

Comparative Analysis of Mathematical Models for Turbofan Engine Weight Estimation

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Abstract. Accurate mathematical models for turbofan engine weight estimation are necessary requirement for optimization of the working process parameters at the initial design stage. Open-access publications provide necessary information on eight models that may be used at this design stage. Information on 77 modern turbofan engines was gathered using the available sources: publications, official websites, reference books etc. Data gaps were filled using the mathematical model identification. Gathered data cover wide range of working process parameters, thrust levels and air flow rates and was used to assess the accuracy of the abovementioned weight models. Only four models (Torenbeek, Svoboda, Raymer, Kuz'michev) provide adequate accuracy. Kuz'michev model uses the highest number of input parameters and provide the most precise results, although it must be noted that no correlation between the number of input parameters and accuracy was determined in general.

1 Introduction

Assessment of turbofan weight is of exceptional importance at all design stages. For example, correct weight and fuel efficiency estimation at the initial design stage is required to optimize the engine parameters, select its design architecture and features. The initial design stage that covers all design elements up to the detailed design and engineering of engine elements is characterized by inherent uncertainty of initial design data, limiting the use of mathematical models and providing the high level of inaccuracy. As the project evolve and the information get more detailed, the models for weight estimation become more sophisticated and precise.

Correlation-regression models for weight prediction, developed using the statistical information on existing engines are usually used at the conceptual design stage.

The main goal of this work is to provide the recommendations for selection of weight models at the initial design stage on the basis of comparative analysis.

2 Weight models

Weight models of the following authors were analyzed: Torenbeek E. (Delft University of Technology, Nederland) [1], Raymer D. P. (Conceptual Research Corporation, USA) [2], Jenkinson L. R. (Loughborough University, UK) [3], Svoboda C. (The University of Kansas, USA) [4], Clavier J. (Cranfield University, UK and France) [5], Guha A. (Indian Institute of Technology, India) [6], Byerley A. R. (USAF Academy, USA) [7] and

the model of Samara National Research University, developed under the guidance of V. Kuz'michev.

For the following models information on how many engines were used for their development if available: Svoboda – 70 turbofans, Guha – 30 turbofans, Byerley – 7 turbofans, Kuz'michev – 120 gas turbine engines.

Abovementioned mathematical models comprise of the following equations.

1) Torenbeek model ($W_{\text{eng}} = f(m, \pi_{\Sigma}, G_a, P_{t-o})$) – developed in 2008:

$$W_{\text{eng}} = \frac{10 \cdot \pi_{\Sigma}^{0,25} \cdot G_a}{1 + m} + 12,24 \cdot P_{t-o} \left(1 - \frac{1}{\sqrt{1 + 0,75m}} \right). \quad (1)$$

The model may be used for $m < 8$.

2) Guha model ($W_{\text{eng}} = f(D_f)$) – developed in 2012:

$$W_{\text{eng}} = 1,203 \cdot (110452 \cdot D_f^3 - 30690 \cdot D_f^2)^{0,5833}. \quad (2)$$

3) Svoboda model ($W_{\text{eng}} = f(P_{t-o})$) – developed in 2000:

$$W_{\text{eng}} = 113,398 + 17,844 \cdot P_{t-o}. \quad (3)$$

The model may be used for $m > 2$.

4) Raymer model ($W_{\text{eng}} = f(m, P_{t-o})$) – developed in 1989:

$$W_{\text{eng}} = 14,7 \cdot P_{t-o}^{1,1} \cdot e^{-0,045 \cdot m}. \quad (4)$$

The model may be used for $m < 6$.

5) Jenkinson model ($W_{eng} = f(m, P_{t-o})$) – developed in 1999:

$$W_{eng} = (8,7 + 1,14 \cdot m) P_{t-o}. \quad (5)$$

The model may be used for $5 < m < 14$.

6) Clavier model ($W_{eng} = f(m, \pi_{\Sigma}, G_a)$) – developed in 2008:

$$X = 10^{-6} \cdot \pi_{\Sigma}^2 \cdot m \cdot G_a, \quad (6)$$

$$W_{eng} = -19,821 \cdot X^2 + 720,325 \cdot X + 1524,945, \quad (7)$$

for $X < 5$,

$$W_{eng} = -49,219 \cdot X^2 + 864,891 \cdot X + 1543,161, \quad (8)$$

for $5 < X < 7$,

$$W_{eng} = -5,009 \cdot X^2 + 287,787 \cdot X + 3418,538, \quad (9)$$

for $X > 7$.

7) Byerley model ($W_{eng} = f(m, \pi_{\Sigma}, D_F)$) – developed in 2013:

$$W_{eng} = 37,256 \cdot \pi_{\Sigma} \cdot D_F^2 + 122,45, \quad (10)$$

for $m < 2$ with mixing chamber,

$$W_{eng} = 14,059 \cdot \pi_{\Sigma} \cdot D_F^2 + 1138,32, \quad (11)$$

for $m > 2$ without mixing chamber.

8) Kuz'michev model ($W_{eng} = f(m, \pi_{C\Sigma}^*, G_a, T_4^*, \pi_F^*)$).

Turbofan mass is calculated as follows:

$$W_{eng} = (W_I + W_{II} + W_{m.ch} + W_{ab}) k_e k_{lf}, \quad (12)$$

where: $W_I = B(G_{a1sp,F})^{k_1} \left[\left(\frac{\pi_{C\Sigma}^*}{\pi_F^*} \right)^{0,286} - 1 \right]^{k_2} k_{T_4^*}$ – weight of the engine core;

$$G_{a1sp,F} = G_{a1} \frac{1}{\pi_F} \sqrt{1 + (\pi_F^{0,286} - 1) \frac{1}{\eta_F}} \quad \text{– primary air}$$

mass flow rate at take-off, reduced by the parameters at the fan exit;

$W_{II} = 2,865 \cdot G_{a\Sigma t-o}^{0,903} m^{0,104} \pi_F^{1,193}$ – weight of the fan, fan turbine and bypass ducting;

$W_{m.ch} = 2,316 \cdot G_{a\Sigma t-o}^{0,753}$ – weight of the mixing chamber (if present);

$W_{ab} = 2,9 \cdot G_{a\Sigma t-o}$ – weight of the afterburner (if present);

k_e – coefficient of engine sophistication impact (changes over the years) (Fig. 1);

k_{lf} – coefficient of engine life impact:

$$k_{lf} \begin{cases} 1,0 \dots 1,07 \text{ ? for subsonic airliners;} \\ 1,0 \text{ ? for military long-range aircraft;} \\ 0,9 \text{ ? for fighters.} \end{cases}$$

$k_{T_4^*}$ – coefficient of turbine cooling system impact:

$$k_{T_4^*} = 1 + 2 \cdot 10^{-4} (T_{4 \max}^* - 1200).$$

Values of B , k_1 , k_2 were obtained statistically and are shown in Table 1.

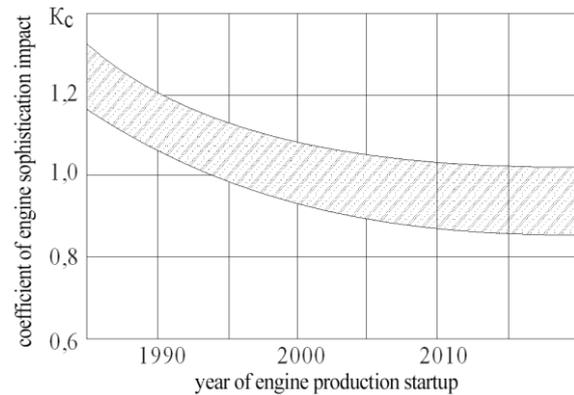


Figure 1. Coefficient of engine sophistication impact against the year of engine production startup.

3 Collecting and processing the engines' data

A databank of information on 77 turbofan engines' parameters was collected to analyze the models accuracy. This bank included engines for both military and civil aircraft set into the production after 1992, with a wide range of working process parameters, thrust levels and air flow rates (Table II). The year of production was chosen so that the engines sample was large enough and the engines were modern.

The following sources of information were used:

- 1) official websites of the engine-developing companies (Rolls-Royce plc, General Electric, Pratt & Whitney, UEC-Saturn and others);
- 2) reference books [8, 9, 10, 11];
- 3) publications [4, 12];
- 4) unofficial internet sources.

The information on engine parameters published in open access is usually not consistent, and in the most cases include just a several values of working process parameters, thrust and SFC. In some cases the flow parameters correspond to poorly identified characteristic sections (it is impossible to determine if the temperature at the turbine inlet is given at the turbine nozzle entrance or at the nozzle throat), some sources provide a mixture of parameters corresponding to various modes of operation etc.

Table 1. Values of coefficients for engine weight estimation.

Type of gas turbine engine	$0,5 < G_{a\text{IspF}} < 5 \text{ kg/s}$			$5 < G_{a\text{IspF}} < 50 \text{ kg/s}$			$G_{a\text{IspF}} > 50 \text{ kg/s}$		
	B	k_1	k_2	B	k_1	k_2	B	k_1	k_2
Turbojet, turbofan $\pi_{c\Sigma t-o}^* > 5$	20,9	0,8	0,5	15,2	1	0,5	6,96	1,2	0,5
Turbojet, turbofan $\pi_{c\Sigma t-o}^* < 5$	16,0	0,8	0	11,6	1	0	5,32	1,2	0

Table 2. Investigated limits of working process parameters.

Parameter	$G_{a\text{t-o}}$, kg/s	P_{t-o} , kN	$\pi_{c\Sigma}^*$	T_4^* , K	m	W_{eng} , kg	D_f , m	π_F^*	Year
min	20	8,45	9,8	1291	0,16	180	0,452	1,44	1992
max	1436	406	50	2273	11	7893	3,124	7	2016

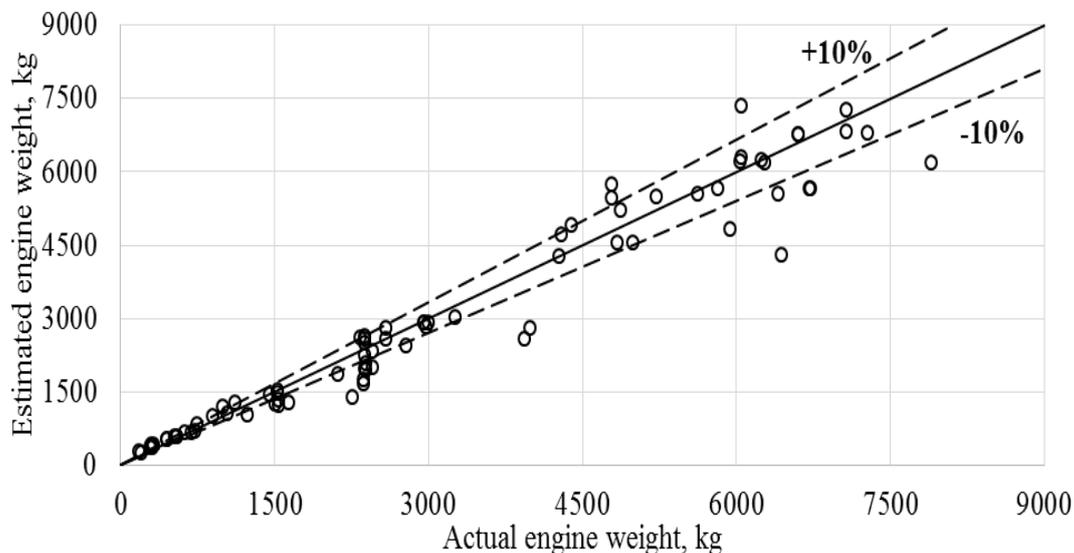


Figure 2. Estimated weight against the actual engine weight (Svoboda model).

Providing the consistent information on engine parameters (regularization) task is a stochastic task of obtaining the most probable values set of working process parameters of engine as a whole [13]. Regularization was carried out using the developed at the Samara National Research University software “ASTRA” [14, 15] as task of minimization of discrepancies between the published and calculated values (e.g. SFC, thrust, air mass flow rate, etc.), while the efficiency ratios, losses coefficients and other parameters with unknown values were optimizable variables.

Parameters’ values of the modern turbofan engines in general do not exceed the limits shown in Table II, so the results of mass estimation and the analysis of

abovementioned mass models would be correct for the most of modern turbofan engines.

4 Weight estimation, accuracy assessment and analysis

Weight was estimated for each engine using every weight model listed above. The results of estimation were then plotted against the actual engine weight in Fig. 2-9. Standard deviation for each weight model was calculated as well and is shown in Fig. 10. The weight models was arranged in order of increasing standard deviation.

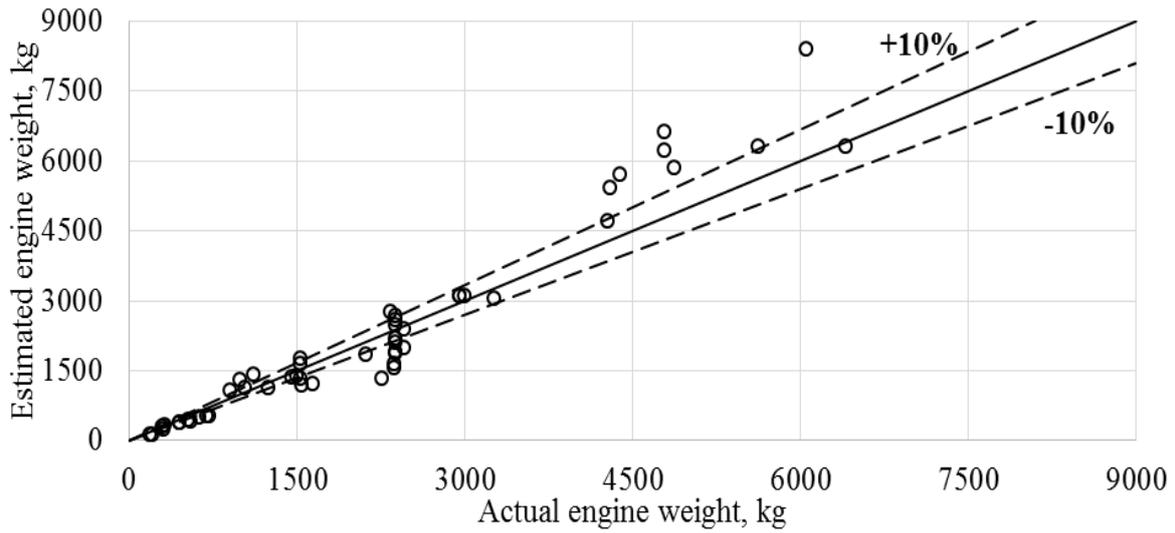


Figure 3. Estimated weight against the actual engine weight (Raymer model).

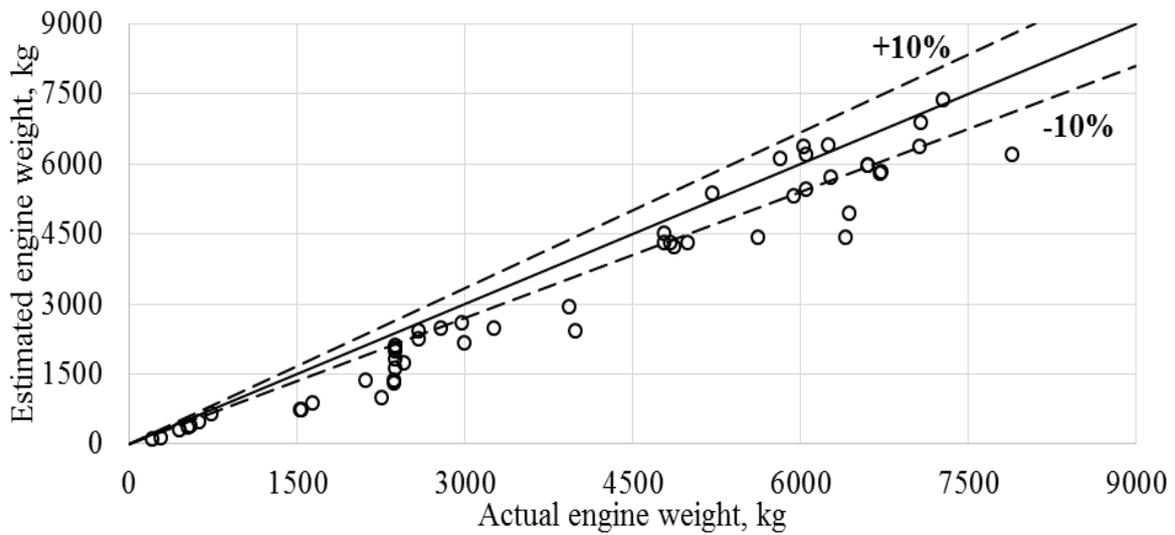


Figure 4. Estimated weight against the actual engine weight (Jenkinson model).

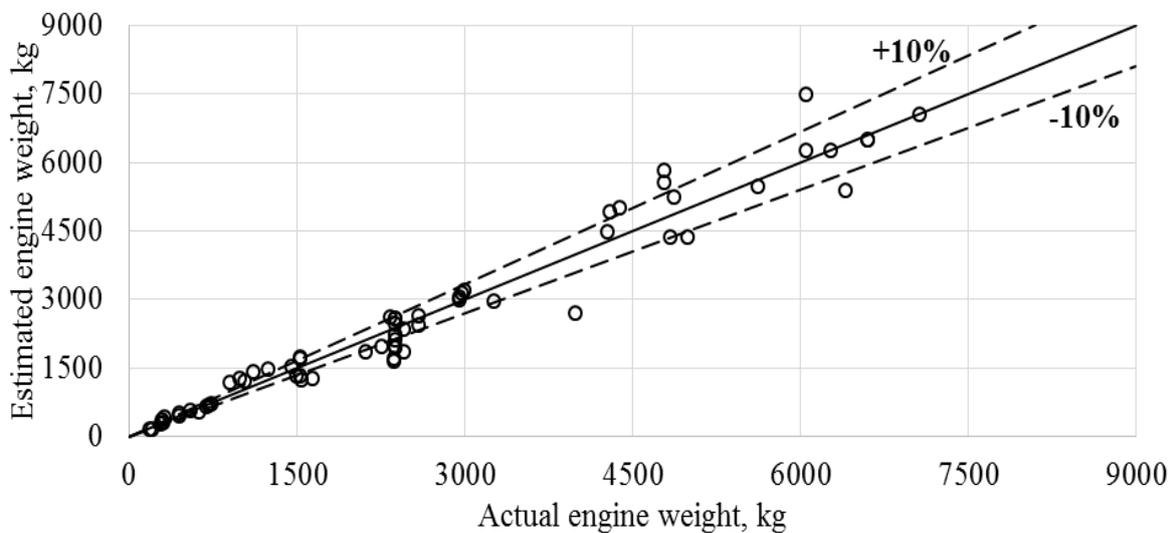


Figure 5. Estimated weight against the actual engine weight (Torenbeek model).

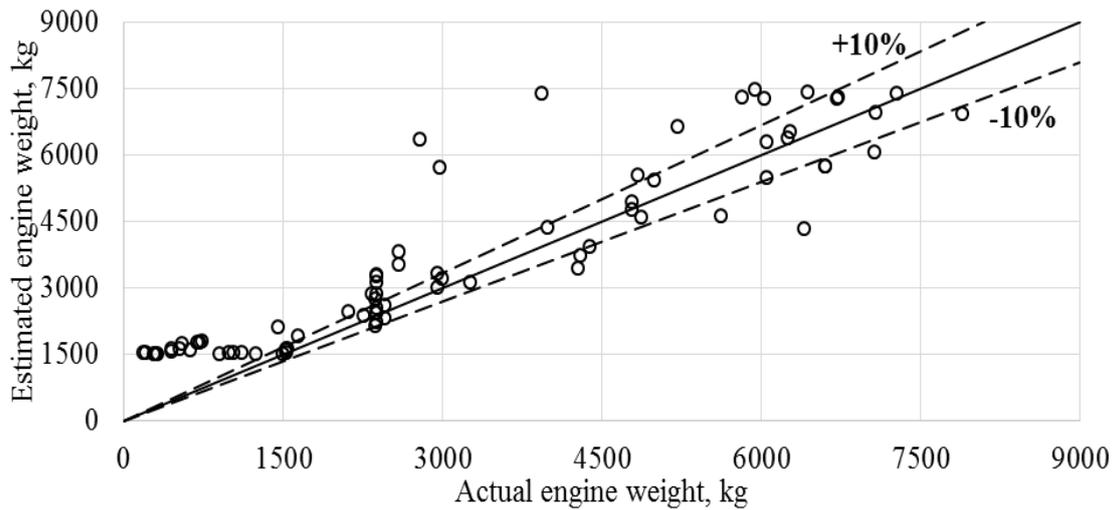


Figure 6. Estimated weight against the actual engine weight (Clavier model)

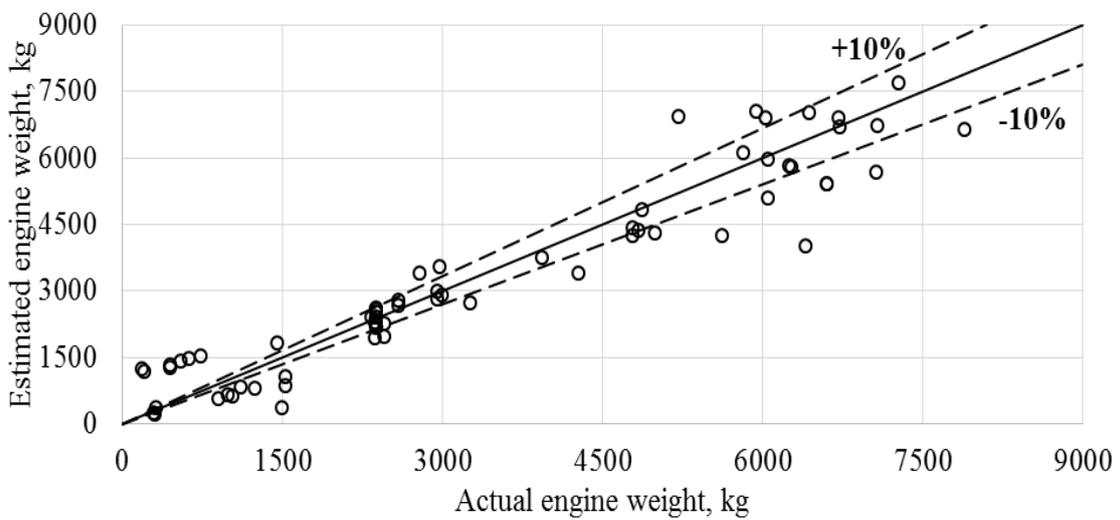


Figure 7. Estimated weight against the actual engine weight (Byerley model)

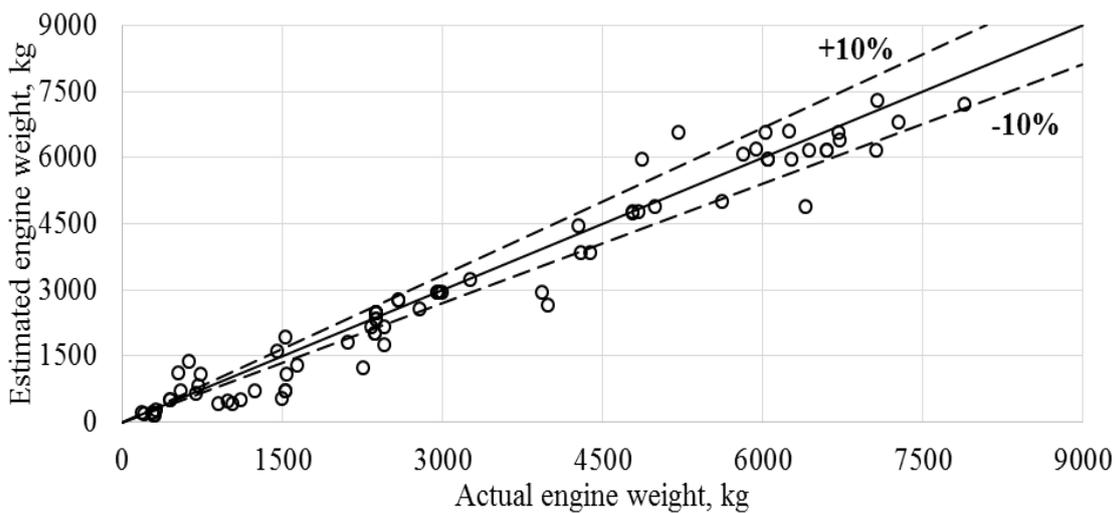


Figure 8. Estimated weight against the actual engine weight (Guha model)

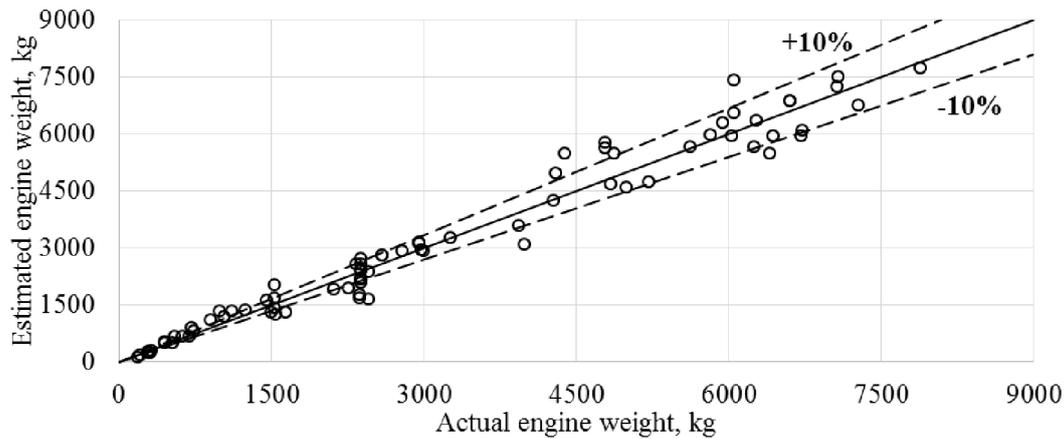


Figure 9. Estimated weight against the actual engine weight (Kuz'michev model)

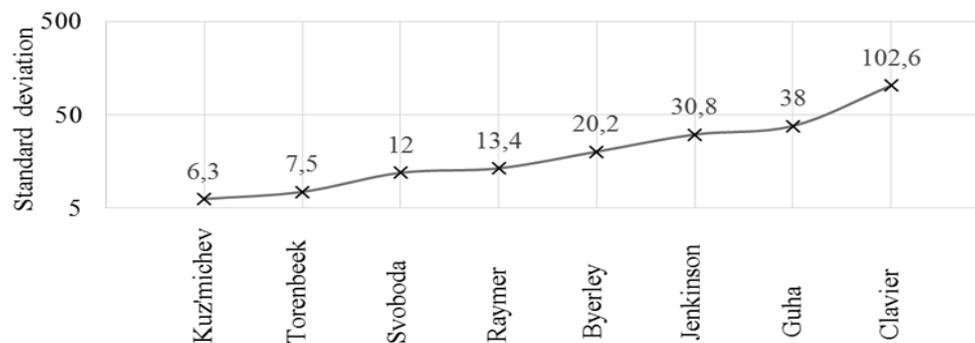


Figure 10. Standard deviation for the investigated weight models.

Design expertise [5] suggests the acceptable level of inaccuracy for the initial design stage at 10-15%.

Analysis lead to the following conclusions.

Four of the examined weight models may be used only for the big-scale engines [16], as the inaccuracy increases substantially for the low-scale engines. The limits of applicability (inaccuracy less than 12%) are: Guha ($D_F > 1.2$ m), Byerley ($D_F > 1$ m), Jenkinson ($P_{t-o} > 100$ kN) and Clavier ($G_{F t-o} > 150$ kg/s).

Inaccuracy of all weight models increases as the engine scale decreases, e.g. Kuz'michev model has inaccuracy of about 20% for the engines with weight less than 1500 kg, while its average inaccuracy is about 6.3%. Apparently, the coefficients of this model should be corrected for the low-scale engines with take-off thrust less than 50kN.

Svoboda and Guha models, that take only one input parameter (thrust or fan diameter), were additionally investigated using the information on engines having similar scale factor but substantially different working process parameters. The inaccuracy for these engines reached 25% (although the average value for Svoboda model is 12%), thus Svoboda and Guha models should be used at the stage of requirements specification drafting.

5 Conclusion

Svoboda and Raymer models should be used at the initial phases of designing, as they are able to provide the

engine weight estimation for a limited amount of information.

As the design information is accumulated, Torenbeek and Kuz'michev models become preferable, as they provide more accurate and detailed interrelation between engine features and its weight.

All the described above models lack accuracy if used for the low-scale engine weight estimation, specialized models should be developed for such applications.

Nomenclature

- D = diameter
- G = mass flow rate
- m = bypass ratio
- P = thrust
- T^* = total temperature
- W = weight
- π = pressure ratio
- σ = standard deviation
- a = air
- ab = afterburner
- C = compressor
- eng = engine
- F = fan
- m.ch = mixing chamber
- t-o = take-off
- Σ = overall
- 4 = section after combustion chamber
- I = engine core
- II = low-pressure spool

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