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Semi-Markov model of a technical system with the component-wise instantly replenished time reserve

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Abstract. Time redundancy is one of the methods to increase the reliability and efficiency of technical systems. When it is used, the system is given additional time (a time reserve) for restoring characteristics. In this paper we construct a semi-Markov model of a two-component system with a component-wise instantly replenished time reserve. In this paper we construct a semi-Markov model of a two-component system with a component-wise instantaneous replenishment of the time reserve. For an approximate determination of the stationary characteristics of the reliability of the system, the phase merging scheme algorithm is used.

1 Introduction

When designing and operating technical systems, great attention is paid to the reliability and efficiency of both the system as a whole and its individual components. There are various ways to improve the reliability and efficiency of technical systems, one of which is time redundancy.

About the time redundancy [1-8] say in cases where the system during its operation is given additional time (time reserve) for the restoration of its technical characteristics. In production systems, the time reserve sources can be different: warehouses, various types of inter operational storage devices, a stock of productivity, etc. Systems with functional inertia have a reserve of time.

For systems with a time reserve, a malfunction (failure of an object) does not yet mean a failure of the system itself, if the restoration of the operability of the object ends before the use of the time reserve. At present, semi-Markov processes are often used to model systems for various purposes [9-13].

In this paper, using the theory of semi-Markov processes with a common phase space of states [9-13], a model of a two-component system with a component-wise instantly replenished time reserve is constructed. The reliability characteristics of the system under consideration are determined; the effect of the time reserve on the characteristics obtained is analyzed.

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2 The system description

The system S, consisting of two components K_1 and K_2, and the component K_1 has an instantly replenished time reserve equal to $h = const$. Components K_1, K_2 time to failure are random variables (RVs) α_1, α_2 with distribution functions (DFs) $F_1(t) = P(\alpha_1 \le t)$, $F_2(t) = P(\alpha_2 \le t)$, component restoration times are RVs β_1, β_2 with DFs $G_1(t) = P(\beta_1 \le t)$, $G_2(t) = P(\beta_2 \le t)$. After the restoration of the first component, the time reserve is replenished to the level *h*. RVs α_1, α_2, β_1, β_2 are assumed to be independent in aggregate, having finite mathematical expectations, functions $F_1(t)$, $F_1(t)$, $G_1(t)$, $G_2(t)$ have distribution densities $f_1(t)$, $f_1(t)$, $g_1(t)$, $g_2(t)$. The case of parallel connection (in reliability sense) of components without their disconnection is considered. The failure of system S occurs if both components are restored and the time reserve is fully used up.

3 Semi-Markov model building and stationary characteristics definition

To describe the system S operation let us introduce the following set E of system semi-Markov states:

$$E = \{1, 210x, 1\bar{0}1x, 1\bar{0}0x, 211x, 111x, 2\bar{0}0xz, 101x_1x_2, 100x_1x_2, 2\bar{0}1xz, 200x, 110x, 201x\}.$$

The conceptual sense of codes is:

1 – components K_1, K_2 begin to operate, time reserve is full and equal to h;

$210x$ – component K_2 has failed, K_1 is in up-state, time $x > 0$ is left till component K_1 failure, time reserve is not used;

$1\bar{0}1x$ – component K_1 has failed but continues to function due to the time reserve, time $x > 0$ is left till component K_2 failure;

$1\bar{0}0x$ – K_1 has failed but continues to function due to the time reserve, K_2 continues restoration, time $x > 0$ is left till K_2 begins operate;

$211x$ – K_2 begins to operate, time $x > 0$ is left till K_1 failure, time reserve is not used;

$111x$ – K_1 has been restored and operates, time reserve replenished to the level of *h* and doesn't use, time $x > 0$ is left till K_2 failure;

$110x$ – K_1 begins operate, time reserve replenished to the level of *h* and doesn't use, time $x > 0$ is left till K_2 begins operate;

$201x$ – K_1 continues restoration, the time reserve is fully used up, K_2 begins operate, time $x > 0$ is left till K_1 begins operate;

$200x$ – K_1 continues restoration, the time reserve is fully used up, K_2 begins restoration, time $x > 0$ is left till K_1 begins operate, system failure;

$2\bar{0}0xz$ – K_1 continues restoration and continues to function due to the time reserve, K_2 begins restoration, time $x > 0$ is left till K_1 begins operate, z is the value of the remaining time reserve;

$101x_1x_2$ – K_1 continues restoration, the time reserve is fully used up, time x_1 is left till K_1 begins operate, time x_2 is left till K_2 failure;

$2\overline{0}1xz$ – K_1 continues restoration and continues to function due to the time reserve, K_2 begins operate, time $x > 0$ is left till K_1 begins operate, z is the value of the remaining time reserve;

$100x_1x_2$ – both components continue restoration, the time reserve is fully used up, time x_1 is left till K_1 begins operate, time x_2 is left till K_2 begins operate, system failure.

Time diagram of the functioning of the initial system is shown in Figure 1:

1(2) corresponds to the functioning of the K_1 (K_2), 1p corresponds to the state of the time reserve.

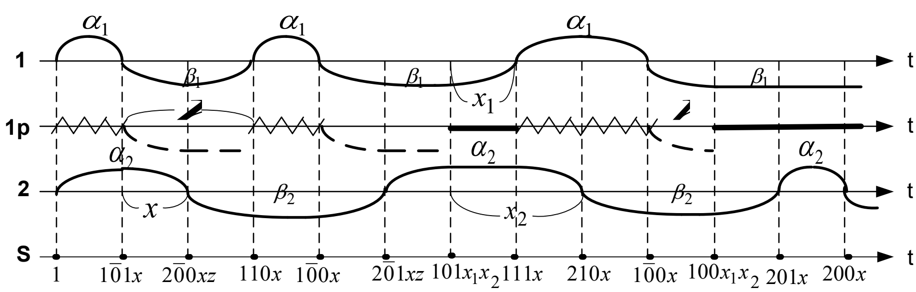

Fig. 1. Time diagram of the functioning of the initial system S.

We proceed to determine stationary characteristics of the system reliability. For this, we use phase merging algorithms developed in the works [10-12].

Suppose that the stochastic kernel of the embedded Markov chain (EMC) $\{\xi_n; n \ge 0\}$ of a semi-Markov process $\xi(t)$ [11,13] is close to the stochastic kernel of the EMC $\{\xi_n^{(0)}; n \ge 0\}$ of supporting system *S₀* with a unique stationary distribution $\rho(dx)$. Then for an approximate calculation of the mean stationary operating time of the system to failure T_+, the mean stationary restoration time T_- and stationary availability factor K_a of the initial system S we can use the following approximate formulas [14]:

$$T_+ \approx \frac{(\rho, \overline{\mathrm{m}}_1)}{(\rho, P^{(\mathrm{r})}\overline{\mathrm{1}}_0)}, \qquad T_- \approx \frac{(\rho, P^{(\mathrm{r})}\,\overline{\mathrm{m}}_0)}{(\rho, P^{(\mathrm{r})}\overline{\mathrm{1}}_0)}, \qquad K_a = \frac{T_+}{T_+ + T_-}, \tag{1}$$

$$\overline{\mathrm{m}}_1(x) = \begin{cases} m(x), x \in E_+, \\ 0, x \in E_-, \end{cases} \quad \overline{\mathrm{m}}_0(x) = \begin{cases} 0, x \in E_+, \\ m(x), x \in E_-, \end{cases} \quad \overline{\mathrm{1}}_0(x) = \begin{cases} 0, x \in E_+, \\ 1, x \in E_-, \end{cases} \quad (\rho, f) = \int_X f(x)\rho(dx),$$

$\rho(dx)$ is stationary distribution of supporting EMC $\{\xi_n^{(0)}; n \ge 0\}$; $m(x)$ is mean residence time of the SMP in the state $x \in E$ of the initial system; $P^{(\mathrm{r})}(x, B)$ – transition probabilities of the EMC $\{\xi_n; n \ge 0\}$ of the initial system, *r* is the minimum number of steps for which the system can go to a subset of fault states E_- from the operable ones E_+ entering the ergodic class E^0.

Choose a supporting system *S₀*. Let's assume that the initial system's uptime is significantly longer than the restoration time. Then, the reference system will be the system

S_0, in which the components are recovered instantaneously, that is, the superposition of the two renewal processes.

Time diagram of the functioning of the support system is shown in Figure 2.

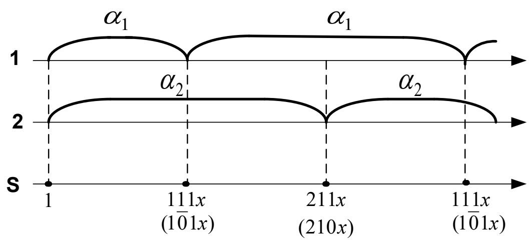

Fig. 2. Time diagram of the functioning of the supporting system S_0.

Let us find EMC $\{\xi_n^0, n \ge 0\}$ transition probabilities:

$$p_{111x}^{1\bar{0}1y} = f_1(x-y),\quad 0<y<x,\quad p_{111x}^{210y} = f_1(x+y),\ y>0,\quad p_{1\bar{0}1x}^{111x} = 1,$$
$$p_{211x}^{1\bar{0}1y} = f_2(x+y),\quad y>0,\quad p_{211x}^{210y} = f_2(x-y),\quad 0<y<x,\quad p_{210x}^{211x} = 1.$$

Denote by $\rho(111x)$, $\rho(1\bar{0}1x)$, $\rho(211x)$, $\rho(210x)$ the values of EMC $\{\xi_n^0, n \ge 0\}$ stationary distribution in states $111x$, $1\bar{0}1x$, $211x$, $210x$. Construct the system of integral equation for them [11]:

$$\begin{cases} \rho(111x) = \rho(1\bar{0}1x), \qquad \rho(211x) = \rho(210x), \\ \rho(1\bar{0}1x) = \int_x^\infty \rho(111y) f_1(y-x)dy + \int_0^\infty \rho(211y) f_2(y+x)dy, \\ \rho(211x) = \int_x^\infty \rho(211y) f_2(y-x)dy + \int_0^\infty \rho(111y) f_1(y+x)dy, \\ \int_0^\infty \rho(111x)dx + \int_0^\infty \rho(1\bar{0}1x)dx + \int_0^\infty \rho(211x)dx + \int_0^\infty \rho(210x)dx = 1 \end{cases} \tag{2}$$

As shown in [11], the system of equations (2) has the following solution:

$$\rho(111x) = \rho(1\bar{0}1x) = \rho_0 \bar{F}_2(x),\ \rho(211x) = \rho(210x) = \rho_0 \bar{F}_1(x), \tag{3}$$

where the value of ρ_0 can be obtained from the normalization requirement.

The class of ergodic states of the support system S_0 has the form:

$$E^0 = \{111x, 1\bar{0}1x, 211x, 210x\}.$$

For the initial system, the sets of operable E_+ and faulty states E_- have the form:

$$E_+ = \left\{1, 210x, 1\bar{0}1x, 1\bar{0}0x, 211x, 111x, 2\bar{0}0xz, 101x_1x_2, 2\bar{0}1xz, 110x, 201x\right\},\ E_- = \left\{100x_1x_2, 200x\right\}.$$

Let us find EMC $\{\xi_n, n \ge 0\}$ transition probabilities of the initial system which are used in the applied method:

$$P_{210x}^{1\bar{0}0y} = g_2(x+y), y>0;\ P_{1\bar{0}1x}^{2\bar{0}0yh-x} = g_1(x+y), y>0, 0<x<h;\ P_{110x}^{1\bar{0}0y} = f_1(x-y), 0<y<x;$$

$$P_{\bar{1}01x}^{101yx-h}=g_1(y+h),y>0,x>h\,;P_{\bar{1}00x}^{110y}=g_1(x-y),0<y<x,x>h\,;P_{111x}^{210y}=f_1(x+y),y>0\,;$$

$$P_{\bar{1}01x}^{110y}=g_1(x-y),0<y<x,0<x<h\,;\quad P_{\bar{1}00x}^{100yx-h}=g_1(y+h),y>0,x>h\,;$$

$$P_{111x}^{1\bar{0}1y}=f_1(x-y),0<y<x\,;\ P_{211x}^{210y}=f_2(x-y),0<y<x\,;\ P_{201x}^{200y}=f_2(x-y),0<y<x\,. \quad (4)$$

The mean values of sojourn times in the states of the initial system are represented by formulas:

$$E\theta_{210x}=\int_0^x\bar{G}_2(t)dt\,,\ E\theta_{\bar{1}01x}=E\theta_{\bar{1}00x}=\int_0^{x\wedge h}\bar{G}_1(t)dt\,,\ E\theta_{211x}=\int_0^x\bar{F}_2(t)dt\,,\ E\theta_{\bar{2}\bar{0}0xz}=\int_0^{x\wedge z}\bar{G}_2(t)dt\,,$$

$$E\theta_{\bar{2}01xz}=\int_0^{x\wedge z}\bar{F}_2(t)dt\,,\ E\theta_{101x_1x_2}=E\theta_{100x_1x_2}=x_1\wedge x_2\,,\ E\theta_{200x}=\int_0^x\bar{G}_2(t)dt\,. \quad (5)$$

For the considered system $r = 2$, because initial system can in two steps go into a subset of failure states E_- from a subset of up-states entering the ergodic class E^0.

We find stationary characteristics of the system T_+, T_-, K_a by using the formulas (1), (3-5).

$$(\rho,\bar{m}_1)=\rho_0\left(E\alpha_1E\alpha_2+\int_0^h\bar{F}_2(x)dx\int_0^x\bar{G}_1(t)dt+\int_0^h\bar{G}_1(t)dt\int_h^\infty\bar{F}_2(x)dx+\int_0^\infty\bar{F}_1(x)dx\int_0^x\bar{G}_2(t)dt\right). \quad (6)$$

$$(\rho,P^{(2)}\bar{1}_0)=\rho_0\bar{G}_1(h)\left[\int_0^h\bar{F}_2(x)\bar{G}_2(h-x)dx+\int_0^\infty\bar{F}_1(x)\bar{G}_2(x+h)dx\right]. \quad (7)$$

$$(\rho,P^{(2)}\bar{m}_0)=\rho_0\int_h^\infty\bar{G}_1(y)dy\int_0^\infty\bar{F}_1(x)\bar{G}_2(x+y)dx+\rho_0\int_h^\infty\bar{G}_1(y)dy\int_0^h\bar{F}_2(x)\bar{G}_2(x+y)dx. \quad (8)$$

$$T_+\approx\frac{(\rho,\bar{m}_1)}{(\rho,P^{(2)}\bar{1}_0)}=\frac{E\alpha_1E\alpha_2+\int_0^h\bar{F}_2(x)dx\int_0^x\bar{G}_1(t)dt+\int_0^h\bar{G}_1(t)dt\int_h^\infty\bar{F}_2(x)dx+\int_0^\infty\bar{F}_1(x)dx\int_0^x\bar{G}_2(t)dt}{\bar{G}_1(h)\left(\int_0^h\bar{F}_2(x)\bar{G}_2(h-x)dx+\int_0^\infty\bar{F}_1(x)\bar{G}_2(x+h)dx\right)}. \quad (9)$$

$$T_-\approx\frac{(\rho,P^{(r)}\bar{m}_0)}{(\rho,P^{(r)}\bar{1}_0)}=\frac{\int_h^\infty\bar{G}_1(y)dy\int_0^\infty\bar{F}_1(x)\bar{G}_2(x+y)dx+\int_h^\infty\bar{G}_1(y)dy\int_0^h\bar{F}_2(x)\bar{G}_2(x+y)dx}{\bar{G}_1(h)\left(\int_0^h\bar{F}_2(x)\bar{G}_2(h-x)dx+\int_0^\infty\bar{F}_1(x)\bar{G}_2(x+h)dx\right)}. \quad (10)$$

As an example of the use of formulas (1,9-10), let us consider a system in which K_1 operating time $E\alpha_1 = 8$ h, K_2 operating time $E\alpha_2 = 6$ h, K_1 recovery time $E\beta_1 = 0.71$ h, K_2 recovery time $E\beta_2 = 0.83$ h, RV $\alpha_1, \alpha_2, \beta_1, \beta_2$ have 5th order Erlang distribution. The time reserve h varies from 0 to 0.5 hours in 0.1 increments. The corresponding values of mean stationary operating time of the system to failure T_+, mean stationary restoration time T_- and stationary availability factor K_a of the system for the specified distribution were calculated. The results are presented in Figure 3.

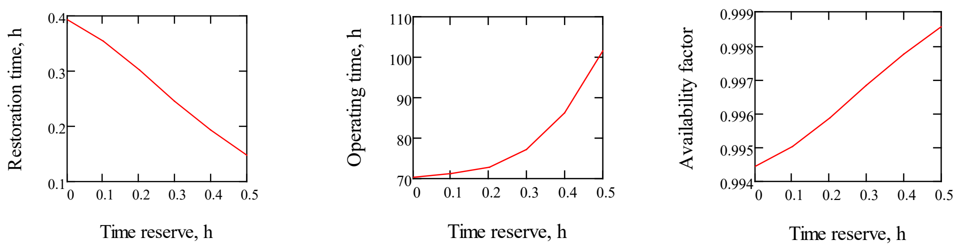

Fig. 3. Mean stationary reliability characteristics of the system.

4 Conclusion

Time redundancy is widely used to ensure the reliability and efficiency of the operation of technical systems. Unlike other kinds of redundancy, the reserve here is the time reserve that occurs during the system operation. The time reserve can be used to switch the structural reserve, detect and eliminate failures, repetition works impaired failures, standby load in working condition. Time redundancy is widely used in technical and information systems, computer networks, communication systems.

In this paper, we construct a semi-Markov model of a two-component technical system with a component-wise time reserve in the case when the reserve has only one component. Using the phase merging scheme algorithm, stationary reliability characteristics of the system under consideration are approximately found. The analysis of the effect of the instantly replenished time reserve value on the obtained reliability characteristics is carried out.

In the future, it is planned to build two-component and multi-component models of systems with a random component-wise time reserve and various strategies for its use.

The results of this work can be used to build models of technical systems with different types and strategies for using the time reserve, engineering calculations and solving optimization problems.

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References

1. G.N. Cherkesov, *Reliability of Technical Systems with Time Redundancy* (Sovietskoe Radio, Moscow, 1974) (in Russian)
2. B.P. Kredentser, *Prediction of Reliability of Systems with Time Redundancy* (Naukova Dumka, Kiev, 1978) (in Russian)
3. V.Y. Kopp, Yu. E. Obzherin, A.I. Peschanskiy, *Stochastic models of automized system with time reservation* (Publishing of Sevastopol State Technical University, Sevastopol, 2000) (in Russian)
4. I.A. Ushakov, *Probabilistic Reliability Models* (Wiley, San Diego, Calif., 2012)
5. Yu.N. Rudenko, I.A. Ushakov, *Reliability of Energy Systems* (Nauka, Novosibirsk, 1989) (in Russian)

6. Yu.E. Obzherin, S.M. Sidorov, S.N. Fedorenko, MATEC Web of Conferences **129**, 03009. (2017)
7. Yu.E. Obzherin, A.I. Peschansky, Cybern. Syst. Anal., Vol. **40**, No. 5. (2004)
8. Yu. E. Obzherin, A.V. Skatkov, J. Math. Sci., Vol. **57**, No. 5. (2010)
9. N. Limnios, G. Oprisan, *Semi-Markov Processes and Reliability* (Springer Science+Business Media, New York, 2001)
10. V.S. Koroluk, A.F. Turbin, *Markovian restoration processes in the problems of system reliability* (Naukova Dumka, Kiev, 1982) (in Russian)
11. V.S. Koroluk, *Stochastic System Models* (Naukova Dumka, Kiev, 1989) (in Russian)
12. V.S. Korolyuk, N. Limnios, *Stochastic Systems in Merging Phase Space* (World Scientific, Imperial Coledge Press, 2005)
13. Y.E. Obzherin, E.G. Boyko, *Semi-Markov Models: Control of Restorable Systems with Latent Failures* (Elsevier Academic Press, London, 2015)
14. A.N. Korlat, V.N. Kuznetsov, M.M. Novikov, A.F. Turbin, *Semi-Markov Models of Recoverable Systems and Queuing Systems* (Stiinta, Chisinau, 1991)