Robust Adaptive Nonlinear Dynamic Inversion Control for an Air-breathing Hypersonic Vehicle

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Abstract. This paper presents a robust adaptive nonlinear dynamic inversion control approach for the longitudinal dynamics of an air-breathing hypersonic vehicle. The proposed approach adopts a fast adaptation law using high-gain learning rate, while a low-pass filter is synthesized with the modified adaptive scheme to filter out the high-frequency content of the estimates. This modified high-gain adaptive scheme achieves a good transient process and a nice robust property with respect to parameter uncertainties, without exciting high-frequency oscillations. Based on input-output linearization, the nonlinear hypersonic dynamics are transformed into equivalent linear systems. Therefore, the pole placement technique is applied to design the baseline nonlinear dynamic inversion controller. Finally, the simulation results of the modified adaptive nonlinear dynamic inversion control law demonstrate the proposed control approach provides robust tracking of reference trajectories.

1 Introduction

Since the winged-cone hypersonic vehicle model [1] was presented by NASA Langley Research Center, the air-breathing hypersonic vehicles (AHVs) have attracted much interest around the world. What’s more, the successful flight of X-43A [2], X-51A, and Hyshot2 [3] has given a great number of test data on the aerodynamics, structure, guidance, and control of AHVs, enriching the knowledge of hypersonic theory. However, the highly coupled dynamics make the AHVs sensitive to flight condition [4]. As a result, a robust control system is important to the AHVs.

The feedback linearization method has been widely used in AHVs. Wang and Stengel [5] gave the condition of applying nonlinear dynamic inversion (NDI) for a generic AHV. In their work, the nonlinear vehicle systems were transformed into equivalent linear systems. Afterward, the genetic algorithm was applied to optimize the design parameters of the LQR control. But the feedback linearization requires an accurate model because it’s sensitive to uncertainties. To enhance the robustness of the NDI control, Xu et al. [6] adopted sliding mode control to design a NDI controller for the AHV. In parker [7], the elevator-to-lift coupling was canceled by adding an additional canard. What’s more, adaptive technique is synthesized with the feedback linearization method. Chen and Ai [8] proposed a NDI based L1 adaptive control design for the AHV. The L1 adaptive term was used to estimate the parametric uncertainties and external disturbances. Fiorentini et al. [9] applied the canard deflection to control the outer-loop and utilized the elevator deflection to control the inner-loop, resulting in low-order subsystems. Based on these low-order subsystems, a robust adaptive dynamic inversion approach was implemented to achieve robust tracking performance.

In this study, the canard included configuration is adopted to create low-order subsystems. Based on NID, the nonlinear systems are transformed into low-order linear subsystems. We applied the pole placement method [10] to design the control parameters of the equivalent linear systems. To enhance the stability of the control system, a modified adaptive scheme [11] was applied to estimate the parametric uncertainty. In the adaptive term, a low-pass filter was introduced to the estimate parameters. As a result, the high frequency content of the adaptive process is canceled. In the end, Monet Carlo simulation was conducted to demonstrate the robustness of the proposed method.

The main contributions of this study are: (a) with an additional canard, the relative degree of the system is well defined without dynamic extension; (b) the pole placement technique is synthesized with the NDI to design the feedback gains of the equivalent linear system; (c) the modified adaptive scheme is adopted to improve the performance of the AHV. The remainder of this paper is organized as follows: In Sec. 2, a nonlinear model of the AHV is presented and the control-oriented equations are obtained. The control design of the NDI is proposed in Sec. 3. Finally, simulation results and conclusions are presented in Sec.4 and Sec. 5, respectively.
\[ T = \pi z \left[ C_{\gamma \phi} (\alpha) \phi + C_{1} (\alpha) \right] \]
\[ L = \pi S C_{1} (\alpha) \delta \]
\[ D = \pi S C_{2} (\alpha) \]
\[ M = z_{f} T + \pi S C_{m} (\alpha) \delta \]

\[ \delta = \left[ \delta_{\phi}, \delta_{\phi} \right]^T \]
\[ C_{\gamma \phi} (\alpha) = n C_{\gamma \phi}^{\text{SW}} \alpha^2 + v C_{\gamma \phi}^{\text{SW}} \alpha^3 + v C_{\gamma \phi}^{\text{SW}} \alpha + v C_{\gamma \phi}^{\text{SW}} \]
\[ C_{1} (\alpha) = n C_{1} \alpha^3 + v C_{1} \alpha + v C_{1} \alpha + v C_{1} \]
\[ C_{2} (\alpha) = n C_{2} \alpha^2 + v C_{2} \alpha + v C_{2} \alpha + v C_{2} \]
\[ C_{m} (\alpha) = n C_{m} \alpha^2 + v C_{m} \alpha + v C_{m} \alpha + v C_{m} \]
\[ m = \nu \cdot m \]
\[ I_{yy} = \nu \cdot I_{yy} \]

\[ \dot{V} = (T \alpha - D) \gamma^2 - g \gamma \]
\[ \dot{h} = V \gamma \]
\[ \dot{\gamma} = (L + T \phi) (mV) - g \gamma \gamma \]
\[ \dot{\theta} = q \]
\[ \dot{q} = M \dot{\gamma} \]

\[ V = \theta^2 \phi + \theta \phi \phi - g \gamma \]
\[ \dot{h} = V \gamma \]
\[ \dot{\theta} = \theta^2 \phi + \theta \phi \phi \]

\[ \theta = (\pi \nu) \left[ \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu \right]^T \]
\[ \phi = (\pi \nu \ m) \left[ \alpha, \alpha, \alpha C_{\gamma} \alpha, \alpha C_{1} \alpha, \alpha C_{2} \alpha, \alpha C_{m} \alpha \right] \]
\[ \psi = (\pi \nu \ m) \left[ \alpha C_{\gamma} \alpha, \alpha C_{1} \alpha, \alpha C_{2} \alpha, \alpha C_{m} \alpha \right] \]
\[ \phi = (\pi \nu \ m) \left[ \alpha C_{\gamma} \alpha, \alpha C_{1} \alpha, \alpha C_{2} \alpha, \alpha C_{m} \alpha \right] \]
\[ \theta = (\pi \nu \ m) \left[ \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu \right]^T \]
\[ \phi = (\pi \nu \ m) \left[ \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu \right]^T \]

\[ \theta = (\pi \nu \ m) \left[ \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu \right]^T \]
\[ \phi = (\pi \nu \ m) \left[ \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu \right]^T \]

\[ \theta = (\pi \nu \ m) \left[ \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu \right]^T \]
\[ \phi = (\pi \nu \ m) \left[ \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu \right]^T \]

\[ \theta = (\pi \nu \ m) \left[ \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu \right]^T \]
\[ \phi = (\pi \nu \ m) \left[ \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu \right]^T \]

\[ \theta = (\pi \nu \ m) \left[ \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu \right]^T \]
\[ \phi = (\pi \nu \ m) \left[ \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu, \nu \right]^T \]
\[
\begin{align*}
\theta_c &= (\mathbf{v}_s^T \mathbf{v}_w)\mathbf{v}_w^T \\
\phi_c &= (\mathbf{b} \mathbf{s}^- \mathbf{J}_{y_{\mathbf{x}}}) \text{diag}(\mathbf{C}^T_{\mathbf{M}} \mathbf{C}^T_{\mathbf{M}}) \\
\end{align*}
\]

3. Control design

3.1 NDI control for the velocity subsystem

\[
\dot{V} = V - V_{\text{ref}}
\]

\[
\dot{V} = \theta_c \phi_c + \theta_c \phi_c \phi_c - g \implies \dot{V} = \theta_c \phi_c
\]

\[
\phi = -k_p \dot{V} - \sigma \dot{V}_{\text{ref}}
\]

\[
\hat{\theta}_c = \Gamma_c (\dot{V} \phi_c - \sigma (\dot{V} - \dot{V}_c))
\]

\[
\hat{\theta}_s = \Gamma_s (\dot{V} \phi_c - \sigma (\dot{V} - \dot{V}_c))
\]

\[
\begin{align*}
\dot{\hat{\theta}}_s &= \sigma (\dot{V} - \dot{V}_c) \\
\dot{\hat{\theta}}_s &= \sigma (\dot{V} - \dot{V}_c)
\end{align*}
\]

\[
\begin{align*}
\dot{\hat{\theta}}_c &= \Gamma_c (\dot{V} - \dot{V}_c) \\
\dot{\hat{\theta}}_c &= \Gamma_c (\dot{V} - \dot{V}_c)
\end{align*}
\]

\[
\dot{V} = \hat{V} + \theta_c \phi_c + \theta_c \phi_c \phi_c - g \implies \dot{V} = \theta_c \phi_c
\]

\[
\begin{align*}
W_D &= \dot{V} + \sigma \dot{V} - \sigma \dot{V}_c \\
W_D &= \dot{V} + \sigma \dot{V} - \sigma \dot{V}_c
\end{align*}
\]

3.2 NDI control for the altitude and pitch subsystems

\[
\hat{h} = h - h_{\text{ref}} \implies \dot{\hat{h}} = V \implies V = \hat{h} - \hat{h}_{\text{ref}}
\]

\[
\dot{\hat{h}} = \theta_c \phi_c + \theta_c \phi_c \phi_c - g - \hat{h}_{\text{ref}}
\]

\[
\begin{align*}
u_b &= \theta_c \phi_c \phi_c - g + \hat{h}_{\text{ref}} \\
u_b &= \theta_c \phi_c \phi_c - g + \hat{h}_{\text{ref}}
\end{align*}
\]

\[
\begin{align*}
\dot{\hat{h}} &= \dot{h} + \sigma (\dot{h} - \dot{h}_c) \\
\dot{\hat{h}} &= \dot{h} + \sigma (\dot{h} - \dot{h}_c)
\end{align*}
\]

\[
\sigma (\dot{h} - \dot{h}_c) \leq \dot{i} = \sigma (\dot{h} - \dot{h}_c)
\]

\[
\begin{align*}
\hat{h} &= \gamma_{\phi_c} h - \gamma_{\phi_c} \hat{h} \\
\hat{h} &= \gamma_{\phi_c} h - \gamma_{\phi_c} \hat{h}
\end{align*}
\]

\[
\begin{align*}
\dot{\hat{h}} &= \Gamma_{\phi_c} (\dot{h} - \dot{h}_c) \\
\dot{\hat{h}} &= \Gamma_{\phi_c} (\dot{h} - \dot{h}_c)
\end{align*}
\]
\[
\dot{\theta} = \Gamma \cdot \left( \phi \cdot e^T_P \cdot P_b - \sigma \cdot \left( \dot{\theta} - \dot{\theta}_f \right) \right)
\]

\[
\dot{\theta}_f = \Gamma \cdot \left( \dot{\theta} - \dot{\theta}_f \right)
\]

\[
W = e^T_P \cdot e = \dot{\theta}^T \cdot \Gamma \cdot \dot{\theta} + \dot{\theta}^T \cdot \Gamma \cdot \dot{\theta} - \sigma \cdot \dot{\theta}^T \cdot \Gamma \cdot \dot{\theta} - \sigma \cdot \dot{\theta}^T \cdot \Gamma \cdot \dot{\theta}
\]

\[
\dot{W} \leq -e^T_P \cdot e \cdot -\sigma \cdot \left( \dot{\theta} - \dot{\theta}_f \right)^T \left( \dot{\theta} - \dot{\theta}_f \right)
\]

\[
G(x) = \left[ \dot{\theta}^T \cdot \phi \cdot \dot{\theta}^T \cdot \phi \right]^T
\]

Figure 1.
\[ G(x) = V \frac{\gamma q}{S \tau} \]

\[ (v, v_c, C_b^p \tau_c^2 - v_c, v_c, C_b^p \tau_c^2) = u \]

\[ \delta = G^{-1} [u_h, u_{\theta}]^T \]

3.3 Pole placement technique

\[ k_{p} = \omega_n^2, k_{d} = -\zeta_{\theta} \omega_n \]

| \( v \) | 0 | 0.2 |
| \( h \) | 20 | 40 |
| \( \gamma \) | 0.5 | 0.7 |
| \( \alpha \) | 0.02 | 0.04 |
| \( Q \) | 0.05 | 0.1 |
| \( \phi \) | 0.01 | 0.02 |

Table 3.

4. Simulation analysis
5. Conclusion

In this paper, we have designed a robust adaptive NDI control law for an AHV. The pole placement technique is applied to design the feedback gains of the equivalent linear system, which is convenient to set the gains for the clear physical meanings. The modified adaptive scheme applied a low-pass filter to cut off the high-frequency oscillations arising from the high-gain learning rate. The high gain learning rate of the modified adaptive scheme accelerates the convergence rate and is robust to parametric uncertainties. Simulation results show the effectiveness of the modified adaptive NDI control law, which provides a good tracking performance and a nice robust property for the AHV. The adaptive gains are chosen by trial and error. In the future work, we will study how to select the adaptive gains properly.

References


