

# Mathematical Modelling and Transient Thermal Analysis of Coupler-Rocker Bearing

*Abdullah Jamil*<sup>1,\*</sup>, *Masri B. Baharom*<sup>1</sup> and *Tamiru Alemu Lemma*<sup>1</sup>

<sup>1</sup>Universiti Teknologi PETRONAS, 32610 Seri Iskandar, Perak Darul Ridzuan, Malaysia

**Abstract.** Friction is the resisting force between two bodies having relative motion. These bodies can be solid surfaces, fluid films or elements sliding against each other. There are many devices used to overcome sliding friction which include wheels and bearings. Ball bearings are used in many high speed and high precision machine tools because of their high productivity. A Crank-Rocker four bar mechanism consists of 4 linkages and 4 nodes. These nodes perform complex motions especially the coupler-rocker joint. In order to reduce the friction between these relatively moving links, ball bearings can be introduced. The coupler-rocker bearing oscillates about some axis as well as the raceways have some relative motion. Heat generation rate is not known for bearings performing this type of complex motion. This paper describes the mathematical modelling and thermal analysis of coupler-rocker bearing. Heat generation in the bearing can be estimated using this model. This can be countered by having proper lubrication and speed of bearing.

## 1 Introduction

Friction has its advantages and disadvantages. It is due to friction that joining bodies remain intact and does not slip away. On the other side, excess frictional motion generates heat at contact point due to atoms getting excited within the bodies. Also, due to friction, the contacting surfaces wear out and damage the machinery. Therefore, a number of antifriction techniques are used to reduce friction and improve life and efficiency of machines.

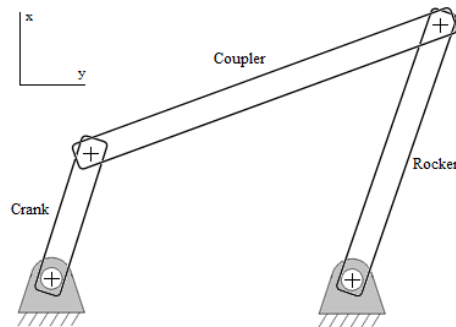
There are many devices used to overcome sliding friction which include wheels and bearings. Bearing types include: Ball bearing, Roller bearing and Fluid bearings. Ball bearings have several small metallic or ceramic balls within inner and outer races. Due to this, there is very small contact area between two relatively rotating shafts and journals.

A novel crank-rocker engine was developed in Universiti Teknologi PETRONAS (UTP), Malaysia which is a single curved cylinder, IC Engine [1]. This engine is based on fourbar mechanism with one crank and one rocker (Figure 1). These links are joined together using ball bearings. During operation of engine, these bearings perform complex motions especially the coupler-rocker bearings. This bearing performs oscillatory and rocking motion at the same time. Therefore, the joint failure of engine is most critical at this point. There needs to perform a thermal analysis of this oscillatory Coupler-Rocker Ball Bearing to understand its Heat generation and dissipation behavior.

Researchers have performed many thermal and structural analyses on roller element bearings. They studied the effect of radial or axial load on ball bearings and the heat

generation rate due to friction between balls, races and cages within the bearing. Chao Jin et al. [2] performed heat generation modelling of Angular contact ball bearing and validated using experiment at variable bearing rpm and load. Y-R Jeng and CC Gao [3] investigated the temperature rise of Angular contact ball bearing under oil-air lubrication system. He found experimentally that as the oil supply decreases, the minimum oil film thickness between the bearing elements decreases which results in increase in frictional force and heat generation within the bearing.

Takabi and Khonsari [4] performed thermal analysis by applying radial load on Deep Groove Ball Bearing and verified the results by thermal resistance network modelling. Ball bearing was tested for two important parameters i.e. speed and external radial load using a designed testing rig. A set of experiments were carried out to test the bearing by keeping one parameter constant and varying different values of the other.



**Fig. 1.** Crank-Rocker Mechanism

The thermal analysis of the roller bearings has not been discussed comprehensively because of their fundamental geometric and dynamic complexity. During operation of ball bearing, the frictional heat is generated due to excessive surface rubbing of balls with cages, inner and outer races. This generated heat is calculated by many researchers [4-8] using a set of equations. The variables used in calculations depend upon the bearing and lubricant used. Their values can be acquired from datasheets from respective distributors. Harris and Khonsari [9-10] explained in detail the heat generation within bearings which depends on the Total Frictional Moment which is the sum of Frictional moment due to applied load on bearing and Frictional moment due to viscous forces of lubricant.

The calculated Heat Generation in the bearing is dissipated to the surrounding through modes of heat transfer. Most researchers [11-15] have designed thermal resistance network model to calculate the heat transfer through conduction and convection from the bearing. This method comprises of the network of resistances constructed among different components of test bearing assembly. These components (nodes) include inner and outer races, balls, shaft, ambient air, lubricating oil and bearing housing etc. Depending on the mode of heat transfer, the respective heat transfer coefficients are calculated using the conduction and convection formulas. Harris [9] gives a set of expressions to calculate the heat transfer from bearing through conduction and convection. The convective heat transfer coefficient depends upon the density, viscosity, temperature, flow rate, thermal coefficient of volume expansion and conductivity of lubricating fluid which can be acquired from lubricant datasheet.

Radiation is another mode of heat transfer, but a small amount of heat is radiated to the environment even at high surface temperature. Therefore, heat transfer through radiation is ignored if the surface temperature is not high enough. J. Takabi and M. M. Khonsari [4]

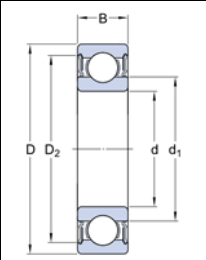
neglected heat transfer through radiation due to relatively low bearing surface temperature. L. Q. Wang et al. [16] also ignored the radiation in the bearing because the temperature differences among the bearing components are very small.

## 2 Methodology

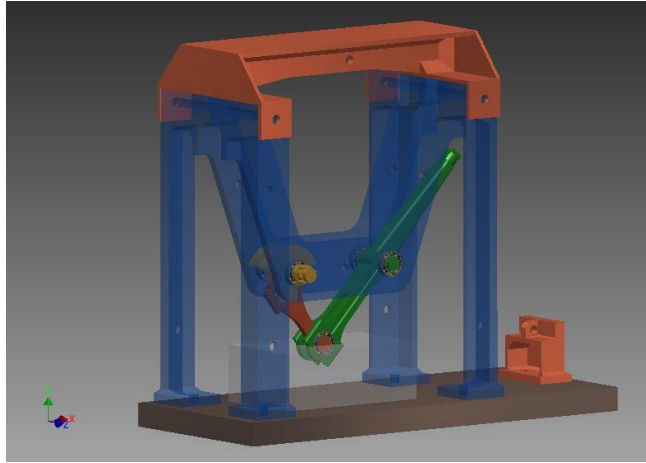
The link lengths of the bearing testing rig are benchmarked from the modified crank-rocker engine [1]. A CAD model of C-R bearing testing mechanism was designed (Figure 2) in widely used software Autodesk Inventor Professional based on the dimensions of Crank-Rocker Engine. The selected oscillatory bearing (Coupler-Rocker) Ball Bearing is 61902 Deep Groove Ball Bearing based on the dimensions of Coupler and Rocker links (Table 1).

A constant load cannot be applied on a moving bearing using weights, load cell or hydraulic/pneumatic cylinders. Therefore, mechanical springs were opted as best choice. Extension Springs were preferred over Compression Springs because of their easy to use nature. The two extremes of the four-bar mechanism are when crank and coupler links are inline. At these two positions, the free-end of rocker link are at extreme locations. Therefore, the two extension springs of same spring rate can be attached to the rocker link on the opposite ends at these extreme positions in such a way that when one spring is completely free, the other spring is completely stretched.

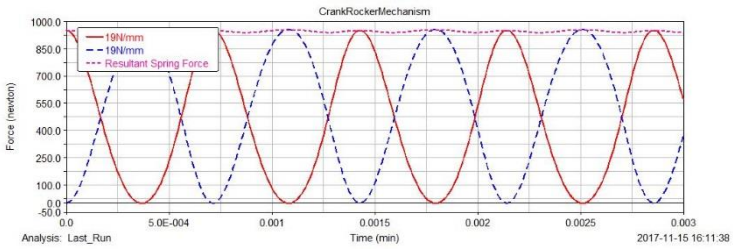
**Table 1.** 61902 Deep Groove Ball Bearing [21].

Dimensions	Symbol	Value (mm)
	d	15
	D	28
	B	7
	d1	18.8
	D2	25.3

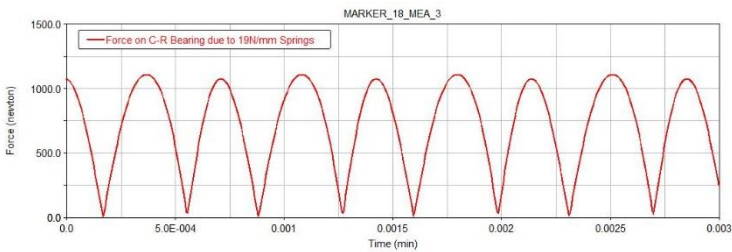
During the operation of Testing Rig, one spring starts stretching, the spring starts releasing. In other words, when one spring starts exerting restoring force on rocker link, the other spring starts releasing the force but the overall force on rocker link will be constant. This force is the vector sum of the two spring forces as shown in Figure 3. This constant load on rocker is transferred to the coupler-rocker bearing joint and can be measured using ADAMS software (Figure 4). In this way, ball bearings can be loaded when performing oscillatory motion for thermal analysis.



**Fig. 2.** Coupler-Rocker Bearing Testing Rig



**Fig. 3.** Two Spring Forces and their Resultant



**Fig. 4.** Force on Coupler-Rocker Bearing

## 2.1 Frictional heat generation model

The total frictional torque  $M$  is given by Stein and Tu [17].

$$M = M_l + M_v + M_f \quad (1)$$

Where  $M_l$ ,  $M_v$ , and  $M_f$  are the frictional torques due to the applied load, viscous forces and that due to the flanges respectively. However, the frictional torque due to roller end-flange  $M_f$  is not considered for the case of ball bearings [4]. The formulas for individual frictional torques are acquired from Harris [9].

$$M_l = f_1 F_\beta d_m \tag{2}$$

$$f_1 = z \left( \frac{F_s}{C_s} \right)^y \tag{3}$$

$$M_v = 10^{-7} f_o (v_o n)^{\frac{2}{3}} d_m^3 \tag{4}$$

Where  $f_l$  is a factor depending upon bearing design and relative load.  $F_\beta$  depends on the magnitude and direction of applied load.  $d_m$  is the mean diameter of bearing.  $z$  and  $y$  are constants depending on ball bearing type.  $F_s$  and  $C_s$  are static equivalent load and basic static load rating respectively.  $f_o$  depends upon type of bearing and method of lubrication.  $v_o$  is the kinematic viscosity of lubricant. Values of these variables are acquired from SKF bearing catalogue and lubricant datasheet. Total frictional heat generation in ball bearings is calculated from the following formula.

$$Q_f = 1.047 \times 10^{-4} n M \tag{5}$$

Where  $n$  is the rotational speed of bearing. It is noted that this formula is valid for stationary bearing with races performing relative rotatory motion with proper lubrication.

## 2.2 Heat generation model

A resistance network is modelled according to the experimental bearing setup. This network specifies the resistances in conduction and convection from balls to races, shaft, link, air and oil. In order to have simplified equations, resistances at each node are calculated.

$R_{12}$  and  $R_{23}$  are axial conduction through the shaft.  $R_{34}$  is the radial conduction between inner race and shaft.  $R_{45}$  is the radial conduction between balls and inner race.  $R_{56}$  is radial conduction between balls and outer race.  $R_{67}$  is the radial conduction between outer race and coupler link.  $R_{c,oil}$  is the oil convection from races, balls and shaft.  $R_{c,air}$  is the air convection from shaft and coupler link. Calculated Heat Generation  $Q_f$  is incorporated in node 5 i.e. balls. Each energy balance equation tells about the change of heat generation with time at that node.

The conductive and convective resistances have the form  $R_r = \frac{\ln\left(\frac{R_1}{R_2}\right)}{2\pi k l}$  for radial conduction,  $R_a = \frac{\Delta L}{Ak}$  for axial conduction and  $R_{conv} = \frac{1}{hA}$  for convection. Where  $l$ ,  $R_1$  and  $R_2$  are the width, inner and outer radii of the annular structure,  $\Delta L$  is the distance between two points,  $k$  is the thermal conductivity of material,  $A$  is the cross-sectional area through which heat transfers and  $h$  is the heat transfer coefficient. For calculating convective resistances, value of heat transfer coefficient  $h$  needs to be calculated. Reynolds number, Nusselt number, Prandtl number and Grashof numbers are calculated to find  $h$  values. The equation used to calculate heat transfer coefficients for different geometrical objects are given by Harris [9]. For rotating ball,

$$\frac{h_v D}{k} = 0.33 Re^{0.5} Pr^{0.4} \tag{6}$$

$$Re = \frac{uD}{\nu} \tag{7}$$

$$Pr = \frac{\nu}{\alpha} \tag{8}$$

$$A = n * 4\pi r^2 \tag{9}$$

Where  $u$  is the average translational velocity of bearing,  $D$  is the diameter of ball,  $\nu$  is kinematic viscosity of lubricant,  $\alpha$  is the thermal diffusivity of lubricant,  $n$  is the number of balls and  $r$  is the radius of bearing element.

For Cylindrical Ring (Races),

$$\frac{h_v D}{k} = 0.19(Re^2 + Gr) \tag{10}$$

$$Gr = \frac{Bg(T_s - T_\infty)D^2}{\nu^2} \tag{11}$$

Where  $D$  is the Diameter of outer race,  $B$  is the volume expansion coefficient of oil,  $g$  is the gravitational constant,  $T_s$  is Temperature at surface and  $T_\infty$  is Temperature of oil. The convection through air are calculated using the following equations [4].

$$R_{c,air} = \frac{1}{hA} \tag{12}$$

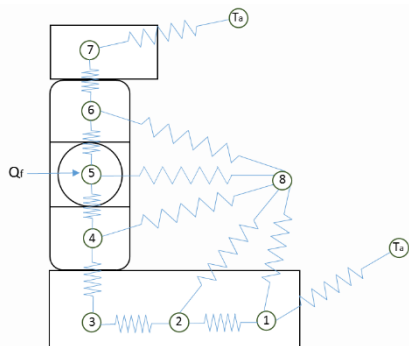
$$\frac{hD}{k} = Nu = 0.119Re^{\frac{2}{3}} \tag{13}$$

$$Re = \frac{\omega b^2}{\nu} \tag{14}$$

Where  $\omega$  is the bearing speed,  $b$  is the characteristic linear dimension,  $\nu$  is the kinematic viscosity of air. Low temperature variation has negligible effect on the viscosity of air [18]. Therefore, viscosity of air is calculated at ambient air temperature and kept constant throughout the calculations. Viscosity of oil, however, varies significantly with temperature and therefore, can be acquired from the lubricant datasheets. Adrian Bejan [18] provided the values of Thermal Volume Expansion Coefficient, Thermal Diffusivity, Specific Heat and Thermal Conductivity of Engine Oil at different temperatures.

### 2.3 Mathematical modelling of heat transfer (thermal resistance network)

A network of thermal resistance elements was used to represent the bearing testing rig whereas the balls and raceways were modeled as lumped thermal masses connected by resistances.



**Fig. 5.** Thermal Resistance Network for Bearing Assembly

Figure 5 shows the entire thermal network of test bearing consisting of 8 thermal nodes for the bearing assembly with 3 nodes for the shaft, 3 nodes for the ball, the inner and outer raceways, 1 for rocker link and lubricant in the oil bath each. The number of nodes implemented in the thermal network is found to be sufficient for prediction of bearing temperature. Due to the symmetry of the model, only quarter of the actual model is used in this study.

Considering energy balance equation for each node, one can write a set of 8 partial differential equations with 8 unknown temperatures at each node (15). Energy balance equations for all 8 nodes with unknown Temperature variables are mentioned below.

$$\begin{aligned}
 \frac{T_2 - T_1}{R_{12}} + \frac{T_a - T_1}{R_{c,air}} &= m_1 c_1 \frac{\partial T_1}{\partial t} \\
 \frac{T_1 - T_2}{R_{12}} + \frac{T_3 - T_2}{R_{23}} + \frac{T_8 - T_2}{R_{c,oil}} &= m_2 c_2 \frac{\partial T_2}{\partial t} \\
 \frac{T_2 - T_3}{R_{23}} + \frac{T_4 - T_3}{R_{34}} &= m_3 c_3 \frac{\partial T_3}{\partial t} \\
 \frac{T_3 - T_4}{R_{34}} + \frac{T_5 - T_4}{R_{45}} + \frac{T_8 - T_4}{R_{c,oil}} &= m_3 c_3 \frac{\partial T_4}{\partial t} \\
 \frac{T_4 - T_5}{R_{45}} + \frac{T_6 - T_5}{R_{56}} + \frac{T_8 - T_5}{R_{c,oil}} + Q_f &= m_5 c_5 \frac{\partial T_5}{\partial t} \\
 \frac{T_5 - T_6}{R_{56}} + \frac{T_7 - T_6}{R_{67}} + \frac{T_8 - T_6}{R_{c,oil}} &= m_6 c_6 \frac{\partial T_6}{\partial t} \\
 \frac{T_6 - T_7}{R_{67}} + \frac{T_a - T_7}{R_{c,air}} &= m_7 c_7 \frac{\partial T_7}{\partial t} \\
 \frac{T_2 - T_8}{R_{c,oil}} + \frac{T_4 - T_8}{R_{c,oil}} + \frac{T_5 - T_8}{R_{c,oil}} + \frac{T_6 - T_8}{R_{c,oil}} &= m_8 c_8 \frac{\partial T_8}{\partial t}
 \end{aligned} \tag{15}$$

Since these are PDEs which need to be solved at each time step simultaneously, therefore a Finite Difference Method (FEM) is used to solve these energy balance equations iteratively in MATLAB. Initial temperature of bearing and oil is the ambient temperature of testing rig i.e. 20°C or 293K. Once the resistance values are calculated, the energy balance equations are entered in MATLAB for calculations. Time-step of the iterations is kept very small for relatively accurate results through Finite Difference Method. The code was written in such a way that at each iteration, all the 8 energy balance equations were simultaneously solved for that time step. Once the temperature values for all 8 nodes are calculated for that time-step, the values were then used as initial conditions for the next iteration and the process continues until steady state is achieved.

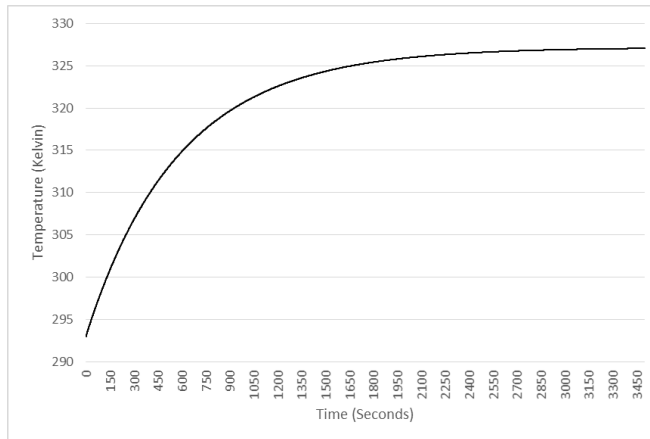
The lubricant used in the experiments was Shell Advance 4T AX5 15W-40. The viscosity depends considerably on temperature of oil [19]. Since heat transfer coefficient values depends upon viscosity of oil, therefore, viscosity of oil is varied successively after each iteration. The viscosity-temperature trend of 15W-40 oil was measured using SVM™ viscometer [20]. The formula generated from the graph was used to vary the viscosity of oil in each iteration.

$$\nu = 28.65179 + \frac{4458102000000}{2^{\left(\frac{T}{8.662826}\right)}} \tag{16}$$

Where  $T$  and  $\nu$  are the Temperature and Viscosity of oil respectively.

### 3 Results and discussions

A comprehensive mathematical model to analyze the transient thermal behavior of a ball bearing is presented. Predictions based on the model described is shown in figure 6. As seen the temperature of the bearing outer raceway increases from the ambient temperature exponentially until it reaches the steady state temperature of about 327K approximately after 50 min. It is useful to note that the evolution of the other nodes temperatures is qualitatively similar to the outer raceway temperature trend.



**Fig. 6.** Predicted Outer Raceway Temperature for the Test Conditions

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## References

1. M. Salah Eldin, M. B. Baharom, A. Rashid A. Aziz, *Journal of Mechanical Science and Technology*, Volume **31**, Issue 7, pp 3563–3571, (2017)
2. C. Jin, B. Wu, Y. Hu, *Tribology International*, **45**(1), pp 8-15, (2012)
3. Y-R Jeng, C-C Gao, *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, **215** (2), pp 139-148, (2001)
4. J. Takabi, M. M. Khonsari, *Tribology International*, **60**, pp 93-103, (2013)
5. X. Min, J. Shuyun, C. Ying, *International Journal of Machine Tools and Manufacture*, **47**(1), pp 53-62, (2007)
6. S. R. Babu, S. Dhamotharan, *International Journal of Mechanical & Mechatronics Engineering*, **14**(6) pp 99-103, (2014)
7. L. Wang, G. Chen, L. Gu, D. Z. Zheng, *Proceedings of the Institution of Mechanical Engineers Part C Journal of Mechanical Engineering Science* 1989-1996, **221**(3) pp 353-359, (2007)
8. R. S. Moorthy, V. P. Raja, *Arabian Journal for Science and Engineering*, **39**(11), pp 8111–8119, (2014)
9. T. A. Harris, *Rolling Bearing Analysis*, Fourth Edition, John Wiley & Sons, Inc. ISBN 0-471-35457-0, (2001)
10. M. M. Khonsari, E. R. Booser, *Applied Tribology: Bearing Design and Lubrication*, Second Edition, John Wiley & Sons, Ltd. ISBN: 978-0-470-05711-7, (2008)
11. W. O. Winer, S. Bair, B. Gecim, *ASLE Transactions*, **29**(4), pp 539-547, (1986)
12. F. Pouly, C. Changenet, F. Ville, P. Velex, B. Damiens, *Tribology Transactions*, **53**(6), pp 957-967, (2010)
13. F. Pouly, C. Changenet, F. Ville, P. Velex, B. Damiens, *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, **224**(9), pp 925-933, (2010)
14. B. R. Jorgensen, Y. C. Shin, *Journal of Tribology*, **119**(4), pp 875-882, (1997)



15. L. Qi, G. Zhang, Y. Liu, *The 2010 IEEE International Conference on Information and Automation*, Harbin, pp 201-205, (2010)
16. L. Q. Wang, G. C. Chen, L. Gu, D. Z. Zheng, *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, **221**(3), pp 353-359, (2007)
17. J. L. Stein, J. F. Tu, *Journal of Dynamic Systems, Measurement and Control*, **116**(3), pp 372-386, (1994)
18. Bejan, *Convection Heat Transfer*, Fourth Edition, John Wiley & Sons, Ltd. ISBN: 978-0-470-90037-6, (2013)
19. Shell Advance 4T AX5 15W-40 Datasheet. Available: [http://www.tg-shell.com/uploads/product/20/Shell\\_Advance\\_4T\\_AX5\\_Scooter\\_15W\\_40\\_TDS.pdf](http://www.tg-shell.com/uploads/product/20/Shell_Advance_4T_AX5_Scooter_15W_40_TDS.pdf)
20. Anton Paar SVM<sup>TM</sup> Viscometer 15W-40 Viscosity Data Measurement. Available: <https://wiki.anton-paar.com/en/engine-oil/>
21. SKF 61902 Deep Groove Ball Bearing Specs Sheet. Available: <http://www.skf.com/group/products/bearings-units-housings/ball-bearings/deep-groove-ball-bearings/deep-groove-ball-bearings/index.html?designation=61902-2RS1>