MATEC Web of Conferences **224**, 02066 (2018) https://doi.org/10.1051/matecconf/201822402066
ICMTMTE 2018

Distributed impact elements and their use in modelling vibration fields in machines structures

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Abstract. To model the vibration fields established in composite multidimensional machine structures, distributed impact elements are used, which take into account the wave processes occurring in colliding subsystems (shock pairs), which, in turn, quite often determine the structure of these fields. Two types of corresponding models are considered: with simple and complex structures. Specific examples of dynamic models are given. The results of the analysis are given.

1 Introduction

Modeling of vibration fields in machine structures is an important and actual problem, since their features and structure on the one hand determine the vibratory activity of machines and machine systems, and on the other hand can determine many aspects of the useful use of vibration processes. One of the most important factors in the formation of the vibratory field is the systematic collisions of structural elements that generate broadband shock-vibrating processes [1, 2]. In many cases, it is shock-vibrating processes that are the determining factors that influence the structure of vibratory fields. The description of the global vibratory fields that are installed in spatial regions can be divided into two sections. First, for many problems it is important to take into account the wave properties of the colliding bodies themselves. For example, in cases where the vibratory fields are determined, in particular, by the collisions of any strings, threads, beams, chains, wires, cables, membranes, plates, panels, gratings and the like extended, flat or more complex objects.

Secondly, a large number of systems can be represented in the form of any supportive elastic, viscoelastic, resilient-plastic and the like structures with an elastically amortized equipment containing any shock pairs.

In the first case, the modeling should be carried out by describing the shock process in the structures themselves. Models of distributed strongly nonlinear systems of simple structure can be used here [3]. In the second case, it may be expedient to conduct some idealization ("smearing" the equipment over space) and use models of highly nonlinear continuous media of complex structure [2, 4-6].

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Note that the selection of two cases is made for convenience. They have a close nature, they are described in many ways similar models and they exhibit a manifestation of similar dynamic effects, perhaps the main one is the emergence of "claps" (the appearance of synchronization of shocks at remote points). It is systems of two types will be considered in the article.

2 Forced flat oscillations of a string with a double limiter

2.1 Statement of the problem

Let`s consider the system shown in Fig. 1. Absolutely flexible thread (string), jammed at the ends, makes flat oscillations, limited by point and flat obstacles. We assume that the flat wall vibrates according to the law $\Delta_0(t) = -\Delta - 2\varepsilon - \varepsilon d(t) < 0$; ε - small parameter $(\Delta_0 > \varepsilon > 0)$

We denote the required deflection (vibratory field) as $u(x,t)$. Having chosen the units of measurement so that the density of the string material and the tension are single, in the absence of touching the limiters, we will have: $u_{tt} - u_{xx} = 0$.

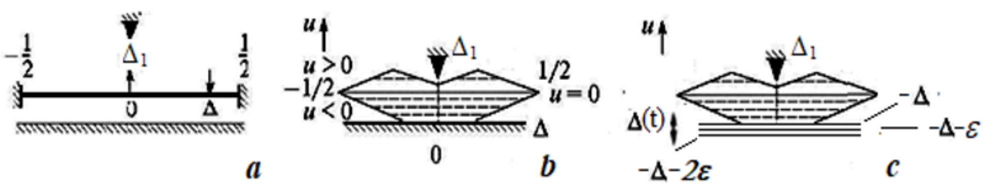

Fig. 1. A string with a double obstacle.

Suppose $x \in [-½, ½]$; $t \in \mathrm{R}$. The boundary conditions and the relations describing the obstacles are written in the form

$$u(-½, t) = u(½, t) = 0;\ u(x,t) \geq \Delta(t),\ \Delta < 0;\ u(0,t) \leq \Delta_1,\ \Delta_1 > 0. \quad (1)$$

Here Δ_1 is the coordinate of the point limiter located in the middle of the string. The problem under consideration can be written in the form of a nonlinear Klein-Gordon equation

$$u_{tt} - u_{xx} - \Phi(u) = 0,$$

where the generalized function $\Phi(u)$ is the density of the interaction force of a string with obstacles: $\Phi(u) = \Phi\Delta(u) + \Phi\Delta1(u)$ - can be represented as the sum of the densities of interaction forces with a straight line and point delimiters (Fig. 1, a). The introduction of generalized functions is necessary in view of the discontinuities arising from shocks [1, 3, 4].

The case of a conservative system ($\varepsilon = 0$) was considered in the article [3]. The form of the strength of the interaction force with a point limiter $\Phi\Delta 1\ (u)$ was determined from the following considerations. When interacting, when x = 0, for some time there are string expansions. In this case, the force of the reaction of the point obstacle $r_j(t)$ under $u(0, t) = \Delta_1$ ($t \in [t_j, \theta_j]$) acts on the string for during the projections. Here, t_j and θ_j are calculated moments of the beginning and end of the dwell; the index *j*- corresponds to some *j*-th interaction. In this way

$$\Phi_{\Delta 1}(u)=R_j(t)\delta(x)[\eta(t-t_j)-\eta(t-\theta_j)],\ R_j(t)=u_x(-0,t)-u_x(+0,t)\le 0,;\ t\in[t_j,\theta_j], \tag{2}$$

where η (*t*) is the unit function. When interacting with the flat part of the limiter in the conservative case ($\varepsilon = 0$) a continuum analogue of Newton's conjecture is adopted:

$$u_t(x,t-0)=-u_t(x,t+0),\ u(x,t)=\Delta. \tag{3}$$

Relations (3) define a generalized function $\Phi\Delta(u)$. For some n-th approach of the string to the limiter [3]

$$\Phi_\Delta\ (u)=J(x)\delta[t-t_n(x)]\gamma(x;\Delta). \tag{4}$$

Here $J(x)$ is the density of the shock pulse, $t_n(x)$ is the distribution of the n-th "phase" of the shock, determined from the equation $u[x,\ t_n(x)]=\Delta<0$; $\delta(t)$ - δ- Dirac function : The indicator function $\gamma(x;\Delta)=0$ for those x when the string does not interact with a flat limiter and $\gamma(x;\Delta)=1$ when it interacts.

Thus, the problem is to study the solution of equation (2) with the additional conditions (1). When $\varepsilon = 0$ such a study was carried out in the paper [3]. It was shown, in particular, that if the initial conditions in a certain sense are similar to those that generate the first standard form of natural oscillations of a string in the case of motions without constraints, when the initial conditions of the problem are $u(x,0)=0$; $u(x,0)=0$; $u_t(x,0)=-2V_0\pi\sin[\pi(x+{}^1/_2)]$ this is the only periodic vibratory shock process that preserves structural stability, i.e. does not qualitatively depend on small changes in parameters, will refer to the class of synchronous claps [3, 7] and have the structure:

$$u(x,t)=W(J_0;x,t)=\sum_{k=1}^{\infty}D_{1k}(J_0;x,t)D_{2k}(J_0;x,t),\ J(\omega)=\omega(2|\Delta|+\Delta_1)(2\pi-\omega)^{-1} \tag{5}$$

Here, the function $W(J_0;x,t)$ corresponds to the profile of a standing wave of the "clap with two teeth" type (Fig. 1, b); $D_{1k}(J_0;x,t)$ и $D_{2k}(J_0;x,t)$ periodic, with a $2\pi\omega^{-1}$ period, rather cumbersome functions, depending on all the parameters of the system appearing in formulas (1), (3) and (4), as well as the proper properties of the string. We describe the motion using Fig. 1, *b*.

Suppose that from a position of equilibrium the string, taking a trapezoidal profile uniformly, moves with velocity $-\frac{1}{2}J(\omega)$ toward a flat obstacle. At time t_1, the upper base of the trapezium collides with the obstacle and is reflected in accordance with condition (3). Then the upper part of the string begins to move to the sharpened limiter, which it reaches at some moment t_2. After that, the midpoint makes a high point, and the profile acquires a "two-trapezoidal" shape. After reaching the final "two-tooth" configuration, the process proceeds in the reverse order. It should be emphasized that all the parameters of this profile are defined in [3]. We also note that the second formula (5) determines the frequencies of existence of structurally stable periodic standing waves of a given type. These frequencies fill the segment, and the frequency $\omega\to 2\pi$ corresponds to infinitely high energies.

2.2 Forced oscillations

Suppose the parameter $\varepsilon > 0$. For sufficiently small values of it, we will seek forced oscillations in the form of claps (5), since this mode is the only structurally stable one. For these modes, basically, it is legitimate to use the analogue of the Newton hypothesis on the recovery coefficient.

Suppose $0 < R \leq 1$. Without changing conditions (1) and (3), we put instead of (2)

$$u_t(x_0, t_0 + 0) = -Ru_t(x_0, t_0 - 0) + (1+R)\Delta_0'(t_0) \tag{6}$$

Claps can be characterized by a pair of variables (v_k, t_k), where v_k is the velocity modulus of each point of the impact segment before the k_{th} collision occurring at time t_{k}. Between strikes, the string is linear (the period of oscillations equals to 2). Therefore

$$v_{k+1} = Rv_k + (1+R)\Delta'(t_k)\,;\, t_{k+1} = t_k + 1 + 2\,|\,\Delta_0'(t_k)\,|\, v_{k+1}^{-1}. \tag{7}$$

The function $\Delta_0(t)$ is sinusoidal; using the periodicity conditions: $v_k = v_k \equiv v$; $t_k = kt + \varphi$ ($\varphi-$unknown phase), we obtain from (7):

$$v(1-R) = (1+R)\varepsilon\omega\sin\varphi \qquad \tfrac{1}{2}v(2\pi n - \omega) - \omega(\Delta+\varepsilon) = \varepsilon\omega\cos\varphi \tag{8}$$

Eliminating the phase from equations (8) and introducing the notation $\beta \equiv 2\pi - \omega$; $\gamma \equiv \omega(\Delta+\varepsilon)$; $R_0 \equiv (1+R)(1-R)^{-1}$, we find two values of the velocity:

$$v_{1,2} = \{2\beta\gamma \pm \sqrt{[\beta^2\gamma^2 - (4R_0^2 + \beta^2)(\gamma^2 - \varepsilon^2\omega^2)]}\}(4R_0^2 + \beta^2)^{-1} \tag{9}$$

So, as for traditional vibratory shock systems, there are two modes of motion with shocks. The values of the phases $\varphi_{1,2} \in [0, \pi]$. Impulse densities $J_{1,2}(\omega) = 2v_{1,2}$. The values of the interacting trapezoidal bases are found from the form of standing wave profiles (Fig. 1, *c*): $l_{01,2} = 1 - 2(\Delta + \varepsilon + \varepsilon\cos\varphi_{1,2})$.

Therefore shock impulses $I_{1,2}(\omega, \varphi_{1,2}) = 2[1 - 2(\Delta + \varepsilon + \varepsilon\cos\varphi_{1,2})]v_{1,2}$.

The nature of the relationship $I(\omega)$ is shown in Fig. 2; curves a and b correspond to the absence of dissipation (R = 1: $\varphi=0, \pi$); curve c - the value of $\varphi = \pi / 2$.

In the case of R <1, as the velocity ($\omega \rightarrow 2\pi$) increases: $\varphi_{1,2} \rightarrow \frac{1}{2}\pi$ (the boundary of the conditions of existence is reached). Both modes merge into one. A detailed analysis of the exact solutions found shows that these regimes are characterized by nonlinear resonant effects ("tightening" in frequency and amplitude, "resonance mode failure", etc.), manifested in traditional systems with classical shock pairs [1, 3, 4]. This circumstance was also noted during experimentation [1]. Thus, the dynamics of trapezoidal standing waves is similar to the dynamics of particles colliding with rigid walls. We note that these solutions are accurate in the framework of the accepted assumptions. Similarly, one can consider subharmonic regimes 1: n, and also obtain other generalizations.

To study the stability of the solutions found, the system is analyzed by the method of finite differences. The unstable mode corresponding, as in the traditional case to a smaller value of the momentum, is indicated by a dotted line in Fig. 2.

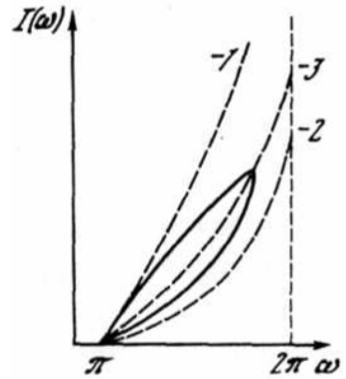

Fig. 2. Dependence of the pulse on the wave frequency.

3 Models of systems with complex structure

3.1 General model

For objects with a complex structure, it is assumed that at each point of any spatial regions a shock pair is placed. So the vibratory field is described by a set of displacement functions $\{u_\alpha(x,t)\}$, and the index α can change on a finite, countable or even a continual set [7], and the variable x is a scalar or a vector.

Let us give an example of a model of a strongly nonlinear object of a complex structure. First, we postulate the existence of some elastic, elastic-viscous, or some other medium, which we will call a carrier. For example, it is described by the dynamic Lame equation (Navier-Cauchy) with addition terms of the form:

$$\rho u_{tt} = \mu\Delta u + (\lambda + \mu)\nabla\nabla \cdot u + B\left[u\right] + \mathrm{G}\{t,u,[u_\alpha]\} - \Phi[u_\alpha]. \qquad (10)$$

Here the state of the carrier part is described by the displacement field $u(x,t);\ u \in \mathbf{R}^3;\ \ x \in \mathbf{Z} \subset \mathbf{R}^3;\ t \in \mathbf{R}.$. It is denoted that: the Hamilton (nabla) and Laplace operators; ρ - is the density of the carrier part of the medium; λ and μi are its elastic parameters; B [...] is the operator of dissipative losses in the carrier medium; $[\mathrm{u}_\alpha]$ - a set of displacement functions of the attached parts of the medium; $\mathrm{G}\{....\}$ - the corresponding power characteristic; $\Phi[\mathrm{u}_\alpha]$.- the impact of attached objects containing percussion elements. We write the boundary conditions symbolically in the form $\Gamma u = X$, where Γ is the operator of the boundary conditions, and X is a given vector.

When considering the media of a complex structure of this type, the boundary conditions are set only for the carrier part.it should be noted that that here for simplicity, some attributes related to purely mathematical aspects of the formulation have been and will be omitted.

Let the depreciated equipment be elastically connected to the carrier medium:

$$\Phi[u_\alpha] = c_1\left(x\right)[u(x,t) - u_1^I(x,t)] + c_2(x)[u(x,t) - u_1^{II}(x,t)] \qquad (12)$$

This modeling method assumes that a shock pair is connected to each point of the region $\mathbf{Z}$ through an elastic element, determined by the contacting linear subsystems

$\boldsymbol{A}^{(\mathrm{I})}(x)$ and $\boldsymbol{A}^{(\mathrm{II})}(x)$, each of which corresponds to the displacement fields $u_q^{I,II}(x,t)$; q=1, 2,..,N. In formula (12) $u_1^{I,II}(x,t)$ the points of addition of subsystems; the points of contact are denoted as $u_n^{I,II}(x,t)$, $n \le N$. Subsystems $\boldsymbol{A}^{(\mathrm{I,II})}(x)$ are described by families of distributed dynamic compliance: the indices q, k run through values from 1 to N; N; $i\omega = \partial / \partial t$.

We will assume that the collisions are straight, central and one-dimensional. Suppose that $u_0 = u_n^{II} - u_n^{I}$ is the the approach of contact points. The force of interaction under shock $\Phi(u_0, u_{0t}) = \lambda\Phi(u_0) + \Phi_1(u_0, u_{0t})$; $\lambda >> 1$ is a large parameter []. Here, on the right-hand side, the first term determines its elastic component, and the second - dissipative. Function F is a hypothesis, generally speaking, of a non-instantaneous impact. When $\lambda \to \infty$ the impact model becomes Newtonian, the force Φ is written in terms of the delta function.

For suspension and interaction points, we have operator equations:

$$u_1^{\mathrm{I,II}} = L_{11}^{\mathrm{I,II}}(i\omega)c_{1,2}(x)u \pm L_{n1}^{\mathrm{I,II}}(i\omega)\Phi\ (u_0, u_{0t}) + h_1^{(I,II)}, \tag{13}$$

$$u_n^{\mathrm{I,II}} = L_{nn}^{(\mathrm{I,II})}(i\omega)c_{1,2}(x)u \pm L_{nn}^{\mathrm{I,II}}(p)\Phi(u_0, u_{0t}) + h_n^{(I,II)}. \tag{14}$$

For system $A^{(\mathrm{I})}$, "plus" is chosen in equations (3) and (4); and for $A^{(\mathrm{II})}$- "minus". In (13) and (14) additional external forces $h_{n,1}^{(I,II)}$ are introduced.

Features of the interaction of the carrier and the attached parts form the structure of the global vibratory field. At the same time, the use of models of this type leads to loss of information about the behavioral features of individual elements of the system, as well as the effects that are manifested only when addressing discrete models.

3.2 Examples

In Fig. 3 the schemes of the models of the type considered above are shown. In Fig.1. *a*. as the carrier part, a longitudinally oscillating rod is chosen, with each point of which two linear stationary subsystems forming the shock element. Fig. 3, *b* shows a discrete model with a two-dimensional carrier medium (string lattice [4, 8]), which, upon continuation, becomes a membrane, and when additional hypotheses are made, into a plate. This figure denotes the possibility of considering non-Newtonian blows mentioned above. Fig. 3, *c* represents a discrete chain of mechanisms. Located on vibrating bases and, finally, on Fig. 3, shows a very specific model of the vibrator with "internal discontinuities". All these models describe the mechanisms for generating broadband vibrational processes that exist in representative spatial areas.

The analysis of such systems is carried out with the help of special modifications of the methods of time-frequency analysis of vibratory impact processes and special modifications of modern numerical methods. The vibration fields established in these systems depend on the manifestation of the two main physical mechanisms. Amortized systems act on the load-bearing part of the structures in much the same way as dynamic shock (or with rare combinations of parameters by linear) absorbers of vibration processes [1, 4, 5]. The carrier part performs filtering and spectral transformations of the transmitted vibration, and also determines the global resonance properties. These mechanisms, are manifested in different points of the bearing structures in different ways. This circumstance makes it possible to manifest many very specific dynamic effects. Let's name the main ones.

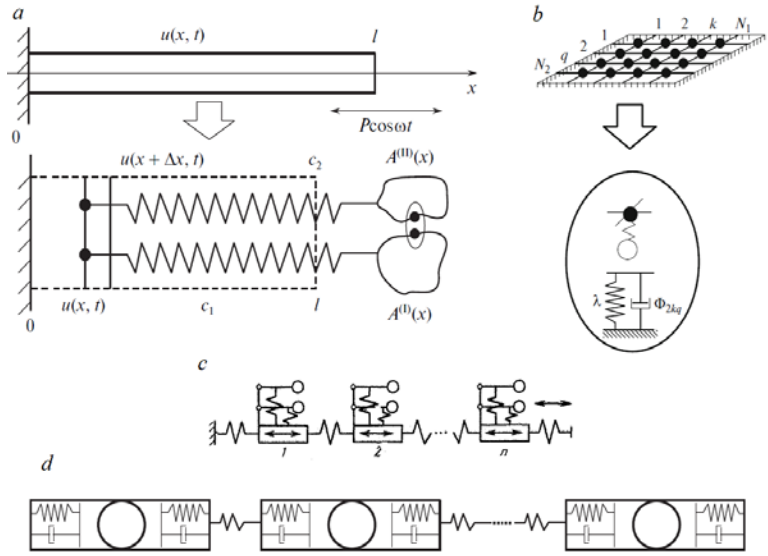

Fig. 3. Dynamic models of objects with complex structure.

1. Occurrence of alternating zones of transparency and opacity to pass through the construction of the fundamental vibration tone.

2. The manifestation of the effect of synchronization of intense resonant mode with collisions in certain spatial regions. During synchronization, motion modes of the various types of "claps" types are realized. Resonance regimes preserve the main properties of nonlinear resonances [1] (phenomena of "tightening" in frequency and amplitude, the effect of "hard start", the phenomenon of "failure", the emergence of multi-valued amplitude-frequency characteristics, and others).

3. The emergence of complex modes of motion: subharmonic, combinational, modes with "bounce", chaotic, rare-shock and others.

This work was supported by the Russian Foundation for Basic Research (project 18-08-00168).

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