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Analysis of oscillations of a mechanical system with inertial exciters at an alternating position of it's mass center

Sergey Eremeykin[1], *Grigory* Panovko[1, 2], and *Alexander* Shokhin[1, *]

[1]Mechanical Engineering Research Institute of Russian Academy of Sciences, Moscow, Russia
[2]Bauman Moscow State Technical University, Moscow, Russia

Abstract. Numerical simulation results of dynamics of a mechanical system with self-synchronizing unbalance exciters with displaced center of mass relative to the initial centered state are described in application to analysis of dynamics of vibrating machines with variable position of operation load. Features of self-synchronization of exciters are revealed depending on position of the center of mass. Influence of position of the center of mass on resonance frequencies and modes is presented.

1 Introduction

Vibration technologies and vibration machines are widely used in various industries, in particular, in mining and processing minerals, processing agricultural products, construction and road works, metallurgy, machine building, etc. [1-4].

Among the various types of vibrating machines, the machines with unbalanced exciters of the working body oscillations are widely used. Depending on arrangement of exciters, various oscillation modes and trajectories of working body are realized. A special place in this class of machines is occupied by machines with self-synchronizing exciters. Extensive scientific literature is devoted to study of dynamics of such machines [3-7].

One of the most important problems in design of effective vibrating technological machines is to provide stable operating at resonant mode when relatively high levels of oscillations of working body are realized with low power consumption. Essentially resonant adjustment allows to reduce power consumption and simultaneously improve both dynamics and operation quality of a machine. However, in practice, resonant adjustment is difficult due to parameters changes in time, primarily operation load value and its location on a working body. In papers [8-10] it was proposed to use automatic control system to maintain resonant mode of oscillations for centered schemes of vibrating machines (when the resultants of the inertia forces, exciting forces and resilient forces are collinear to each other and pass through the system's center of mass). In this paper dynamics of similar centered systems was analyzed in detail, taking into account possible changes in mass of operation load.

[*] Corresponding author: shohinsn@mail.ru

However, when considering real technological machines, it is also necessary to take into account possible displacement of operation load on the working body.

This paper presents results of numerical simulation for one of possible scheme of vibrating machine with variable center of mass.

2 Design scheme

Design scheme is a one-mass mechanical system excited by two self-synchronizing unbalanced exciters driven by AC motors (see Fig. 1).

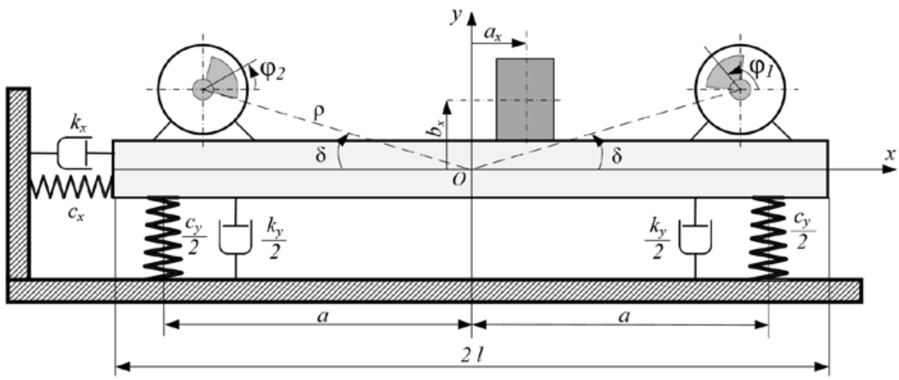

Fig. 1. Design scheme.

The platform, simulating the working body of the machine, is considered as rigid body on viscoelastic supports with linear elastic and damping characteristics. The motion of the system is considered in stationary coordinate system *y0x*, whose origin coincides with static equilibrium position of the platform's center of mass. Two identical AC motors with equal unbalanced masses are placed on the platform symmetrically relative to the vertical axis *Oy*. The rotor axes are parallel to each other and perpendicular to the *y0x* plane. Both motors are connected to three-phase AC mains via a single inverter so that its rotors rotate in opposite directions.

The operation load is simulated by rigid body (weight) fixed on the platform. Position of the weight is described by parameter a_x - distance from central symmetry axis (see Fig. 1).

The principal feature of this design scheme is that it takes into account interaction of the oscillating system with exciters driven by limited power motors. As a result of this interaction, different types of synchronous rotation of exciters are possible depending on power supply frequency. On the one hand, synchronization type is determined by oscillation mode, and on the other hand oscillation mode depends on synchronization type.

Motion of the system is described by displacements of platform's center of mass in directions *Oy* and *Ox*, rotation angle φ of the platform and rotation angles of rotors φ_1 and φ_2. All angular coordinates mentioned here are measured from the *Ox* axis counterclockwise.

Differential equations of motion of the system are:

$$
\begin{aligned}
&M\ddot{x} + k_x \dot{x} + c_x x = m_r r\left[\left(\dot{\varphi}_1^2 \cos\varphi_1 + \ddot{\varphi}_1 \sin\varphi_1\right) + \left(\dot{\varphi}_2^2 \cos\varphi_2 + \ddot{\varphi}_2 \sin\varphi_2\right)\right] + \\
&+2m_r \rho\, \ddot{\varphi} \sin\delta + b_x\, m_b\, \ddot{\varphi}, \\
&M\ddot{y} + k_y \dot{y} + c_y y = m_r r\left[\left(-\dot{\varphi}_1^2 \cos\varphi_1 + \ddot{\varphi}_1 \sin\varphi_1\right) + \left(-\dot{\varphi}_2^2 \cos\varphi_2 + \ddot{\varphi}_2 \sin\varphi_2\right)\right] + a_x\, m_b\, \ddot{\varphi}, \\
&J\ddot{\varphi} + k_\varphi \dot{\varphi} + c_\varphi \varphi = \left(m_b b_x + 2m_r \rho \sin\delta\right)\ddot{x} - m_b a_x \ddot{y} + \qquad (1) \\
&+m_r \rho\, r\left[\dot{\varphi}_1^2 \sin\left(\varphi_1 - \delta\right) - \ddot{\varphi}_1 \cos\left(\varphi_1 - \delta\right) - \dot{\varphi}_2^2 \sin\left(\varphi_2 + \delta\right) + \ddot{\varphi}_2 \cos\left(\varphi_2 + \delta\right)\right], \\
&J_1 \ddot{\varphi}_1 - m_r r\left[\ddot{x} \sin\varphi_1 - \left(\ddot{y} + g\right)\cos\varphi_1 - \ddot{\varphi}\rho\cos\left(\varphi_1 - \delta\right)\right] = \sigma_1\left(M_1 - M_C\right), \\
&J_2 \ddot{\varphi}_2 - m_r r\left[\ddot{x} \sin\varphi_2 - \left(\ddot{y} + g\right)\cos\varphi_2 - \ddot{\varphi}\rho\cos\left(\varphi_2 + \delta\right)\right] = \sigma_2\left(M_2 - M_C\right),
\end{aligned}
$$

here m_r – mass of each debalance, r – corresponding eccentricities, $J_1 = J_2 = J_r + m_r r^2$, J_r - moment of inertia of rotors; $M = m_b + 2m_r$ - full mass of the system; $m_b = m_0 + m_x$ - mass of the platform with the weight; a_x, b_x - horizontal and vertical coordinates of weight's center of mass, c_x, c_y, c_φ, k_x, k_y, k_φ - stiffness and damping coefficients for viscoelastic supports in horizontal, vertical and angular directions respectively; ρ - distance from the platform's center of mass to the axes of rotors; δ - angles between *x* axis and the axis, which pass through platform's center of mass and axis of rotor in plane *xOy*, $J = J_0 + 2\,m_r\,\rho^2 + \left(a_x^2 + b_x^2\right)m_b$, J_0 - moment of inertia of the platform; g - gravitational acceleration; $\sigma_1 = +1$, $\sigma_2 = -1$ - constants that define the direction of rotors' rotation; M_C - moment of resistance for the rotors.

Torques M_1, M_2 in right-hand side of Eq. 1 could be described by static characteristic of motors. The characteristics are obtained using simplified Kloss formula:

$$
M_1 = M_1\left(s_1\right) = \frac{2\,M_{cr1}}{s_1/s_{cr1} + s_{cr1}/s_1},\; M_2 = M_2\left(s_2\right) = \frac{2\,M_{cr2}}{s_2/s_{cr2} + s_{cr2}/s_2}, \qquad (2)
$$

here M_{cr1}, M_{cr2} - critical (maximum) torques for each motor, s_{cr1}, s_{cr2} - slip at critical torque, $s_1 = 1 - P\left|\dot{\varphi}_1/f\right|$, $s_2 = 1 - P\left|\dot{\varphi}_2/f\right|$ - current slip determined by frequency *f* and angular velocity of rotors $\dot{\varphi}_1, \dot{\varphi}_2$, $P = 2$ is number of poles pairs of motors.

3 Simulation results

Equations (1)-(2) have been solved numerically for these parameters values $m_r = 0.0666$ kg, $r = 0.005$ m, $J_1 = J_2 = 1.665 \cdot 10^{-6}$ $\mathrm{kg \cdot m^2}$, $m_0 = 12.02$ kg, $m_x = 4$ kg, $c_x = 624000$ N/m, $c_y = 286\,704$ N/m, $c_\varphi = 1750.52$ N·m, $k_x = 20$ N·s/m, $k_y = 18.75$ N·s/m, $k_\varphi = 0.2\,\mathrm{N \cdot s^2/m}$, $\rho = 0.128$ m; $\delta = 23°$, $J_0 = 0.02847$; $M_C = 0.02$ N·m, $M_{cr1} = 2.64$ N·m, $M_{cr2} = 1.01 M_{cr1}$, $s_{cr1} = s_{cr2} = 41.6\%$. The solution was obtained with stepwise increase in power supply frequency (0.5 Hz increase on each step). Thus transition to each next step was made only if oscillations are stable on the previous step. Simulation was carried out independently for each position of the weight a_x in the range $[0; l]$ with increment $l/10$. Fig. 2 shows frequency response for vertical (Fig. 2,a) and angular (Fig. 2, b) components of oscillations at three different positions of the weight $a_x = \{0, \ l/2, \ l\}$ for the point corresponding to initial centered position of platform's center of mass. For the horizontal component of oscillations, there are no significant changes in frequency responses with displacement of mass center, therefore, corresponding plots are not given here.

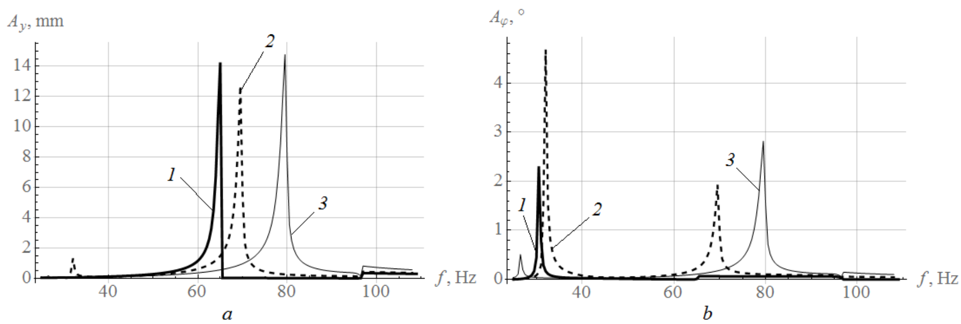

Fig. 2. Frequency responses: 1 - $a_x = 0$, 2 - $a_x = 0,5l$, 3 - $a_x = l$.

Analysis of the obtained results shows that displacement of mass center leads to significant change in resonant frequencies and oscillation modes. For the symmetric system, first resonant frequency corresponds to simultaneous excitation of angular and horizontal (in the direction of *Ox* axis) oscillations. In this case, vertical oscillations aren't excited. At the second resonant frequency only vertical vibrations are excited. If the center of mass is shifted, at the first resonance frequency oscillations in all coordinate directions are excited, and at the second resonance frequency vertical oscillations combined with angular oscillations are excited.

The computational results for resonant frequencies depending on position of the weight's mass center are shown in Fig. 3. Note that change in second resonance frequency is more significant.

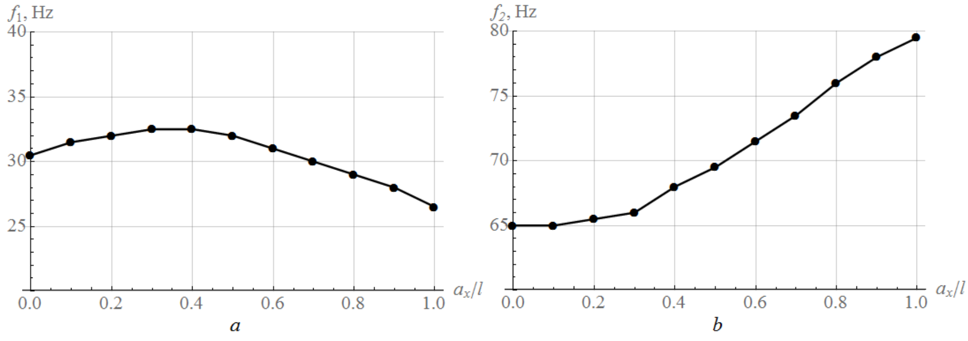

Fig. 3. First (*a*) and second (*b*) resonant frequencies depending on the weight location.

Synchronization type was estimated by relative shift of rotors' phases $\Delta\varphi = \varphi_1 - \varphi_2$ depending on power supply frequency. Computational results for the phase shift $\Delta\varphi$ are shown in Fig. 4 for three positions of weight's center of mass a_x.

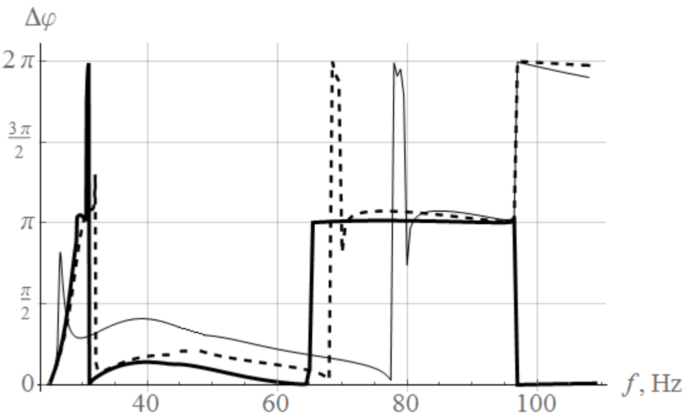

Fig. 4. Relative phase shift depending on supply frequency: 1 - $a_x = 0$, 2 - $a_x = 0,5l$, 3 - $a_x = l$.

One can see that debalances have two main types of synchronization. At frequencies up to the first resonance, there is slow increase in the phase shift up to π that corresponds to antiphase rotation. With the increase of power supply frequency from the first to the second resonance frequency phase shift approaches to zero, that corresponds to in-phase rotation of debalances. Further increase in supply frequency (above the second resonance) leads to stabilization of phase shift near $\Delta\varphi = \pi$. These synchronization types in the specified frequency ranges don't change when the center of mass is shifted. The transition through resonance is accompanied by sharp changes in phase shift (and synchronization type).

4 Conclusions

In the result of the study, impact of the mass center displacement on resonant frequencies and oscillation modes has been described. It is shown that displacement of the mass center leads to the disappearance of a pure vertical oscillations form presented in centered system with synchronous rotation of rotors. Synchronization types practically do not change when position of center of mass changes, and are determined only by resonant frequencies of the system.

Obtained results can be used in development of control systems for resonant vibrating machines.

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