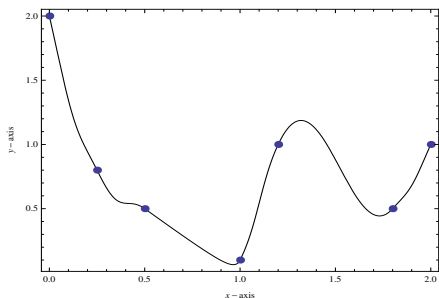
(b) Not-a-knot cubic spline



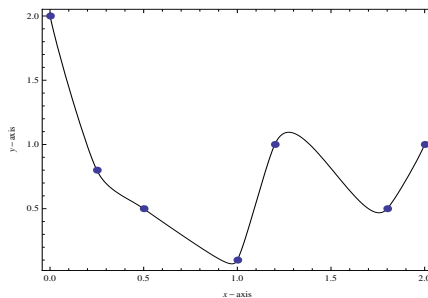
(c) Natural cubic spline



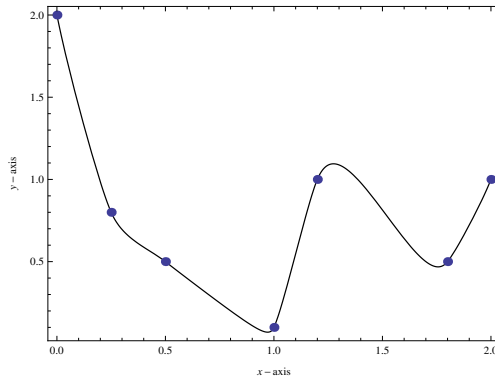
(d) PCHIP from MATLAB



(e) The proposed scheme with $\alpha_i = \beta_i = 1$



(f) The proposed scheme with $\alpha_i = \beta_i = 0.25$



(g) The proposed scheme with $\alpha_i = \beta_i = 0.1$

Fig. 2. Comparison of interpolating curve for wind velocity data.

Discussion and conclusion

The main objective of the present study is to preserve the positivity of wind velocity data by using new rational cubic spline interpolant. This rational spline has three parameters and the positivity of the data can be preserved by finding the suitable sufficient condition on one parameter meanwhile the remaining two free parameters can be used to refine the positive interpolating curve. Based on the comparison against (a) cubic spline interpolations with not-a-knot and natural end boundary conditions, cubic Ball interpolation and PCHIP from [1], we conclude that, the proposed schemes better, notably in the aspects of visually pleasing and the existence of free parameters for shape refining as well as avoiding any derivative modification that has appeared in PCHIP. For future studies, we intend to extend the proposed scheme to surface interpolation. This is very useful for geological surface data interpolation.

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