Estimation of certain parameters of Black-Scholes model in analysing effectiveness of development investments

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Abstract. The option pricing theory has wide applicability in corporate finance, but it is also increasingly used to analyze the effectiveness of non-financial (material) investments. In traditional investment analysis, a project or a new investment should be accepted only if the returns on the project exceed the hurdle rate; in the context of cash flows and discount rates, this translates into projects with positive net present values (NPV). There is no doubt that it does not take full account of the numerous options that usually relate to developer investment. However, in many cases, the valuation of real options is more difficult than the valuation of options for financial assets. In this paper, we will analyze one of the options, which isembedded in capital budgeting projects - the option to delay a project, especially when a the company has exclusive rights to the project. The value of the option is largely derived from the variance in cash flows – the higher the variance, the higher the value of the project delay option. The variance in the present value of cash flows from the project can be estimated in different ways, however, in the case of non-financial investment projects, these methods are very limited. We are analyzing the possibility of estimating this volatility, taking into account the fact that the forecasted cash flows may show varying volatility in individual years. The paper shows, that by using a probability-based valuation model (using the Crystal Ball techniques) it is possible to incorporate uncertainty into the analysis. The method of presented volatility estimation can be applied by taking into account the randomness of many input data to the project.

1 Introduction

Projects are typically analyzed based upon their expected cash flows and discount rates at the time of the analysis; the net present value calculated on that basis is a measure of its value and acceptability at that time. Assuming that a project requires an initial up-front investment of X, and that the present value of expected cash inflows calculated right now is V, the net present value NPV of this project is the difference between the two:

$$NPV = V - X \tag{1}$$

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The company's decision rule on this project can be summarized as follows:

- if V > X → take the project: Project has positive net present value,
- if V < X → do not take the project: Project has negative net present value.

If the expected cash flows on the project were known with certainty and were not expected to change, there would be no need to adopt an option pricing framework, since there would be no value to the option. Expected cash flows and discount rates change over time and so does the net present value. Thus, a project that has a negative net present value now may have a positive net present value in the future (the project may still be a good project if the company can wait. waits).

The changes in the project's value over time (the present value of the cash inflows may change over that time, because of changes in either the cash flows or the discount rate) give it the characteristics of a call option.

In the article, we consider the situation that the company has exclusive rights to a particular project for the next one year, so we are considering the applying the option pricing theory to valuing the option to delay. The uncertainty of expected cash flows on the project should be viewed as the reason for why the project delay option has value.

The inputs needed to valuing this option are the same as those needed for any option; we need the value of the underlying asset, the variance in that value, the time to expiration on the option, the strike price, the riskless rate and the equivalent of the dividend yield (cost of delay). In the case of real options, the underlying asset is the project itself. The current value of this asset is the present value of expected cash flows from initiating the project now.

Assuming that the cost of delay is zero, the value of a call option for a non-dividend-paying underlying stock in terms of the Black-Scholes parameters is:

$$C = V \cdot N(d_1) - X \cdot e^{-rt} \cdot N(d_2) \tag{2}$$

where:

C – Call Value,
V – current value of the underlying asset (present value of expected cash inflows from Project),
X – strike price of the option (an initial up-front investment)
t – life to expiration of the option (period of exclusive rights to Project),
σ – variance (volatility) in the value of the underlying asset (the variance/volatility in the present value of cash flows from Project),
r – riskless interest rate corresponding to the life of the option,
N(d1) – cumulative distribution of the standard normal distribution for the argument d1,
N(d2) – cumulative distribution of the standard normal distribution for the argument d2;

$$d_1 = \frac{\ln\left(\frac{V}{X}\right) + t \cdot \left(r + \frac{\sigma^2}{2}\right)}{\sigma \cdot \sqrt{t}} \tag{3}$$

$$d_2 = d_1 - \sigma \cdot \sqrt{t} \tag{4}$$

Calculating the value of the call option, and then determining the expanded value of NPV, i.e. ENPV, allows the final conclusion about the effectiveness of a given project [2].

$$ENPV = NPV + C \tag{5}$$

where:
ENPV – expanded NPV,
C – call option.

2 The variance in the present value of cash flows from the project and the possibility of estimating it

The value of the option is largely derived from the variance in cash flows – the higher the variance, the higher the value of the project delay option. Thus, the value of an option to do a project in a stable business will be less than the value of one in an environment where technology, competition and markets are all changing rapidly.

Assuming (for the analyzed case study) that:
Project will generate 100 000 EUR in after tax cash flows for the next 5 years, discount rate is 10% (r=10%), we obtain:

$$V = \sum_{i=1}^{5} \frac{CF_i}{(1+r)^i} = \sum_{i=1}^{5} \frac{100}{(1+0,1)^i} = 379,1 \tag{6}$$

Continue to assume, that an initial up-front investment would be equal to the present value of expected cash inflows from Project (X=379,1), NPV of this Project would be zero (NPV=0).

Calculating the exclusive rights to this Project using the Black-Scholes formula for different variance parameters (time to expiration is 1 year, riskless rate is 3,5%) we obtain the results shown in the figure below.

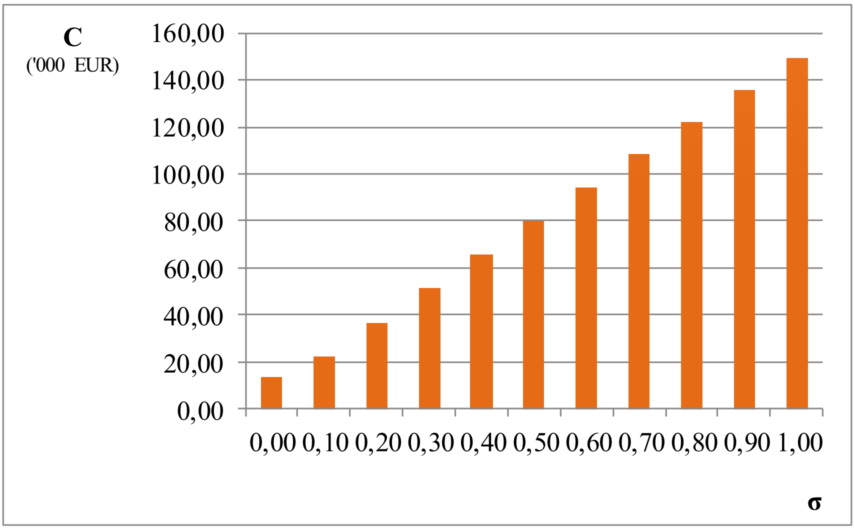


Fig. 1. The value of the call option (the option to Delay a Project), depending on variance in the present value of cash flows from Project

The above chart confirms the hypothesis that the value of the Call option depends very much on the volatility in the underlying project, which confirms that the volatility coefficient estimate is a very important element of the model.

The variance in the present value of cash flows from the project can be generally estimated in one of three ways [1]:

- if similar projects have been introduced in the past, the variance in the cash flows from those projects can be used as an estimate,
- probabilities can be assigned to various market scenarios, cash flows estimated under each scenario and the variance estimated across present values,
- the variance in company value of companies involved in the same business (as the project being considered) can be used as an estimate of the variance.

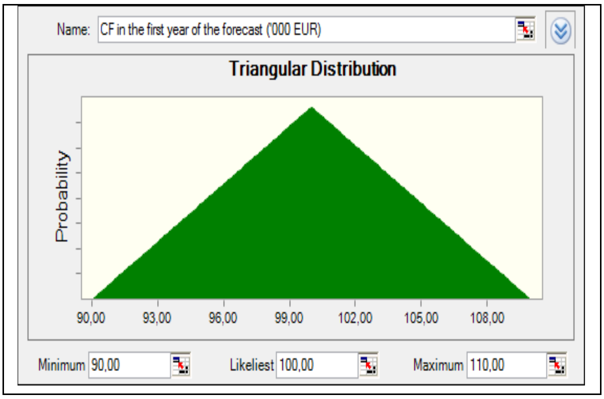
In the case of individual development investments, however, the above methods are useful to a very limited extent.

3 Case study

In our analysis, the uncertainty has been incorporated into the explicit model of NPV. This is done by recognizing that the input variables (in our case, projected cash flow and discount rate) are uncertain and will have a probability distribution pertaining to each of them. Thus by utilizing a probability-based valuation model (using the Crystal Ball technique) it is possible to incorporate uncertainty into the analysis.

The Crystal Ball technique allows incorporating uncertainty into the analysis in a relatively simple form. It is a simulation model (using the Monte Carlo technique) that, instead of taking one defined set of input figures and producing a single point answer (value), carries out multiple calculations through an iterative re-sampling process. Each simulation chooses an input variable from within the probability distribution chosen for each variable and marries these with other randomly chosen inputs to produce a value. The output is expressed as a range of possible values.

In this valuation triangular probability distributions indicated in the literature as appropriate to account for uncertainty in valuation processes [3,4] were used. The manner in which the individual random variables are recognized is indicated below.



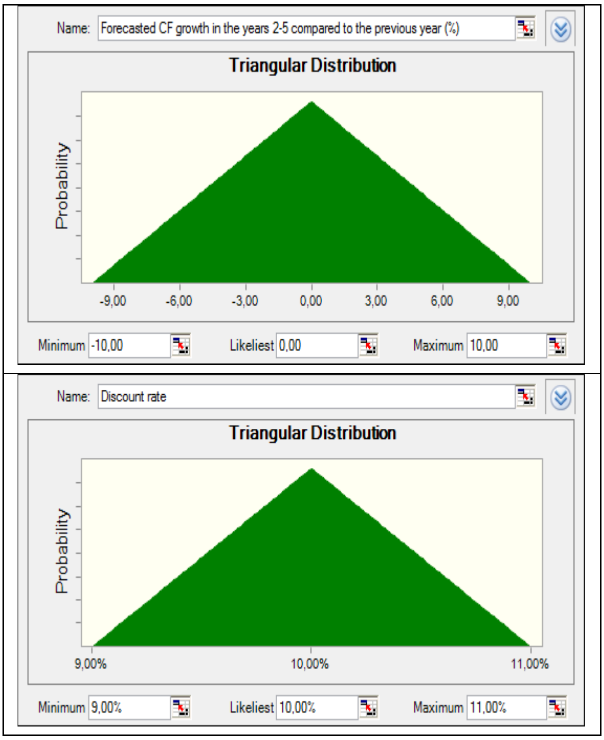


Fig. 2. The form of individual random variables

The results of using the Crystal Ball program were illustrated by the graph below.

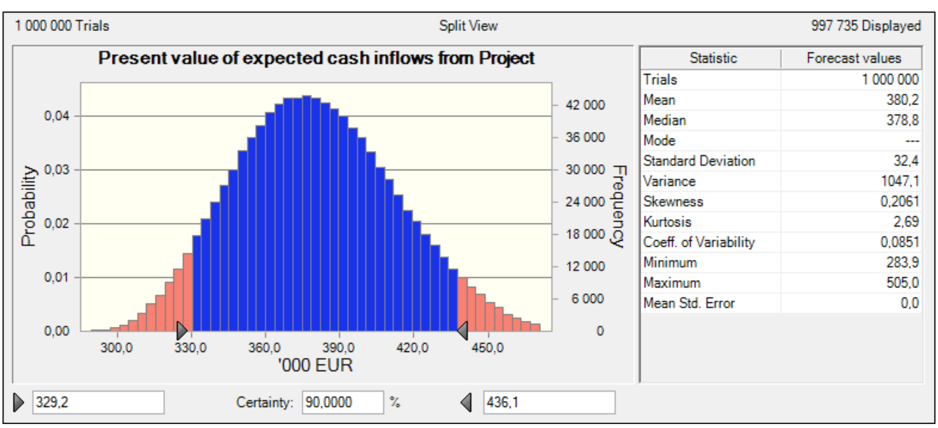


Fig. 3. Simulation results

Using the coefficient of variability given above (s=0.0851, which means variance σ=0,0072), the value of the call option (the option to Delay a Project), calculated based on the formulas (2), (3) and (4), is 20,2 thousand EUR.

In this case it meant that also the expanded NPV of the investment is:

ENPV = NPV+C = 20,2 thousand EUR

In many cases, this is the only way to objectivization of the process of analyzing the effectiveness of development investments using real options.

4 Conclusion

The option pricing theory is widely used in corporate finance, but is also increasingly used to analyze the effectiveness of non-financial investments, including property development investments.

The value of the Call option depends very much on the volatility in the underlying project; in many cases this refers to the variability of the cash flows from the project. Estimation of the coefficient of variation is therefore an important element of the calculation model.

The variance in the present value of cash flows from the project can be estimated in different ways, however, in the case of non-financial investment projects, these methods are very limited.

The simulation process is a universal method of estimation of the coefficient of variation, which is shown in the analyzed example.

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