# RBF NN-Based Backstepping Adaptive Control for a Class of Nonlinear Systems

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**Abstract.** There are many control methods for nonlinear systems, but some of them can not control nonlinear mismatched systems very well. Backstepping control has obvious advantages in controlling nonlinear mismatched systems. Thus we proposed a new radial-basis-function (RBF) neural network-based backstepping adaptive controller combining RBF neural network (RBF NN) and backstepping control for a class of nonlinear mismatched systems. We adopted RBF NN to approximate the system uncertainty. And we analyzed the controller stability using Lyapunov stability theory. Finally we chose sine signal as simulation input signal, simulation results show that the proposed control strategy has better adaptive ability and robustness than PID control, validating the effectivess of the proposed control strategy.

## 1 Introduction

Nonlinear systems have been studied by many scholars, and the control methods used are as follows: iterative learing control [1], feedback linearization [2], fuzzy control [3], sliding mode control [4], etc. However, when the system is a nonlinear mismatched system, the disadvantages of the above methods will be revealed.

Backstepping control can be used to control nonlinear mismatched systems [5]. RBF neural network (RBF NN) has universal approximation capability and can approximate arbitrary nonlinear functions with arbitrary precision [6]. The control algorithm combining RBF NN and backstepping control has been successfully applied by scholars [7-11].

In this paper, we use RBF NN-based backstepping control for a class of nonlinear mismatched systems. RBF NN is used to approximate system uncertainty. Simulation results verify the feasibility of the proposed control algorithm.

## 2 Problem description

Consider the following nonlinear mismatched system with strict feedback [12]:

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1) + \Delta f_1(x_1) \\ \dot{x}_2 = x_3 + f_2(x_1, x_2) + \Delta f_2(x_1, x_2) \\ \cdots \\ \dot{x}_{n-1} = x_n + f_{n-1}(x_1, x_2, \cdots, x_{n-1}) + \Delta f_{n-1}(x_1, x_2, \cdots, x_{n-1}) \\ \dot{x}_n = \psi u + f_n(x_1, x_2, \cdots, x_n) + \Delta f_n(x_1, x_2, \cdots, x_n) \\ y = x_1 \end{cases} \quad (1)$$

where $\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^{\mathrm{T}}$ is a system state variable. $y$ is a system output. $u$ is a system control input. Let $\bar{\boldsymbol{x}}_i = [x_1, x_2, \cdots, x_i]^{\mathrm{T}}$, strict feedback means that the nonlinear terms $f_i$ and $\Delta f_i$ depend only on the system state $x_1, x_2, \cdots, x_i$.

In addition, the nonlinear terms satisfy the following characteristics : (a) $f_i : R^i \to R$ should be smooth enough and $f_i(0, 0, \cdots, 0) = 0$, $i = 1, 2, \cdots, n$; (b) $\Delta f_i : R^i \to R$ is unknown, $\Delta f_i \in C^l$ and $\Delta f_i(0, 0, \cdots, 0) = 0$.

**Assumption 1** Given a smooth nonlinear function vector $\boldsymbol{h} : \Omega \mapsto R^n$ ($\Omega$ is a compact set in $R^m$), there exist an optimal Gaussian base function vector $\boldsymbol{\varphi} : R^m \mapsto R^n$ and weight matrix $\boldsymbol{W}^*_{n\times n}$ such that :

$$\boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{W}^{*\mathrm{T}} \boldsymbol{\varphi}(\boldsymbol{x}) + \boldsymbol{\varepsilon}, \forall \boldsymbol{x} \in \Omega \quad (2)$$

where $\varepsilon$ is the construction error of RBF NN.

In this paper, our main task is to design an adaptive controller so as to make system output $x_1$ asymptotically stable tracking the expected output $x_{1d}$, namely $\lim_{t\to\infty} |x_1 - x_{1d}| = 0$.

## 3 Controller design and stability analysis

We use a RBF NN-based backstepping adaptive control strategy for a class of nonlinear system, Lyapunov stability theory is used to analyze the system stability. The detailed design process is as follows.

**Step 1** Define the tracking error between the actual system output $x_1$ and the expected system output $x_{1d}$ as :

$$e_1 = x_1 - x_{1d} \quad (3)$$

Its derivative is

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_{1d} = x_2 + f_1(x_1) + \Delta f_1(x_1) - \dot{x}_{1d} \quad (4)$$

To eliminate the influence of uncertainty $\Delta f_1$, we use RBF NN to approximate $\Delta f_1$ :

$$\Delta f_1(x_1) = \boldsymbol{W}_1^{*\mathrm{T}} \boldsymbol{\varphi}_1(x_1) + \varepsilon_1 \quad (5)$$

where $x_1$ is a RBF NN input.

If $x_2$ is considered the actual input of system (4), then there exist an ideal control $x_2^*$ [13]:

$$x_2^* = -(f_1 - \dot{x}_{1d} + k_1 e_1 + \boldsymbol{W}_1^{*\mathrm{T}} \boldsymbol{\varphi}_1 + \varepsilon_1) \quad (6)$$

such that $\dot{e}_1 = -k_1 e_1 + (x_2 - x_2^*)$ , $k_1 > 0$ is a design parameter. Because $x_2^*$ is not available, we design a virtual control $x_{2d}$ as follows.

$$x_{2d} = -(f_1 - \dot{x}_{1d} + k_1 e_1 + \hat{\boldsymbol{W}}_1^{\mathrm{T}} \boldsymbol{\varphi}_1) \quad (7)$$

where $\hat{\boldsymbol{W}}_1$ is the estimated value of $\boldsymbol{W}_1^*$.

We choose the following Lyapunov function $V_1$ :

$$V_1 = \frac{1}{2} e_1^2 + \frac{1}{2} tr[\tilde{\boldsymbol{W}}_1^{\mathrm{T}} \boldsymbol{\Gamma}^{-1} \tilde{\boldsymbol{W}}_1] \quad (8)$$

where $\tilde{\boldsymbol{W}}_1 = \boldsymbol{W}_1^* - \hat{\boldsymbol{W}}_1$, $\boldsymbol{\Gamma}_1 = \boldsymbol{\Gamma}_1^{\mathrm{T}} > 0$ is a design parameter.

The derivative of $V_1$ is as follows :

$$\dot{V}_1 = e_1 \dot{e}_1 + tr[\tilde{\boldsymbol{W}}_1^T \boldsymbol{\Gamma}_1^{-1} \dot{\tilde{\boldsymbol{W}}}_1]$$

$$= -k_1 e_1^2 + e_1(x_2 - x_{2d}) + \tilde{\boldsymbol{W}}_1^{\mathrm{T}} \boldsymbol{\varphi}_1 e_1 + \varepsilon_1 e_1 + tr[\tilde{\boldsymbol{W}}_1^{\mathrm{T}} \boldsymbol{\Gamma}_1^{-1} \dot{\tilde{\boldsymbol{W}}}_1] \quad (9)$$

**Step *i*** ( $i = 2, \cdots, n-1$ ) Define the tracking error $e_i$ between the system state $x_i$ and the virtual control $x_{id}$ as

$$e_i = x_i - x_{id} \quad (10)$$

Its derivative is

$$\dot{e}_i = \dot{x}_i - \dot{x}_{id} = x_{i+1} + f_i(\bar{x}_i) + \Delta f_i(\bar{x}_i) - \dot{x}_{id} \quad (11)$$

Similarly, to eliminate the influence of uncertainty $\Delta f_i$, we use RBF NN to approximate $\Delta f_i$ :

$$\Delta f_i(\bar{x}_i) = \boldsymbol{W}_i^{*\mathrm{T}} \boldsymbol{\varphi}_i(\bar{x}_i) + \varepsilon_i \quad (12)$$

where $\bar{x}_i$ is a RBF NN input.

If $x_{i+1}$ is considered the actual input of system (11), then there exist an ideal control $x_{i+1}^*$ :

$$x_{i+1}^* = -(f_i - \dot{x}_{id} + k_i e_i + e_{i-1} + \boldsymbol{W}_i^{*\mathrm{T}} \boldsymbol{\varphi}_i + \varepsilon_i) \quad (13)$$

such that $\dot{e}_i = -k_i e_i - e_{i-1} + (x_{i+1} - x_{i+1}^*)$, $k_i > 0$ is a design parameter. Because $x_{i+1}^*$ is not available, we design a virtual control $x_{i+1,d}$ as follows.

$$x_{i+1,d} = -(f_i - \dot{x}_{id} + k_i e_i + e_{i-1} + \hat{\boldsymbol{W}}_i^{\mathrm{T}} \boldsymbol{\varphi}_i) \quad (14)$$

where $\hat{\boldsymbol{W}}_i$ is the estimated value of $\boldsymbol{W}_i^*$.

We choose the following Lyapunov function $V_i$ :

$$V_i = \sum_{j=1}^{i-1} V_j + \frac{1}{2} e_i^2 + \frac{1}{2} tr[\tilde{\boldsymbol{W}}_i^{\mathrm{T}} \boldsymbol{\Gamma}_i^{-1} \tilde{\boldsymbol{W}}_i] \quad (15)$$

where $\tilde{\boldsymbol{W}}_i = \boldsymbol{W}_i^* - \hat{\boldsymbol{W}}_i$, $\boldsymbol{\Gamma}_i = \boldsymbol{\Gamma}_i^{\mathrm{T}} > 0$ is a design parameter.

The derivative of $V_i$ is as follows :

$$\dot{V}_i = -\sum_{j=1}^{i} k_j e_j^2 + e_i(x_{i+1} - x_{i+1,d})$$

$$+ \tilde{\boldsymbol{W}}_i^{\mathrm{T}} \boldsymbol{\varphi}_i e_i + \varepsilon_i e_i + tr[\tilde{\boldsymbol{W}}_i^{\mathrm{T}} \boldsymbol{\Gamma}_i^{-1} \dot{\tilde{\boldsymbol{W}}}_i] \quad (16)$$

**Step *n*** Define the tracking error $e_n$ between the system state $x_n$ and the virtual control $x_{nd}$ as

$$e_n = x_n - x_{nd} \quad (17)$$

Its derivative is

$$\dot{e}_n = \dot{x}_n - \dot{x}_{nd} = \psi u + f_n(\bar{x}_n) + \Delta f_n(\bar{x}_n) - \dot{x}_{nd} \quad (18)$$

To eliminate the influence of uncertainty $\Delta f_n$, we use RBF NN to approximate $\Delta f_n$ :

$$\Delta f_n(\bar{x}_n) = \boldsymbol{W}_n^{*\mathrm{T}} \boldsymbol{\varphi}_n(\bar{x}_n) + \varepsilon_n \quad (19)$$

$u$ is the actual control of system (18), there exist an ideal control $u^*$ :

$$u^* = -(f_n - \dot{x}_{nd} + k_n e_n + e_{n-1} + \boldsymbol{W}_n^{*\mathrm{T}} \boldsymbol{\varphi}_n + \varepsilon_n) \quad (20)$$

such that $\dot{e}_n = -k_n e_n - e_{n-1} + \psi(u - u^*)$, $k_n > 0$ is a design parameter. Because $u^*$ is not available, we design an actual control $u$ as follows.

$$u = -(f_n - \dot{x}_{nd} + k_n e_n + e_{n-1} + \hat{\boldsymbol{W}}_n^{\mathrm{T}} \boldsymbol{\varphi}_n) \quad (21)$$

where $\hat{\boldsymbol{W}}_n$ is the estimated value of $\boldsymbol{W}_n^*$.

We choose the following Lyapunov function $V_n$ :

$$V_n = \frac{1}{2} \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{\xi} + \frac{1}{2} tr[\tilde{\boldsymbol{Z}}^{\mathrm{T}} \boldsymbol{\Gamma}^{-1} \tilde{\boldsymbol{Z}}] + \frac{1}{2} \eta \tilde{\psi}^2 \quad (22)$$

where $\eta > 0$ is a design parameter, $\boldsymbol{\Gamma} = \begin{bmatrix} \boldsymbol{\Gamma}_1 & & & \\ & \boldsymbol{\Gamma}_2 & & \\ & & \ddots & \\ & & & \boldsymbol{\Gamma}_n \end{bmatrix}$,

$\boldsymbol{\xi} = [e_1 \ e_2 \cdots e_n]^{\mathrm{T}}$ , and $\tilde{\psi} = \psi - \hat{\psi}$ , $\hat{\psi}$ is the estimated value of $\psi$ .

We choose the following adaptive law :

$$\dot{\hat{\boldsymbol{Z}}} = \boldsymbol{\Gamma} \boldsymbol{\varphi} \xi^{\mathrm{T}} - n \boldsymbol{\Gamma} \|\xi\| \hat{\boldsymbol{Z}} \quad (23)$$

where $\boldsymbol{\varphi} = [\boldsymbol{\varphi}_1 \ \boldsymbol{\varphi}_2 \cdots \boldsymbol{\varphi}_n]$, $n > 0$ is a design parameter.

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{W}_1 & & & \\ & \boldsymbol{W}_2 & & \\ & & \ddots & \\ & & & \boldsymbol{W}_n \end{bmatrix}, \ \|\boldsymbol{Z}\|_F \le Z_M$$

$$\hat{\boldsymbol{Z}} = \begin{bmatrix} \hat{\boldsymbol{W}}_1 & & & \\ & \hat{\boldsymbol{W}}_2 & & \\ & & \ddots & \\ & & & \hat{\boldsymbol{W}}_n \end{bmatrix}, \ \tilde{\boldsymbol{Z}} = \boldsymbol{Z} - \hat{\boldsymbol{Z}}$$

The derivative of $V_n$ is as follows :

$$\dot{V}_n = -\boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K}_e \boldsymbol{\xi} + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{\varepsilon} + tr[\tilde{\boldsymbol{Z}}^{\mathrm{T}} \boldsymbol{\Gamma}^{-1} \dot{\tilde{\boldsymbol{Z}}} + \tilde{\boldsymbol{Z}}^{\mathrm{T}} \boldsymbol{\varphi} \boldsymbol{\xi}^{\mathrm{T}}] + \eta \tilde{\psi} \dot{\tilde{\psi}} \quad (24)$$

where

$\boldsymbol{K}_e = diag\{k_1, k_2, \cdots, k_n\}^{\mathrm{T}}$, $\boldsymbol{\varepsilon} = [\varepsilon_1 \ \varepsilon_2 \cdots \varepsilon_n]^{\mathrm{T}}$, $\|\boldsymbol{\varepsilon}\| < \varepsilon_N$.

Since $\dot{\tilde{\boldsymbol{Z}}} = -\dot{\hat{\boldsymbol{Z}}}$, and $\dot{\tilde{\psi}} = -\dot{\hat{\psi}}$, substitute Eq.(23) into (24), we have

$$\dot{V}_n = -\boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K}_e \boldsymbol{\xi} + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{\varepsilon} + n \|\boldsymbol{\xi}\| tr[\tilde{\boldsymbol{Z}}^{\mathrm{T}} (\boldsymbol{Z} - \tilde{\boldsymbol{Z}})] - \eta \tilde{\psi} \dot{\hat{\psi}} \quad (25)$$

According to the actual system, the inequality $\tilde{\psi} \dot{\hat{\psi}} \ge 0$ is clearly true. In addition, according to Schwarz inequality, the following inequality is true :

$$tr[\tilde{\boldsymbol{Z}}^{\mathrm{T}} (\boldsymbol{Z} - \tilde{\boldsymbol{Z}})] \le \|\tilde{\boldsymbol{Z}}\|_F \|\boldsymbol{Z}\|_F - \|\tilde{\boldsymbol{Z}}\|_F^2$$

Since $K_{\min}\|\xi\|^2 \le \xi^T \boldsymbol{K}_e \xi$, where $K_{\min}$ is the minimum eigenvalue of $\boldsymbol{K}_e$. Thus the above Eq.(25) becomes :

$$\dot{V}_n \le -K_{\min}\|\xi\|^2 + \varepsilon_N\|\xi\| + n\|\xi\|(\|\tilde{\boldsymbol{Z}}\|_F\|\boldsymbol{Z}\|_F - \|\tilde{\boldsymbol{Z}}\|_F^2) - \eta\tilde{\psi}\dot{\hat{\psi}}$$

$$\le -\|\xi\|(K_{\min}\|\xi\| - \varepsilon_N + n\|\tilde{\boldsymbol{Z}}\|_F(\|\tilde{\boldsymbol{Z}}\|_F - Z_M)) \quad (26)$$

As we know,

$$K_{\min}\|\xi\| - \varepsilon_N + n(\|\tilde{\boldsymbol{Z}}\|_F^2 - \|\tilde{\boldsymbol{Z}}\|_F Z_M)$$

$$= K_{\min}\|\xi\| - \varepsilon_N + n(\|\tilde{\boldsymbol{Z}}\|_F - \frac{1}{2}Z_M)^2 - \frac{n}{4}Z_M^2$$

Thus if the following condition can be met

$$\|\xi\| > \frac{\varepsilon_N + \frac{n}{4}Z_M^2}{K_{\min}} \quad (27)$$

then $\dot{V}_n \le 0$.

From Eq.(27), we can see that the tracking accuracy is related to $\varepsilon_N$, $n$ and $K_{\min}$.

## 4 Simulation analysis

We adopt the following system to validate the effectiveness of the proposed control strategy.

$$\begin{cases} \dot{x}_1 = x_2 + x_1\sin(x_1) \\ \dot{x}_2 = x_3 + x_1^2 + x_2^2 + \sin(x_1 + x_2) \\ \dot{x}_3 = 2u + \sin(x_1^2 + x_2^2 + x_3^2) \\ y = x_1 \end{cases} \quad (28)$$

The control goal is to make the system output $x_1$ trace the desired output $x_{1d} = 0.5\sin(2\pi t)$. $k_1 = 10$, $k_2 = 8$, $k_3 = 12$, $\boldsymbol{\Gamma}_i (i = 1,2,3)$ is a unit array with proper dimension, $\eta = 115$, $n = 0.01$, $\boldsymbol{x}(0) = [0,0,0]^{\mathrm{T}}$. The following curves are comparision curves between PID control and the proposed strategy in this paper, namely RBF NN-based backstepping adaptive control(RBFNNB). Figure 1 and figure 2 are curves of tracking sine signal and error using PID control, respectively. Figure 3 and figure 4 are curves of tracking sine signal and error using RBFNNB, respectively. From figure 1-4, it is obvious for us that RBFNNB has better dynamic and steady performance than PID control.

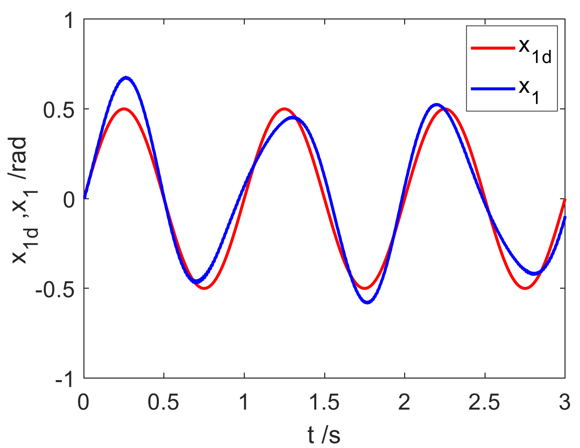


**Figure 1.** Tracking sine signal using PID control.

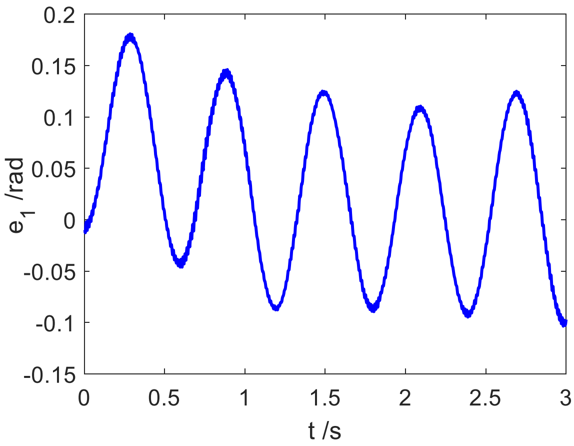


**Figure 2.** Error of tracking sine signal using PID control.

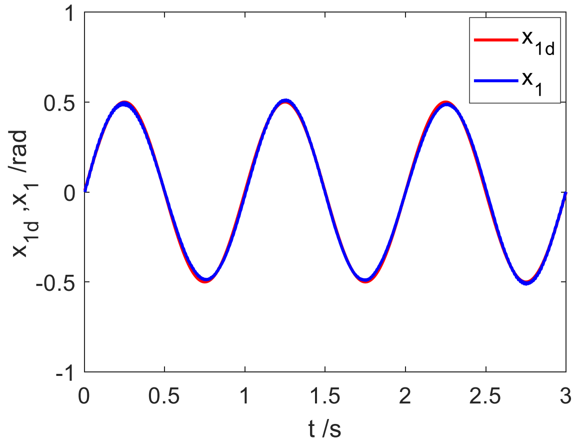


**Figure 3.** Tracking sine signal using RBFNNB.

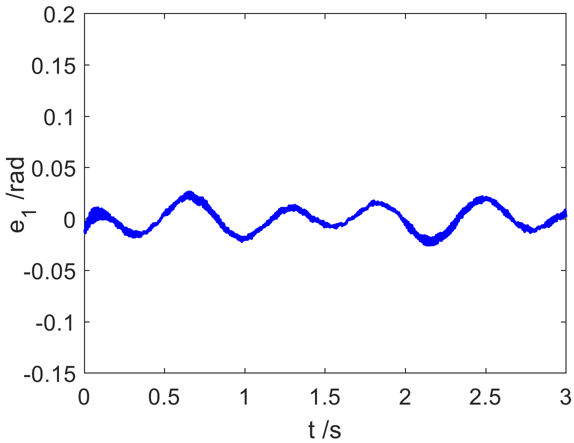


**Figure 4.** Error of tracking sine signal using RBFNNB.

## Conclusion

For a class of nonlinear mismatched systems, we proposed a new RBF NN-based backstepping adaptive control. We designed the adaptive controller combining RBF NN and backstepping control, and analyzed its stability. In simulation analysis, sine signal is chosen as input signal, system has larger tracking error and worse adaptive tracking ability when using PID control, but system has smaller tracking error and better adaptive tracking ability when using RBFNNB, validating the superiority of the proposed control strategy. However, the system studied in this paper is the single input and single output (SISO) system, there is also multiple input and multiple output (MIMO) system in practical engineering. In this paper, we do not consider MIMO system, and this will be the focus of our future work.

## Conflict of interest

The authors confirm that this article content has no conflict of interest.

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