

# Analyzing the dynamics of a single car wheel

*Nikolay Artyomov*<sup>1</sup>, *Mikhail Podrigalo*<sup>1</sup>, and *Aziz Abdulgazis*<sup>2,\*</sup>

<sup>1</sup>Kharkiv Petro Vasylenko National Technical University of Agriculture, Ukraine

<sup>2</sup>Crimean Engineering and Pedagogical University, Lane training, 8, Simferopol, 295015, Simferopol, Republic of Crimea, Russian Federation

**Abstract.** The paper presents an analytical solution to the equation of dynamic energy spending for rectilinear uniform rousset of a drive wheel with an elastic tire when driving on a solid support surface. To that end, the paper proposes different initial calculation charts to analyze the dynamics of the drive wheel; it also finds the efficiency and the additional energy spending in wheel rousset.

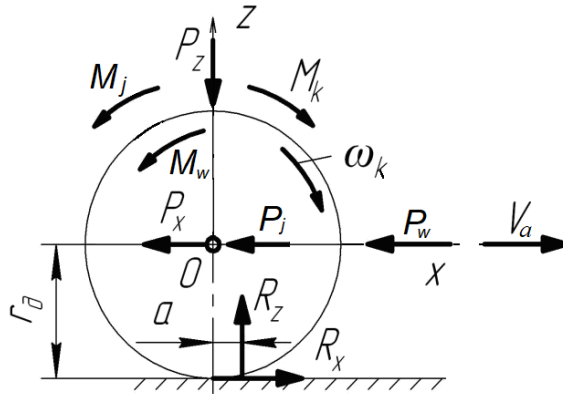
## 1 Introduction

Various approaches and therefore different resulting calculation charts are used when studying car wheels. This results in certain contradictions. Attempts to solve those from the standpoint of classical mechanics are mostly fruitless, as a tire is a deforming body rather than a solid. The paper considers two different representations of the initial drive-wheel calculation diagram for uniform rousset, for which it finds the relevant efficiency expressions. To that end, the study takes into account the parameters that characterize the yielding of tires and the rolling resistance.

State of the Art. Wheel rolling is a process that defines the dynamics of a car. A car wheel that interacts with the support surface is exposed to forces that keep it on the road, move it, stop it, or cause it to change the direction. When exposed to a vertical load, the tire is deformed in the area where it contacts the support surface. The distance from the wheel axis to the support surface becomes smaller than the free radius. In papers [1,2,3] E.A. Chudakov considers the drive-wheel dynamics in accordance with the force and moments chart shown in Figure 1 [1].

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\* Corresponding author: [aziz.abdulgazis@mail.ru](mailto:aziz.abdulgazis@mail.ru)



**Fig. 1.** Forces, moments, and reactions affecting the drive wheels of a car when rolling on a stiff horizontal surface;  $P_z$  is the normal load per wheel,  $R_z$  is the normal road-to-wheel reaction,  $R_x$  is the tangent road-to-wheel reaction,  $P_x$  is the car frame to wheel axis reaction,  $\omega_k$  is the angular velocity of the wheel,  $r_d$  is the dynamic radius of the wheel,  $M_w$ ;  $P_w$  are the moment and the force of air resistance,  $M_j$ ;  $P_j$  is the moment and force of inertia as applied to the wheel,  $a$  is the rolling friction coefficient (shift of the normal reaction  $R_z$ ).

When studying uniform motion of a wheel,  $M_j = 0$  и  $P_j = 0$  The following studies have shown that the air resistance encountered in wheel rolling is negligible, and therefore the values  $M_w$ ;  $P_w$  can be ignored. Papers [4,5] present equations of force and power planace for a uniformly rolling drive wheel, represented as follows:

$$N_f = M_k \cdot \omega_k - P_k \cdot V_a \tag{1}$$

$$M_f = R_z \cdot a = M_k - R_x \cdot r_d, \tag{2}$$

where  $N_f$  is the power the wheel loses in rolling (rolling resistance power).

There is a correlation between the linear wheel-axis velocity ( $V_a$ ) and the angular velocity ( $\omega_k$ ) of the wheel.

$$V_a = \omega_k \cdot r_k, \tag{3}$$

where  $r_k$  is the kinematic radius of the wheel.

Note that by introducing the concept of dynamic ( $r_d$ ) and kinematic ( $r_k$ ) wheel radii [1,2,3] one can effectively reduce an elastic system (which a wheel with a pneumatic tire is) to a rigid system. By definition, the dynamic radius  $r_d$  is the distance from the axis to the support surface, see Figure 1; therefore, it is directly measurable. The dynamic radius  $r_d$  is smaller than the free radius  $r_{cb}$ , which means an increase in traction force and decrease in the linear velocity  $V_a$  as the tire is increasingly deformed when exposed to the increasing force  $P_z$ . The kinematic radius  $r_k$  can be found by indirect management provided the known values  $V_a$  and  $\omega_k$  from the equation (2).

$$r_k = V_a / \omega_k. \tag{4}$$

Therefore, one can argue that losses in energy in a tire depend on the losses in the axis motion speed  $V_a$ . This is typical of yielding systems. A solid has no such losses. Therefore, both the dynamic radius  $r_d$  and the kinematic radius  $r_k$  of a wheel depend on the deformation and the slippage of the tire in the wheel-to-road contact patch and can be used to find the efficiency of the wheel. In papers [4, 5] dividing the left-hand and the right-hand

parts off the equation (1) by  $\omega_k$  while taking into account the expression (3) produces the force balance equation

$$M_f = M_k - P_x \cdot r_k \tag{5}$$

Since  $P_x = |R_x|$ , what makes the above force balance equation (5) different from the equation (2) is the use of the radius  $r_k$  instead of  $r_{\partial}$ . This is not an option, as the equation (5) is a dynamic rather than kinematic equation. The difference in the moments obtained from (2) and (5) respectively equals

$$\Delta M_f = R_x(r_{\partial} - r_k) \tag{6}$$

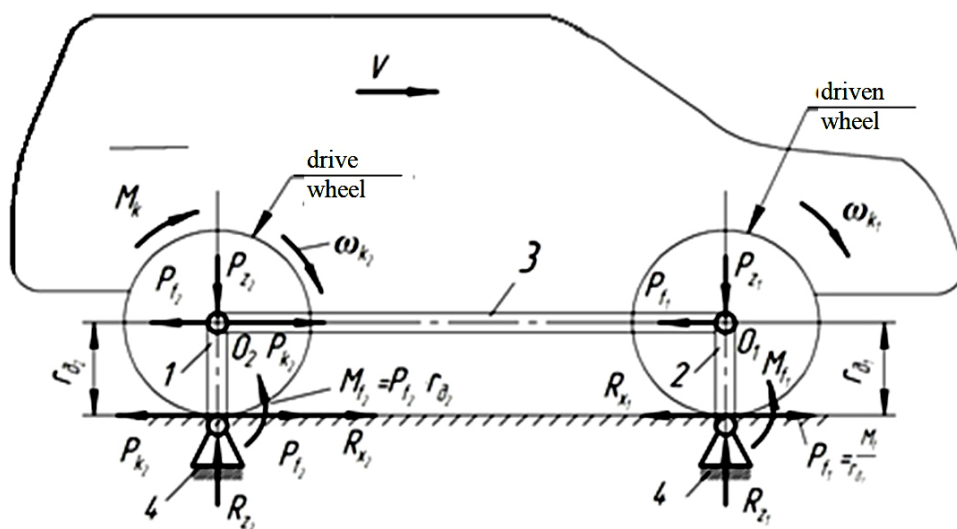
For a rigid zero-slippage wheel,  $\Delta M_f = 0$ , as  $r_{\partial} = r_k = r_{cb}$ . Therefore, it is necessary to refine the force balance equation (1) and the power balance equation (2) for the drive wheel with due account of losses caused by tire yielding.

## 2 Research goal and statement of problem

The research goal consists in finding the dynamic energy spending for rectilinear uniform motion of a drive wheel with an elastic tire when driving on a solid support surface. To that end, one has to solve the following problems: - propose output calculation charts to analyze the dynamics of the drive wheel; - find the efficiency and the additional energy spending of wheel motion.

## 3 Core contents of research

The wheel of a car is an element of the car-road system; when finding its efficiency, one has to clearly classify all the forces into wheel-external and wheel-internal forces. Paper [7] considers a kinematic diagram of the car chassis, see Figure 2.



**Fig. 2.** Reducing the car frame to a replacement mechanism: 1 is the drive rocker (drive wheel), 2 is the idler rocker (idler wheel), 3 is the rod, 4 is the holder.

When a kinematic chain with higher pairs is reduced to a kinematic chain with lower pairs [8], it becomes clear that the torque is a force external to the mechanism [7]. This is contrary to the popular belief [1,2,3,6] that the drive-wheel torque is an internal force, while the tangent reaction in the wheel-to-road contact patch is an external one. There emerges a question whether the moment of rolling resistance is external or external to the wheel as a link of a dual-rocker mechanism, see Figure 2. Two approaches are possible here.

Approach 1. By defining the wheel as the initial link in a dual-rocker mechanism, see Figure 2, we consider the kinematic wheel-holder pair an ideal pair [8]. In that case, the rolling resistance moment  $M_f$  can be considered an external force. Then the instantaneous (power) efficiency can be defined as

$$\eta_k^{MfH} = \frac{P_k \cdot V_a}{M_k \cdot \omega_k} = \frac{P_k}{M_k} \cdot \frac{V_a}{\omega_k} \quad (7)$$

Given the equation (4) as well as the dynamic-radius expression

$$r_\partial = \frac{M_k}{P_k} \quad (8)$$

We obtain the final instantaneous-efficiency expression

$$\eta_k^{MfH} = \frac{r_k}{r_\partial} \quad (9)$$

In that case, the power-balance equation (1) is represented as

$$N_f = \eta_k^{MfH} \cdot M_k \cdot \omega_k - P_x \cdot V_a \quad (10)$$

By dividing the left-hand and the right-hand parts of the equation (10) by  $\omega_k$  and taking into account the equation (9), we obtain

$$M_f = \frac{N_f}{\omega_k} = \frac{r_k}{r_\partial} \cdot M_k - P_x \cdot r_k = \frac{r_k}{r_\partial} (M_k - P_x \cdot r_\partial) \quad (11)$$

We then refine and rewrite the power balance equation (2) as

$$M_f = \eta_k^{сил} \cdot M_k - P_x \cdot r_\partial \quad (12)$$

where  $\eta_k^{сил}$  is the force efficiency of the wheel.

From the equation (12), we find

$$\eta_k^{сил} = \frac{M_f + P_x \cdot r_\partial}{M_k} \quad (13)$$

From the equation (11), we find the wheel torque  $M_k$

$$M_k = \frac{r_\partial}{r_k} \cdot M_f + P_x \cdot r_\partial \quad (14)$$

By substituting the equation (14) into (13), we obtain

$$\eta_k^{сил} = \frac{M_f + P_x \cdot r_\partial}{\frac{r_\partial}{r_k} \cdot M_f + P_x \cdot r_\partial} = \frac{1 + \frac{P_x \cdot r_\partial}{M_k}}{\frac{r_\partial}{r_k} + \frac{P_x \cdot r_\partial}{M_f}} \quad (15)$$

It is clear from (15) that the force efficiency of a wheel depends on the rolling resistance moment  $M_f$ , which is a force external to the wheel. At  $M_f = 0$ ,  $\eta_k^{сил} = 1$ .

Approach 2. Consider the kinematic wheel-road pair a non-ideal pair. In this case, the rolling resistance moment  $M_f$  is the moment of friction in the holder, i.e. an internal force. In this approach, the instantaneous wheel efficiency

$$\eta_k^{MGH} = \frac{P_x \cdot V_a}{M_k \cdot \omega_k} = \frac{\left(P_k - \frac{M_f}{r_\partial}\right) \cdot r_k}{M_k} = \frac{r_k}{r_\partial} \left(1 - \frac{M_f}{M_k}\right). \quad (16)$$

The expression in brackets in the right-hand part of the equation (16) is a wheel efficiency component that takes into account the loss of energy to overcome the rolling resistance

$$\eta_f = 1 - \frac{M_f}{M_k}. \quad (17)$$

Therefore, the power balance equation can be defined as

$$M_k \cdot \omega_k \cdot \eta_k^{MGH} = P_x \cdot V_a. \quad (18)$$

By dividing the left-hand and the right-hand parts of the equation (18) by  $\omega_k$ , we obtain

$$M_k \cdot \eta_k^{MGH} = P_x \cdot r_k. \quad (19)$$

By substituting (16) into (19), we obtain

$$M_k \cdot \frac{r_k}{r_\partial} \left(1 - \frac{M_f}{M_k}\right) = P_x \cdot r_k. \quad (20)$$

or

$$M_k - M_f = P_x \cdot r_\partial. \quad (21)$$

Rewrite the force balance equation as

$$M_k \cdot \eta_k^{CHL} = P_x \cdot z_\partial \quad (22)$$

from which it follows that

$$\eta_k^{CHL} = \frac{P_x \cdot z_\partial}{M_k} \quad (23)$$

By substituting (21) into (23), we obtain

$$\eta_k^{CHL} = \frac{P_x \cdot z_\partial}{M_f + P_x \cdot z_\partial} = \frac{1}{1 + \frac{M_f}{P_x \cdot z_\partial}} \quad (24)$$

Analysis of results obtained by both approaches has shown that aside from energy spent to overcome the rolling resistance of the drive wheels, some losses are due to the tire yielding. This component can be defined as

$$\eta_k^{nsh} = \frac{r_k}{r_\partial} \quad (25)$$

In Approach 1

$$\eta_k^{nsh} = \eta_k^{MGH} \quad (26)$$

Additional yielding-caused energy loss factor

$$\Psi_k^{\text{num}} = 1 - \frac{r_k}{r_0} \quad (27)$$

Let us define the additional losses in power on the basis of (27)

$$\Delta N_k = M_k \cdot \omega_k \left(1 - \frac{r_k}{r_0}\right) \quad (28)$$

Thus, we have obtained analytical expressions that can be used to find the drive-wheel efficiency as well as additional energy losses due to tire yielding.

### 3 Conclusions

1. Classical approach to writing the drive-wheel power and force balance equations for uniform car motion results in certain contradictions. Those are due to regarding the wheel as a solid.

2. The effect of the yielding of tire as the main load-bearing wheel component manifests itself in the effect of the kinematic-to-dynamic radius ratio. The ratio equals the efficiency component that takes into account tire yielding.

3. When analyzing the dynamics of a single drive-wheel of a car, two approaches are possible. In the first approach, the road-wheel kinematic pair is assumed to be ideal, whereas the rolling resistance moment is assumed to be an external force. In the second approach, the kinematic pair is assumed to be non-ideal, the rolling resistance moment is the internal friction moment.

4. Additional power losses are due to tire yielding and become greater at smaller values of the ration  $\frac{r_k}{r_0}$ .

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