Development and research of the rotating lever object as a dynamic model of a cycle mechanism

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Abstract. The operation of high-speed automatic machine mechanisms causes oscillating (dynamic) processes because of the elastic deformation of their elements. This results in a sharp increase in loads in the mechanism links and a decrease in their positioning accuracy. The quantitative estimation of dynamic processes is performed on the basis of modeling the mechanisms by systems of concentrated masses connected by elastic-dissipative and kinematic bonds. One way to develop such systems and describe them mathematically is the method of representing the mechanism as a set of a limited number of objects, each of which represents either a typical mechanism or its separate part. This article considers the development and research of the mathematical model of the rotating lever as one of the most common elements of cycle mechanisms which also include cam-leverage, crank-and-rod and other mechanisms.

1 Dynamic process modeling in mechanisms

The evaluation of the level of dynamic processes influencing the performance and reliability of automatic machine mechanisms [1-4] includes the following stages:

• Creating a dynamic model. The traditional approach [1] implies the substitution of a real object with a certain system of concentrated masses connected by stationary, holonomic piecewise-continuous elastic-dissipative and kinematic bonds. The implementation of such an approach does not always lead to a satisfactory result because it is impossible to provide a correct estimation of the correspondence of a model to a real mechanism [3, 5, 6]. Another approach used in this paper is to represent a real mechanism as a set of interconnected objects – as an object representation [3, 6-8]. Each of the objects is either a group of links, or one link. It is important that the dynamic models of the object be already developed, their applicability limits determined, and their reliability proved.

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• Developing a mathematical model. Traditionally, equations of dynamic model mass motion can be obtained on the basis of the Lagrange equation of the second kind [1, 9]. The process is quite time consuming, but completely formalized. However, if there are mathematical models of each of the objects, then the final model can quite easily be obtained by uniting them into a single system, with certain rules being followed [5, 6, 9].

• Certain difficulties may arise when determining the numerical values of the dynamic model parameters: masses, moments of inertia, rigidity, dissipation coefficients, kinematic functions, characteristics of gaps in kinematic pairs, as well as external forces influencing the links of the mechanism. The 3D model of the mechanism can serve as the basis for determining the model's parameters of elasticity and inertia [10]. The remaining parameters are still being developed alongside with researching the models of objects. If the traditional way of model development is used, then the values of such parameters as dissipation coefficient, friction coefficient and gap characteristics can be assigned only approximately.

• Software solution for the computer calculation of the dynamic processes in the mechanism. There are programs which are known to allow the use of mathematical models of objects, automate the process of their unification into a single model and of their research based on dynamic processes in the mechanism [3, 5, 11].

This article discusses a technique for creating and testing mathematical models of a typical object on the example of the rotating lever object.

2 A mathematical model of the rotating lever object

The rotating lever is among the most common elements in the designs of most mechanisms, in the form of a cantilever beam of variable cross-section, moving in a plane perpendicular to the axis (Fig. 1). Assuming that the rotating lever is an object, the task is to develop and investigate its model on the assumption that the object itself is modeled by a system of masses successively connected by elastic-dissipative bonds [1].

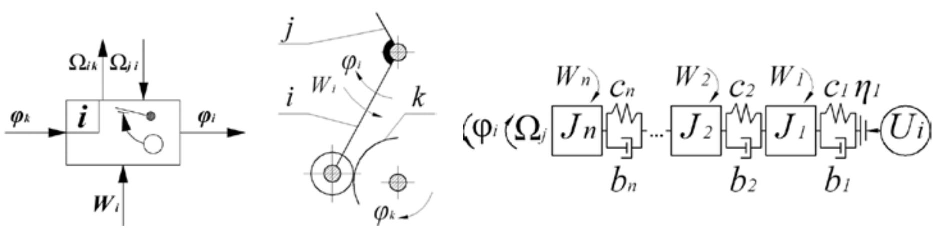

Fig. 1. Object – cam gear lever: a - designation on the object representation of the mechanism (i-th object), b - kinematic scheme, c - dynamic model.

In developing the object model, the following assumptions were made:

• The modeled link (lever) is a continuous homogeneous body divided into *n* fragments by cutting planes perpendicular to its axis;

• Each of the fragments performs only rotary motion about the lever axis. There is neither tension-compression deformation in the lever nor the rotation of the fragments about any other axis except the indicated one.

Different links (groups of links) of mechanisms may correspond to the object, or rather its model, with a certain degree of accuracy. This object (its symbol is given in Fig. 1a) is the cam gear lever (Fig. 1b).

The equation of dynamic model mass motion is (Fig. 1c):

$$\begin{cases} J_1\ddot{\varphi}_1 = -c_1(\varphi_1 - U_i(\varphi_k)) - b_1(\dot{\varphi}_1 - U_i'(\varphi_k)\dot{\varphi}_k) + c_2(\varphi_2 - \varphi_1) + b_2(\dot{\varphi}_2 - \dot{\varphi}_1) + W_1 \\ J_2\ddot{\varphi}_2 = -c_2(\varphi_2 - \varphi_1) - b_2(\dot{\varphi}_2 - \dot{\varphi}_1) + c_3(\varphi_3 - \varphi_2) + b_3(\dot{\varphi}_3 - \dot{\varphi}_2) + W_2 \\ \dots\ \dots\ \dots \\ J_n\ddot{\varphi}_n = -c_n(\varphi_n - \varphi_{n-1}) - b_n(\dot{\varphi}_n - \dot{\varphi}_{n-1}) + W_n + \Omega j \end{cases}, \quad (1)$$

where n is the number of concentrated masses modeling the object under consideration (rotating lever). Obviously, the larger n, the more accurately the model will describe the object, in the natural case when the values of its other parameters are reliable.

Thus, if the equations (1) are used as a mathematical model of the rotating lever, the result will be as follows:

• Input parameters: φ_k, $\dot{\varphi}_k$ - movement and speed of the k-th object preceding it; Ω_j – perturbation from the j-th object following the considered one (2); W_m (m = [1, 2, …, n]) – external loads. It is the moment applied to the m-th mass of the model.

• Output parameters: φ_i, $\dot{\varphi}_i$ - movement and speed of the object; Ωi – response to the perturbation of the preceding object (2); δ, $\delta_{\max}$, σ – accuracy of positioning the lever, its maximum and average values (3).

$$\Omega_i = [c_1(\varphi_1 - U_i(\varphi_k)) + b_1(\dot{\varphi}_1 - U_i'(\varphi_k)\dot{\varphi}_k)] \cdot U_i'(\varphi_k); \quad (2)$$

$$\delta = \varphi_n - Ui(\varphi_k), \quad \delta_{\max} = \max(\varphi_n - Ui(\varphi_k)), \quad \sigma = \frac{1}{T}\int_0^T \left|\varphi_n - Ui(\varphi_k(t))\right| dt. \quad (3)$$

The characteristics of the (3) type are needed to assess the level of dynamic processes occurring in the object. In this paper they will be used to test the models.

• Properties: n – number of dynamic model masses (Fig. 1c); J_m – moments of inertia of the lever fragments about the axes of their rotation ($J_m = const$); c_m, ψ – elastic-inertial characteristics of the bonds among the lever fragments ($c_m = const$) at $m \neq 1$); η_i – gap in the connection of the considered object with the previous one [9, 12]; $U_i(\varphi_k)$, $U_i'(\varphi_k)$ - lever position function and its derivative.

In order to determine the values c_m, b_m, dependences are used given in [1]. $U_i(\varphi_k)$, $U_i'(\varphi_k)$ is the lever position function and its derivative.

3 The applicability limits and stability of the model

Any model's applicability in a system (***dam***) will be tested according to the following indicators: complexity, stability, applicability limits. As a test link, to which a particular investigated object corresponds, a cantilever rotating beam will be chosen, with a rectangular section, twice higher than its width, moving according to a harmonic law of the following kind:

$$U = \frac{\pi}{2} \cdot \sin 2\pi\nu t, \quad U' = \frac{\pi}{2} \cdot \cos 2\pi\nu t. \quad (4)$$

Essentially, it is a rod ($J = 0.1$ kg / m^2, $c = 1.0 \cdot 10^7$ nm / rad) swinging on the axis, passing through one of its ends with an amplitude equal to half the revolution (180 degrees) and frequency of ν times per second ($\nu = 10$ sec^{-1}). The gap η is assumed to equal zero. Resistance forces are not taken into account. The moment ensuring the closure of the kinematic chain is assumed to equal 600 nm. Fig. 2 shows the calculated values of the kinematic characteristics

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(movement - φ, velocity - $\dot{\varphi}$, acceleration - $\ddot{\varphi}$) obtained on the basis of single-mass, two-mass and ten-mass models. In this case, the maximum and the average deviations equal, respectively, to the following values: for the single-mass model 0.0075 and 0.0016 rad, for the two-mass model 0.0075 and 0.0017 rad, for the ten-mass model 0.0089 and 0.0014 rad.

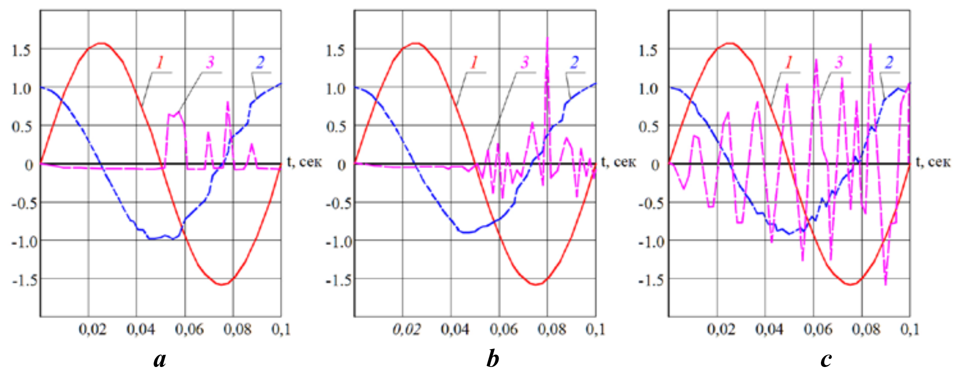


Fig. 2. Characteristics of the cam gear lever object: a - single-mass, b - two-mass, c - ten-mass models. 1 - movement (· 1 rad), 2 - velocity (· 10^2 rad · sec^{-1}), 3 - acceleration (· 10^5 rad · sec^{-2})

If the data obtained on the basis of the ten-mass model are taken as standard, the relative error in the calculations for the single-mass and two-mass models will equal respectively: for δmax - ≈15.9% and ≈16.2%, for σ - ≈16.3% and ≈20% . Thus, as can be seen from the above dependences, a single-mass model is sufficient enough to obtain the results of acceptable accuracy on movements and velocities. The same model makes it possible to estimate the level of loads in the lever and, unfortunately, is not suitable for calculating accelerations. It should be noted that the account of the resistance forces makes it possible to increase sharply the convergence of the results (Fig. 3). So, at $\psi = 0.4$, the relative error for δmax does not exceed 0.4%, and for σ - 6%.

The analysis of the given dependences (see Fig. 2, 3) shows that for the given speed mode, values of elastic-inertial characteristics and the closing torque value, there occurs a loss of contact in the kinematic pair of the object. If this is an actual cam mechanism, then during its operation, the cam lever roller constantly “knocks” along the cam track. Evidently, in this case there is no reason to talk about the operability of the mechanism.

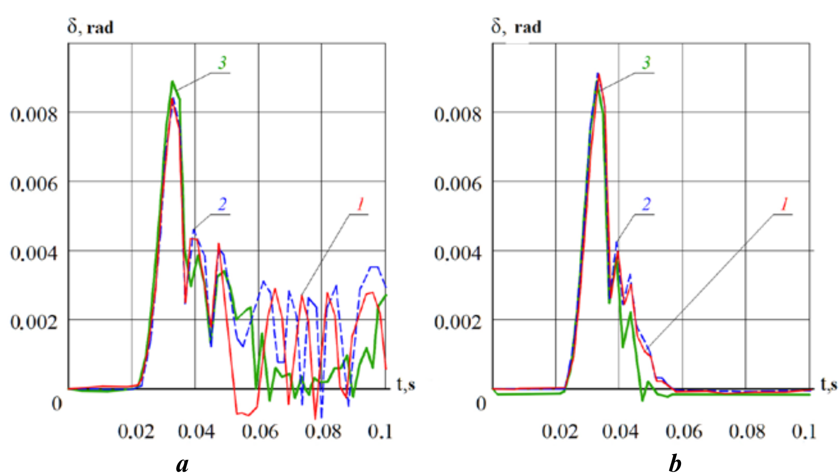
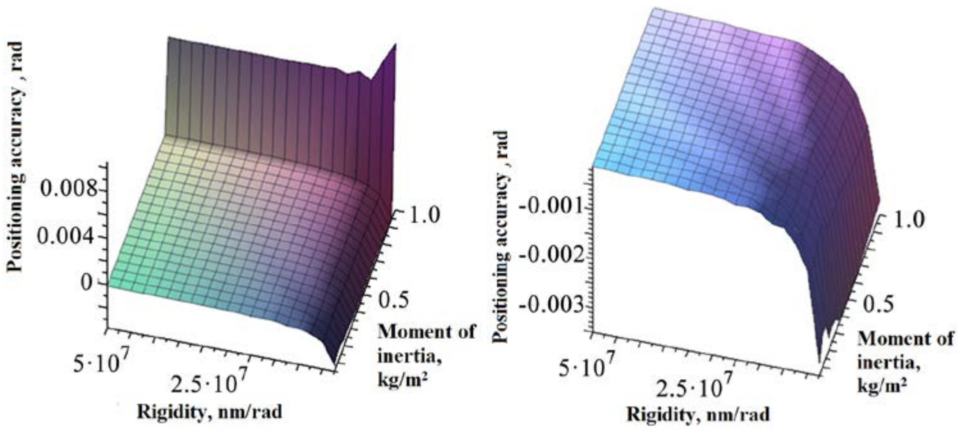


Fig. 3. The accuracy of object positioning: a - $\psi = 0$, b - $\psi = 0.4$; 1 - single-mass model, 2 - two-mass model, 3 - ten-mass model

In order to determine the applicability limits within which the model is stable, its behavior is investigated in the following ranges of value variation of elastic-inertial parameters: moments of inertia - $J = [0.05,\ 1.0]$ kg / m², rigidity - $c = [0.5 \cdot 10^7,\ 5.0 \cdot 10^7]$ nm /rad.



a ***b***

Fig. 4. To calculate the model stability: a - $\nu =\ 10, \quad M = 600$; b) $\nu =\ 10, \quad M = 6000$

Fig. 4 shows the graphs of the function $\delta_{\max}(J,c)$ for various closing torques and speed modes. A two-mass model was used to construct them. The analysis of the given dependences shows that when the model parameters change within the investigated limits of the zone, it does not lose its stability. A fairly sharp increase $\delta_{\max}(J,c)$ (Fig. 4a) is explained by a loss of contact in the kinematic chain of the object, which should be regarded as a loss of operability by the mechanism to which the given object corresponds. The problem can be solved by increasing the closing force (Fig. 4b).

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