

Critical load of steel trusses - graphical methods

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Abstract. The article presents graphical methods for determine critical loads of column and beam elements based on experimental results. The author presents the principles of using each method and the results of author's experimental tests on the instability phenomenon of planar steel truss with imperfections. The discussed methods were used to determine critical load of the tested truss, next compared with the results of numerical analysis. The validity of methods for determining the critical moment in the truss analysis has been confirmed.

1 Introduction

Knowledge of the critical load of compressive and flexural elements is important in the case of design considering global stability of structures: buckling and lateral-torsional buckling [1]. This load can be determined using empirical formulas, e.g. Euler formula, or computer programs, e.g. LT-Beam.

The critical load can also be determined experimentally [2]. However, experimental models always are *imperfect* structures whose equilibrium paths are characterized by limit points rather than bifurcation points [3]. The equilibrium path of the imperfect systems (formula 1a) are characterized by the critical load, which is the horizontal asymptotes and the initial imperfections that affect of the non-linear relationships between load and displacements [4] (figure 1c).

Experimental determination of the critical load of compressed elements is related to the transformation of Southwell, which converts the formula (1a) to the linear form (formula 1b). This function, in the coordinate system (δ / P ; δ), intersects the vertical axis in the point, which is the initial imperfection ($-e_0$), while the tangent of its α angle is the critical force of the elements (figure 2.).

$$\delta = \frac{e_0}{\frac{P_{cr}}{P} - 1}; \delta = \frac{\delta}{P} P_{cr} - e_0 \quad (1a); (1b)$$

where:

δ transverse displacement of the compressed element,

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e_0 initial imperfection,
 P_{cr} critical load of a perfect element,
 P real compressive load.

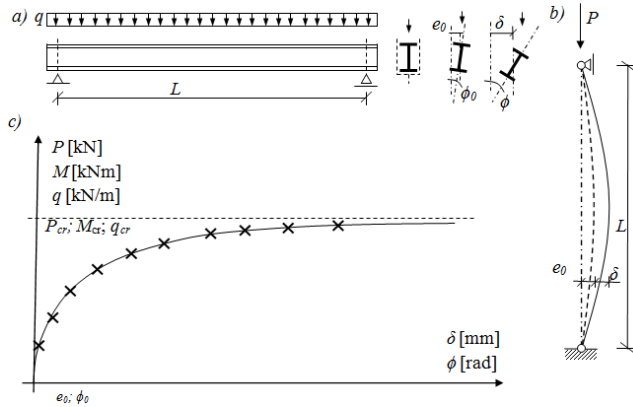


Fig. 1. Imperfect elements: a) beam; b) column; c) equilibrium path.

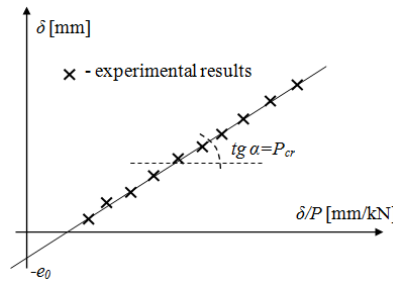


Fig. 2. Southwell's Method.

Similar rules relate to the experimental determination of the critical moment of bending elements. These elements are assumed with initial "out-of-straightness" imperfections e_0 and initial torsional imperfection ϕ_0 . The increment of the twist angle ϕ in the beam is analogous to the increase of deformation δ in the compression element and can be expressed by the formula [3]:

$$\phi = \frac{\phi}{M} M_{cr} - \phi_0 \quad (1c)$$

where:

ϕ angle of torsional deformation,
 ϕ_0 initial torsional deformation,
 M_{cr} critical moment of bending beam,
 M max. moment.

2 Methods to determine the critical moments

Initial attempts to use the Southwell method in the analysis of experimental data of bending beams showed a good agreement between the slope of the line and the empirically determined critical moment [5]. In the 20th century, many researchers proposed their own modifications of the graphic method, e.g.:

- Massey (1963),

- Trahair (*Modified Plot* - 1969),
- Meck (*Skewed Plot* - 1977).

These methods employ the different coordinate systems and different forms of equations (1b) and (1c). The Massey's method should be used for beams at bending with a single initial imperfection e_0 or ϕ_0 [5]. The Massey's plots are constructed in the coordinate system $(\delta/M^2; \delta)$ or $(\phi/M^2; \phi)$, while the critical moment (or critical load) is the square root of the slope angle of the straight line (2a) or (2b). This method involves linear equations in the form of:

$$\delta = \sqrt{M_{cr}} \frac{\delta}{M^2} - e_0; \phi = \sqrt{M_{cr}} \frac{\phi}{M^2} - \phi_0 \tag{2a); (2b)}$$

The Trahair's modification rely on transforming the formulas (1b) and (1c) into the form:

$$\delta \cdot M = \delta \cdot M_{cr} - \delta_0 \cdot M; \phi \cdot M = \phi \cdot M_{cr} - \phi_0 \cdot M \tag{3a); (3b)}$$

This transformation, called *Modified Plot*, creates a straight line in the coordinate system $(\delta; \delta M)$, whose angle of inclination is a critical moment, unfortunately it doesn't give a graphical representation of the initial imperfections in an obvious way [6].

The last variant of the graphical method, discussed in this paper, is called "*Skewed Plot*" due to the specific coordinate system, in which the lateral displacement and the twisting of the cross-section occur simultaneously.

$$\delta = \alpha \frac{\phi}{M} - \delta_0; \phi = \beta \frac{\delta}{M} - \phi_0 \tag{3a); (3b)}$$

where:

$$\alpha = \frac{\pi^2 \cdot EJ}{L^2}; \beta = G \cdot J_T$$

- E modulus of elasticity,
- J second moment of area - weak axis
- L length of elements,
- G shear modulus,
- J_T second polar moment of area.

In this method, the critical load is calculated on the basis of both plots according to the formula:

$$M_{cr} = \sqrt{\alpha \cdot \beta} \tag{3c)}$$

The work of planar trusses are resembles the work of beams at bending (truss is a kind of beams with a castellated web). The experimental determination of the critical load of the compression chord in this elements can be done by different ways: using the equilibrium path of a separate chord (applying the Southwell method); or using the equilibrium path of an entire truss (by the Southwell or Meck method). Typically, in experimental studies, the critical load is firstly determined and next the critical force or critical moment are calculated. It should be noted that the use of the equilibrium path of a separate compression chord does not allow to determine initial imperfections of an entire truss (e_0 i ϕ_0), instead a certain overall imperfection only.

In the experimental tests an external critical load is usually determined. Next the critical force of the compression element or the critical moment of the bending element

is determined based on the known critical load value. The stability problem in beam elements is well recognized and described, e.g. in [7]. The relationship between the critical moment M_{cr} of the bending I-beam and the critical force N_{cr} of the compression chord was presented in the paper [8]. The literature also includes information on equivalent truss models in the analysis of bending I-beams [9]. Thus there is an analogy between buckling of the upper truss chord and lateral-torsional buckling of the bending beam.

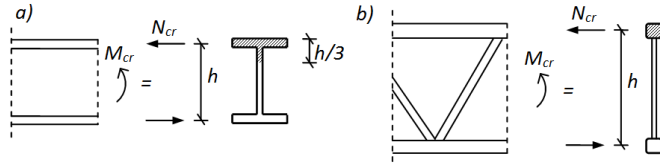


Fig. 3. Relation between the critical moment of the beam and the critical force of the compression chord: a) I-beam; b) truss.

3 Experimental research

Comparison of graphical methods for determining the critical moment in planar trusses was carried out on the results of author's experimental research. That research concerned seven meter long research models shown in Figure 4. The tests were carried out on simply supported single-span models with lateral, rigid support at the ends of the chords. At the beginning of the tests, the measurement was made of real, initial out-of-plane imperfections in the state without external load. During the study, the gravitational load was applied gradually in the nodes of the upper chord. The tests were carried out in the elastic range, limiting the possible deformation of chords up to 150 mm. The research models were made of steel with declared yield point of $f_y = 235$ MPa. The results of experimental tests, in the form of horizontal displacements of the upper and bottom chords, relative to the measured initial imperfections, are presented in Table 1. The displacements were measured at the chord nodes using inductive displacement sensors with a range of 300 mm and an accuracy of 0.001 mm.

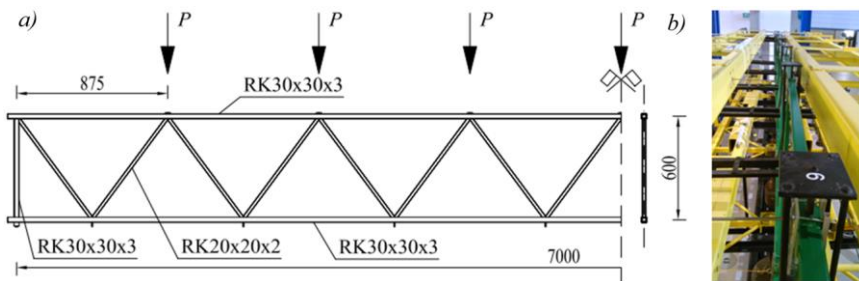


Fig. 4. Research model: a) scheme; b) Lateral-torsional buckling of a truss.

The research studies included also determination of the impact of various, simultaneous systems of chord imperfections on the equilibrium path of the truss. These configurations, e.g. opposite chord imperfections, were induced by means of an additional lateral load. The tests showed a significant effect of the imperfection of the tension chord on the shape of the equilibrium path of the compression chord and therefore also on the limit load on the truss [10]. The article presents only the basic results of experiment obtained for imperfect models without forcing any special imperfection configurations.

For the considered static scheme, the relationship between the load and the maximum bending moment of the equivalent beam can be expressed in the following form (this same is true for critical load and critical moment):

$$M_{max} = L \cdot P \tag{4}$$

Table 1. Experimental results.

Nodal load P [kN]	Model 1		Model 2		Model 3	
	upper chord (comp.)	bottom chord (tension)	upper chord (comp.)	bottom chord (tension)	upper chord (comp.)	bottom chord (tension)
	δ_u [mm]	δ_b [mm]	δ_u [mm]	δ_b [mm]	δ_u [mm]	δ_b [mm]
$e_{0,u}; e_{0,b}$ initial imperfection (middle of chord)	4.803	1.203	5.399	3.304	10.412	9.711
0	0	0	0	0	0	0
0.20	0.024	0.060	0.438	0.223	0.101	0.015
0.40	3.853	1.050	2.871	1.731	1.444	-0.572
0.60	8.271	4.173	10.279	6.933	5.217	1.1815
0.65	11.605	6.582	12.957	8.835	6.717	2.072
0.70	14.623	8.500	17.410	11.947	10.071	4.379
0.75	18.284	11.261	24.741	17.170	14.722	7.524
0.80	25.281	16.249	34.758	24.100	20.057	11.085
0.82	31.504	20.558	39.746	27.424	23.577	13.754
0.84	37.410	24.740	45.217	31.086	28.369	16.692
0.86	42.855	28.413	53.474	36.752	33.364	19.332
0.88	53.036	35.408	60.566	41.475	39.842	23.957
0.90	61.637	41.264	77.964	53.598	46.099	28.441
0.92	69.766	46.502	96.275	66.020	57.653	35.975
0.94	89.142	59.994	132.429	91.225	69.688	43.888
0.96	-	-	-	-	106.986	68.567

On the basis of the obtained results, general deformation of the entire truss was determined: horizontal displacement δ_{truss} and rotation ϕ in accordance with Fig. 5b. These results are summarized in Table 2.

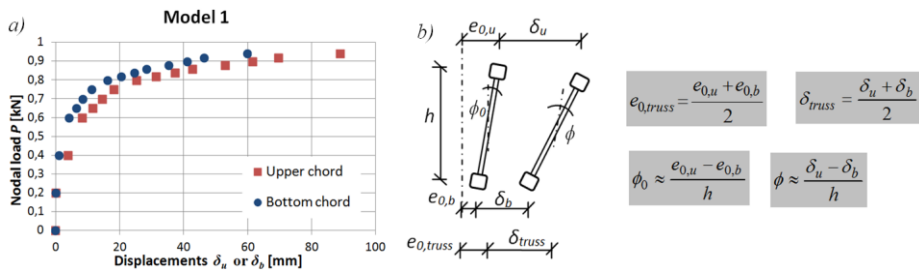


Fig. 5. Measured chord deformations: a) equilibrium path of the chords - Model 1; b) the calculating method of the truss deformation.

Table 2. Deformation of truss.

Nodal load P [kN]	Moment (form. 4) [kNm]	Model 1		Model 2		Model 3	
		Displacements δ_{truss} [mm]	Rotation ϕ [rad]	Displacements δ_{truss} [mm]	Rotation ϕ [rad]	Displacements δ_{truss} [mm]	Rotation ϕ [rad]
initial imperfection $e_{0,truss}; \phi_0$		3.003	0.006	4.352	0.0035	10.062	0.0012
0	0.00	0.000	0.000	0.000	0.0000	0.000	0.0000
0.20	1.40	0.077	0.00016	0.331	0.0004	0.058	0.0001
0.40	2.80	2.452	0.00467	2.301	0.0019	0.436	0.0034
0.60	4.20	6.222	0.00683	8.606	0.0056	3.199	0.0067
0.65	4.55	9.094	0.00837	10.896	0.0069	4.395	0.0077
0.70	4.90	11.562	0.01021	14.679	0.0091	7.225	0.0095
0.75	5.25	14.773	0.01171	20.956	0.0126	11.123	0.0120
0.80	5.60	20.765	0.01505	29.429	0.0178	15.571	0.0150
0.82	5.74	26.031	0.01824	33.585	0.0205	18.666	0.0164
0.84	5.88	31.075	0.02112	38.152	0.0236	22.531	0.0195
0.86	6.02	35.634	0.02407	45.113	0.0279	26.348	0.0234
0.88	6.16	44.222	0.02938	51.021	0.0318	31.900	0.0265
0.90	6.30	51.451	0.03396	65.781	0.0406	37.270	0.0294
0.94	6.44	58.134	0.03877	81.148	0.0504	46.814	0.0361
0.94	6.58	74.568	0.04858	111.827	0.0687	56.788	0.0430
0.96	6.72	-	-	-	-	87.777	0.0640

4 Critical moment of a planar truss

The numerically determined critical loadvalue of the researched structure is 0.971 kN. This load was determined in the SOFiSTiK program by means of linear buckling analysis (LBA) [11]. Numerical analysis was performed on a perfect model with nominal material and cross-section characteristics. The critical force in the compression chord corresponding to the critical load is 11.3 kN, while the appropriate buckling length factor is $\mu = 0.374$.

Based on the values summarized in Tables 1st and 2nd selected plots were developed.

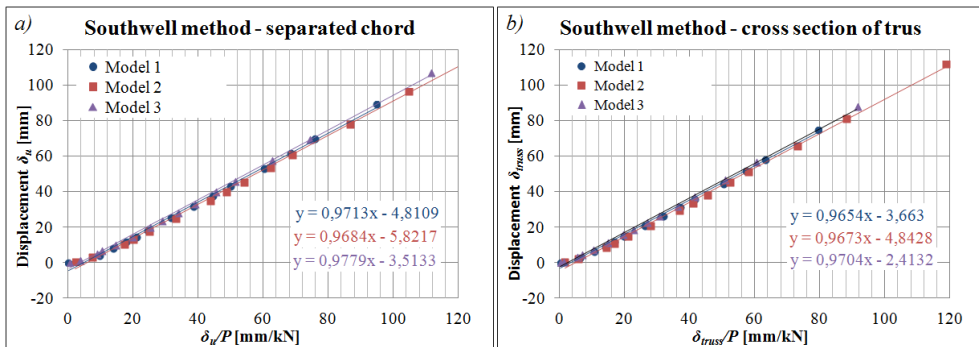


Fig. 6. Graphical results of the Southwell method (all models): a) for the separated upper chord; b) for the whole truss.

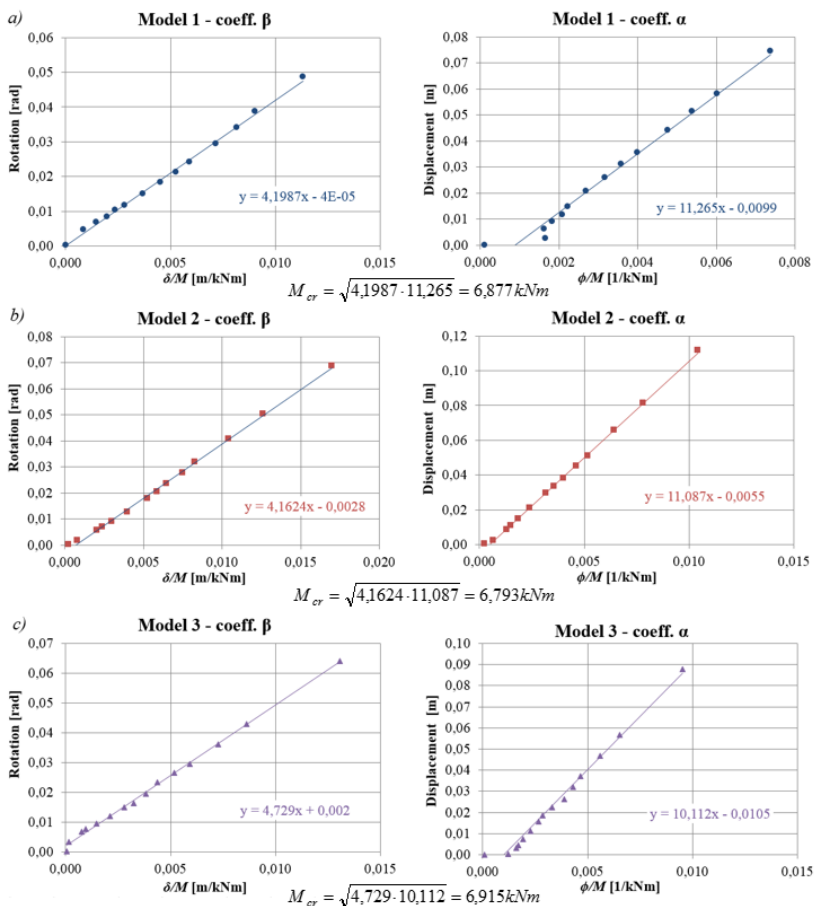


Fig. 7. Graphic results of the Meck method: a) Model 1; b) Model 2; c) Model 3.

The critical loads obtained on the basis of the values from the diagrams are shown in Table 3.

Table 3 Critical load of truss.

Method	Model 1		Model 2		Model 3	
Southwell (separated, upper chord)	0.9713		0.9684		0.9779	
Southwell (whole truss)	0.9654		0.9673		0.9704	
Massey	0.979	1.011	0.987	1.003	0.976	1.034
$(\delta/M^2; \delta)$ $(\phi/M^2; \phi)$						
Trahair	0.950	1.005	0.953	0.974	0.965	1.012
$(M\delta; \delta)$ $(M\phi; \phi)$						
Meck	0.983		0.970		0.988	

Comparing the obtained results, it should be recognized, that all the methods presented at the beginning of the paper allow to properly determine the critical load on the truss due to the lateral-torsional buckling (buckling of the compression belt). The differences between the respective average values and the value obtained on the basis of the numerical analysis do not exceed 10%. The average value obtained of all experimental results is 0.981 kN, while the maximum deviation from the average value is 5.4% (1.034 kN - Massey model 3 - coordinate system $\phi/M^2; \phi$).

5 Analysis of results and conclusions

The article compares the methods of graphical determination of the critical load on the basis of experimental data. The best-known methods and results of the author's experimental tests are presented. Based on the obtained results, the critical load was determined according to the rules each of the described method. Selected methods are presented in the form of graphical plots. The conducted analysis and obtained results allow for the following conclusions:

- The possibility of applying graphical methods, dedicated to beam elements (Massey, Trahair. Meck) to determine the critical load of a planar truss (in the case of the analyzed static scheme) confirms the validity of the use of the term "lateral-torsional buckling of the truss".
- The critical load values calculated on the basis of plots based on the torsion (Massey and Trahair method) are greater than their counterparts calculated on the basis of the horizontal displacement of the truss. This situation may result from neglecting the influence of the vertical displacement of each belt on twisting of the truss.
- Due to small differences between the numerical and experimental value of the critical load for all methods, it can be assumed that for the purposes of engineering practice, these methods can be used interchangeably.
- It can also be assumed that the analysis of the equilibrium path of a separate upper chord is sufficient to assess the critical load of an entire truss structure.

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