

## NUMERICAL AND EXPERIMENTAL INVESTIGATIONS OF NONLINEARITY BEHAVIOUR IN A SLENDER CANTILEVER BEAM

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### ABSTRACT

Nonlinear problem is always occur in slender structures that are usually characterized by large displacements and rotations but small strains. Linear design assumption could lead to premature failure if the structure behaves nonlinearly. In this paper, the static displacement of a slender beam subjected to point load is investigated numerically by incorporating the large amplitude of the displacement. Two types of numerical analyses are performed at a full-scale finite element model which is linear static and geometric nonlinear implicit static. The results of the FEA linear static analysis are compared with the results from the FEA geometric nonlinear implicit static analysis. It shows that very high different load-displacement value response. Experimental static displacement test has been performed to validate both numerical results.

**Keywords:** geometrically nonlinear, structural analysis and linear static.

### INTRODUCTION

Current designs and models in industry normally remain linear. These structural models are normally constructed based on idealized engineering designs and they may not accurately represent all the aspects of actual structures [1]. In some cases, linear models may fail to predict the structural response within the necessary level of reliability. Linear design could lead to premature failure if the structure behaves nonlinearly. Most of cases were assumed as a linear [2]

Nonlinearities in structures can lead to large amplitude responses due to extreme mechanical and environmental loading conditions. The investigation on the nonlinear behavior is very important because it effects to the integrity of the engineering structure. Two types of nonlinearities commonly encountered in structural analysis are geometric and material. Material nonlinearity is associated with nonlinear stress-strain relations whereas non-linear curvature-slope and strain-displacement relations give rise to geometric nonlinearity [3].

The researchers try to consider nonlinear phenomena for structures and devices. Some examples of nonlinear mechanical systems phenomena occur in aerospace sector, guyed tower and rotating wind turbine blade [4, 5]. In aeronautic, a new aircraft designs with light weight and high aspect-ratio wing provides advantage in producing a higher lift-to-drag ratio [6]. These result the wing to be very flexible and cause large

deformations under normal operating loads leading to a geometrical nonlinear behavior and aeroelastic problems [7-9].

Geometrically non-linear problem is always occur at slender structures that are usually characterized by large displacements but small strains [10] and relates with the geometric effect. Large deflection problem of cantilever beams is generally analyzed in curvilinear coordinate system. Euler Bernoulli beam theory in curvilinear coordinate system ( $s, n$ ) is  $1/\rho = M/EI$ , where curvature  $1/\rho = d\varphi/ds$ . So Euler Bernoulli bending moment-curvature relationship is given as follows [3] and  $\varphi$  is the slope  $dy/dx$  at location  $s$ , and it is also the measure of normal direction  $n$ .

$$EI \frac{d\varphi}{ds} = M$$

The objective of this paper is to apply a procedure for investigating the large deflection of a straight cantilever beam subjected to a tip concentrated load. In the present work, we demonstrate the separation between two methods, one is linear static analysis and the other one is geometrically nonlinear implicit static analysis. The paper is organized as follows. Section 1 briefly introduces the geometric nonlinearity in structural modelling. Section 2 describes geometrical and structural features used for experimental and numerical works. Section 3 presents a result of the experimental and numerical analysis performed on the sample. Finally, Section 4 concludes the paper.

## METHODOLOGY

### Experimental Setup

A static load experiment has been specifically designed and performed in laboratory in order to investigate geometric nonlinearity in structural analysis. The experimental set-up consists of a slender beam (standard ruler) with the dimension of 1024 mm × 32 mm × 1.5 mm (length × width × thickness) and a hole of diameter 7.3 mm at one end. The beam was clamped at the left end side by using two G-clamps and the boundary condition is fixed support. This assembly is illustrated in Figure 1.

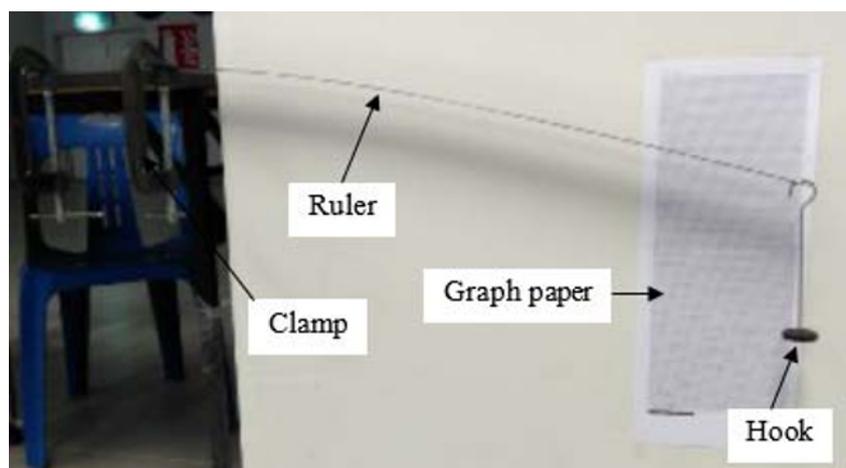


Figure 1. Photograph of the experimental setup

In this work, a graph paper is placed on the wall in order to accurately measure the displacement data. The ruler is point loaded at the right end side by placing several defined weights on the weight hook. The deflection of the ruler for each weight was measured through the marking point on the graph paper. In each step of loading, the deflection profile is captured and recorded manually.

### Finite Element Model and Analysis

The finite element of the beam was developed using Hypermesh with 3813 elements and 8350 nodes. The physical properties used in the finite element modeling are elastic modulus  $E = 193000$  GPa, density  $\rho = 7700$  kg/m<sup>3</sup> and Poisson's ratio  $\nu = 0.33$ . Geometric details and ruler model developed for the present study are shown in Figure 2.

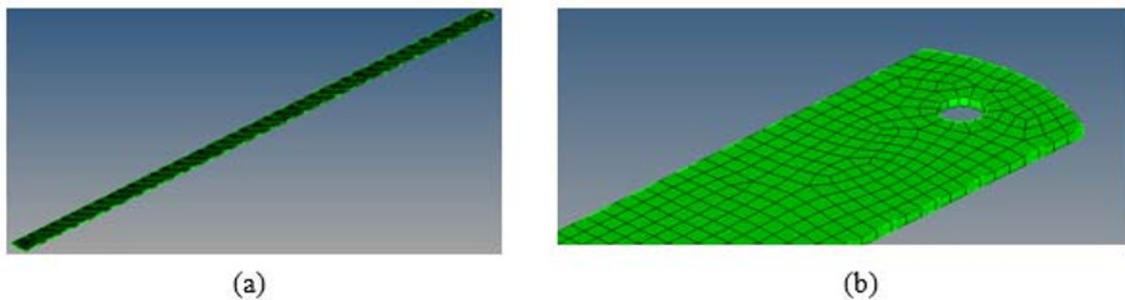


Figure 2. A ruler model (a) isometric view of the CAD model (b) meshed model

The linear and nonlinear effects have been simulated via displacement-based finite element analysis results. The fixed support at one end in the experiment was modelled by fully constrained based on a single point constraint. Eleven sets of different concentrated point load have been assigned to the end of ruler. The large displacement was computed from this model by using two types of loadsteps: linear static analysis and geometrical nonlinear implicit static analysis.

## RESULTS AND DISCUSSION

### Experimental Deflection Profiles

Figure 3 shows the deflection of the cantilever beam for several weight point loads. The deflection comes from the weight of the ruler (distributed load) and weight from the blocks (concentrated load). In zero load condition, the ruler already deflects by exact value due to the weight itself. The experimental data appear to have large deflection during the low value of the applied load due to the geometrically nonlinear behavior of the structure.

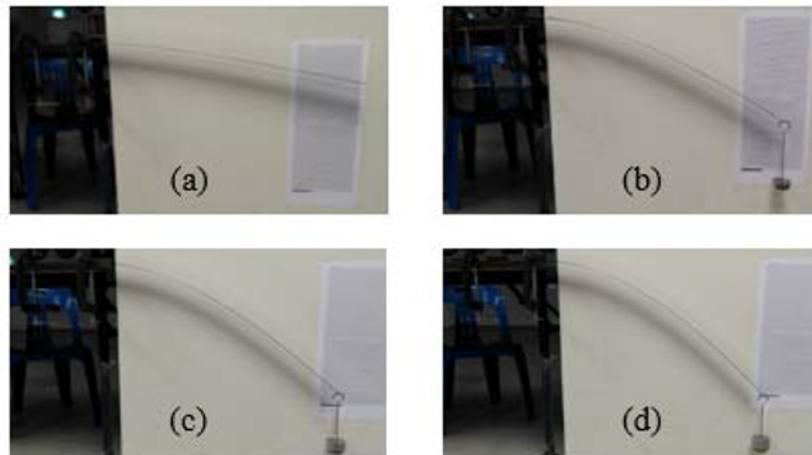


Figure 3. Experimental deflection of a ruler for point load (a) zero (b) 2.206 N (c) 4.206 N (d) 5.206 N

### Comparison between Experimental and Numerical Analysis Results

The deflection of the cantilever beam by numerical analysis for 4.206 N point load using linear static and geometric nonlinear implicit static analyses is shown in Figure 4. Table 1 shows the detail value of the deflection for each of point load by experiment and finite element analysis using linear static and geometric nonlinear implicit static analyses. To perform detail investigation of the deflection, a plot of the displacement values in Table 1 is given in Figure 5.

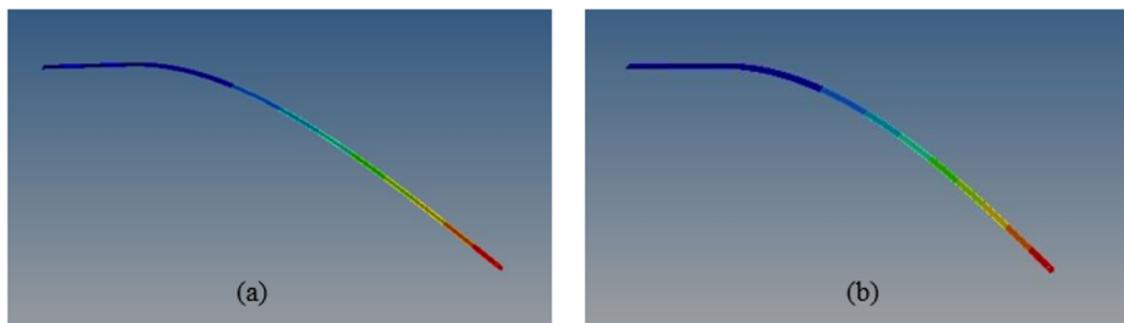


Figure 4. Numerical analysis deflection of a ruler using (a) linear static and (b) geometric nonlinear implicit static analyses

Table 1. Deflection values from linear static and geometric nonlinear implicit static analyses.

	Experiment (Geometric Nonlinear)	FEA (Linear Static)	Error	FEA (Geometric Nonlinear)	Error
Load (N)	Displacement (mm)	Displacement (mm)	%	Displacement (mm)	%
0	126	120	4.6	119	5.9
0.206	153	142	7.2	140	8.8
0.706	197	195	1.0	188	4.4
1.206	243	248	2.0	234	3.7
1.706	282	301	6.7	277	1.9
2.206	316	354	12.0	316	0.0
2.706	350	407	16.3	351	0.5
3.206	379	460	21.4	384	1.5
3.706	402	512	27.6	414	3.1
4.206	427	566	32.6	442	3.5
4.706	447	619	38.6	467	4.5
5.206	464	672	45.0	490	5.6

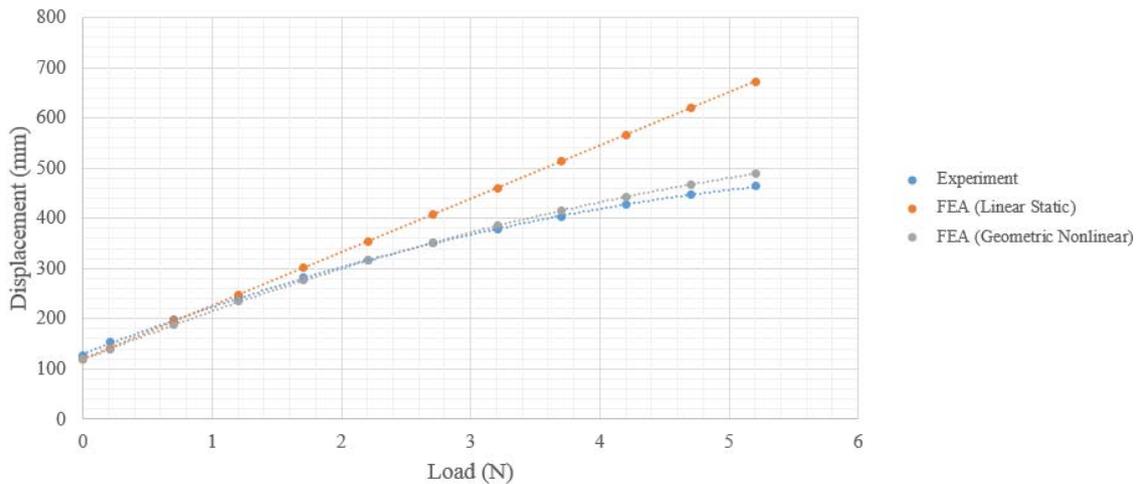


Figure 5. Deflection of a ruler using linear static and geometric nonlinear implicit static

Initial displacement measured is 126 mm experimentally in its free state (weight of ruler only) condition. By using numerically linear static analysis, the displacement is 120 mm. It is observed that this method produces constant increment of displacement values when the point load increases. The increment pattern is consistent with linear characteristic response. The error also increases since the discrepancy between experiment and linear static analysis value increase.

A good agreement of numerical and experimental results has been found for the geometric nonlinear implicit static analysis. Due to the long slender ruler with thin property, the deflection imposed geometric nonlinearity. It is obvious from Figure 5, it can be seen that the geometric nonlinear implicit static analysis shows less discrepancies with real experimental structure and give more precise predictions of

geometric nonlinear behavior for future analyses. The best fit linear load-deflection curve is obtained from the calculated data points and the geometric nonlinear is clearly visible. Hence, this large deflection tip load is considered to capture the effect of geometric nonlinear in actual case.

## CONCLUSION

This paper discusses the deflection behavior of the cantilever beam subjected to the concentrated load. Experiment has been carried out at the long thin stainless steel ruler to measure its deflection. Next, the deflection was calculated by performing numerical modelling and analysis based on two types of static analysis: linear static and geometric nonlinear implicit static. Finally the results were compared and successfully judge the geometric nonlinear deflection behavior in a beam.

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