Hydrodynamic aspects of moving vehicle with sloshing tanks

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Abstract. Freight vehicles with partially filled liquid tanks will be affected by liquid sloshing during transient motion. The sloshing induced by vehicle motion will lead to extra force on the vehicle and sometimes is a potential threat to safety. Previous studies on this problem usually use a mass-spring analogy to represent the sloshing effect of the liquid tank. Its main disadvantage is that CFD analysis or experimental study has to be performed beforehand, so that an equivalent mass-spring model can be constructed by curve fitting. In this paper, frequency domain boundary element method (BEM) and analytical solution to the sloshing problems are used to derive the modal equation and hydrodynamic parameters of the sloshing fluid. The accuracy of the results will be examined by comparison with available CFD results in the literature. The paper then evaluates the accuracy of equivalent mass-spring model and explores the possibility to approximate the sloshing effects inside cylindrical tanks by using analytical solution to the sloshing inside equivalent rectangular tanks.

1 Introduction

The sloshing in partially-filled liquid tank induced by transient freight vehicle motion will impose extra force and moment on the vehicle. If the external excitation meets the natural frequency of the liquid, large force and moment may occur, which has the potential to capsize the vehicle. Numerous studies have been done on identifying the natural frequencies of sloshing, and the force induced by it. McIver [1] presented an analytical solution to the horizontal cylindrical tank in the form of integral equation. Faltinsen [2] gave a simplified solution under shallow water assumption, and compared the difference between cylindrical and rectangular tank. Numerical simulations by boundary element method or CFD have also been carried out.

However, certain difficulties exist when trying to apply the results from fluid dynamic studies to the simulation of vehicle motion. For horizontal cylindrical tank with arbitrary filling levels, a simple analytical solution to its natural frequencies and sloshing forces is hard to find. Numerical approach applies, but is usually time-consuming. As a result, in many

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researchers the sloshing liquid is simplified into a linear mass-spring model [3-4] or a pendulum model [5] linked to the vehicle body. The disadvantage of this method is it requires numerical simulation results to calibrate the model.

The purpose of this paper is to propose a method that utilizes the efficient frequency-domain BEM method and the analytical solution to the sloshing problem to account for sloshing effects in moving vehicles. The outcome will be compared with the CFD results in previous literature to prove the accuracy of the method.

2 Hydrodynamic formulation

In linear potential theory, the motion of incompressible, inviscous and irrotational fluid is controlled by the following dominant equation:

$$\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0 \quad (1)$$

In which ϕ (y,z) is the velocity potential, a scalar function describing the velocity field in fluid domain:

$$v_y=\frac{\partial\phi}{\partial y}, v_z=\frac{\partial\phi}{\partial z} \quad (2)$$

The solution of the equation depends on the boundary conditions of the fluid domain. For liquid in tanks, the solid wall boundary condition (3) applies to the tank surface. On the mean water level, linearised kinematic (4) and dynamic free surface boundary conditions (5) are adopted:

$$\frac{\partial\phi}{\partial n}=v_n \text{ on } S_B \quad (3)$$

$$\frac{\partial\phi}{\partial n}=\frac{\partial\eta}{\partial t}\text{ on MWL} \quad (4)$$

$$\frac{\partial\phi}{\partial t}+g\eta=0 \text{ on MWL} \quad (5)$$

In addition, the fluid should obey mass conservation at all time, from which it can be known that the integration of the free surface elevation along the mean water level should be zero:

$$\int \eta dy=0 \text{ on MWL} \quad (6)$$

In which η(y,t) is the free surface elevation. Without going in details, the motion of the body under lateral excitation can be described by a modal equation as below:

$$\ddot{\beta}_i+\omega_i^2\beta_i=-\frac{\lambda_{2i}}{\mu_i}\ddot{\eta} \quad (7)$$

In which β is a generalized coordinate used to describe the motion of the liquid; η is the external excitation in lateral direction; μ and λ are hydrodynamic parameters in the following form:

$$\mu_i=\frac{\rho g}{\omega_n^2}\int_{-r}^{r} f_i^2\, dy \quad (8)$$

$$\lambda_{2i}=\rho\int_{-r}^{r} f_i y dy \quad (9)$$

i=1,2,...,m is the sloshing mode. The resultant force and moment induced by first sloshing mode is expressed as:

$$F(t)=-M\ddot{\eta}_2-\lambda_{21}\ddot{\beta}_1 \quad (10)$$

$$M(t)=Mz_c\ddot{\eta}_2-\left(g\lambda_{21}\beta_1+\lambda_{011}\ddot{\beta}_1\right) \quad (11)$$

$$\lambda_{01i}=\rho\int_{-r}^{r} f_i\Omega_{01i}dy \quad (12)$$

λ_{011} is the sway-roll coupling term. In particular, if the prescribed motion of the tank is harmonic:

$$\eta_i = A_i \sin(\omega t + \varphi_i) \tag{13}$$

Then the steady-state solution to the modal equation β_i (12) is also harmonic, and the sloshing force can be expressed as:

$$F = -\big(M_l + A_{22}(\omega)\big)\ddot{\eta}_2 \tag{14}$$

In which

$$A_{22}(\omega) = \frac{\lambda_{21}^2}{\mu_1}\frac{\omega^2}{\omega_n^2 - \omega^2} \tag{15}$$

Similarly, the sway-roll coupling term can also be written as A_{24}. As a result, the sloshing terms in equation (14) will have the same form as the first inertia item. It shows that under harmonic excitation, the effect of liquid sloshing is the same as adding an extra mass to the tank. The value of this mass is dependent on excitation frequency, and can be either positive or negative. In this case, an independent modal equation in the form of (7) will no longer be needed, and the sloshing effect can be described by adding a frequency dependent mass item to the original mass of the tank. A similar but slightly more complicated derivation of added mass also applied to the sloshing moment in equation (11). This item A_{ij} is called added mass, and will be used in the studies in the next chapter.

Equation (10) and (11) show the components of the sloshing force and moment. For sloshing force, the first term is the inertia of the liquid mass, while the second term is dependent on hydrodynamic parameter λ_{21}, which is solely dependent on free surface shape. For sloshing moment, it has a similar inertia term, then the second term decided by sloshing, and the last term jointly decided by coupling between translational and rotational degree of freedom.

The following figure 1. shows the tank-fixed coordinate system and the conventions used to describe the 6 DOF rigid motion of the tank. In equation (10) and (11), the subscripts η_2 is the lateral motion along y axis.

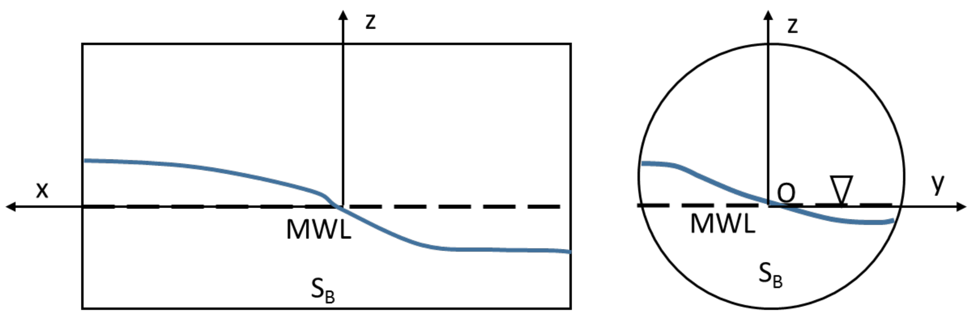


Figure 1. Convention Used for Sloshing Problem

3 Frequency domain numerical simulation of steady-state sloshing

For arbitrary tank geometry, it is hard to get an analytical solution to the free surface elevation and the sloshing velocity potential. In most cases, numerical method has to be applied. It means that the velocity potential has to be solved at every time step. To avoid the time consuming time domain simulation, a widely used approach in offshore engineering is to consider the excitation force and tank motion as the superposition of a series of steady-state harmonic components. For every frequency, there'll be a solution in the form of frequency-dependent added mass as in equation (14). After the calculation of frequency domain response function, the time domain sloshing motion and force is derived by inversed fourier transform.

The frequency domain response of sloshing is solved by boundary element method (BEM) program. The program solves the boundary value problem by utilising Green's second formula. By defining a fundamental solution to the Laplace equation, the problem of solving velocity potential in the fluid domain is converted to calculating the boundary integral along the body surface [6]. BEM method has a significant advantage in computing efficiency since it only requires meshing the surface of the tank. After solving the velocity potential, the frequency-dependent added mass is derived by the pressure integration on the body surface. The work of this paper is completed by a commercial BEM program HydroStar.

$$A_{ij} = \int \varphi_{ij} n dS \tag{16}$$

4 Calculation of transient sloshing and benchmark study

To show the validity of the hydrodynamics approach, and to provide a basic procedure of problem solving, a benchmark study on a turning vehicle with a partially filled gasoline tank is made, based on the work in [3]. A vehicle starts turning from zero velocity, and induces sloshing in its transient period. The acceleration in full curve is 0.6 m/s^2. The tank is considered as with 2D dimension, with length 15.5m, radius 1.5m, filling level 50%. In this section we use direct hydrodynamic parameters to get the sloshing force and moment, and will compare our results with the spring-mass model on the accuracy. The model of the tank is shown below. x is along the longitudinal axis of the tank, y along horizontal axis, and z is at the mean water surface.

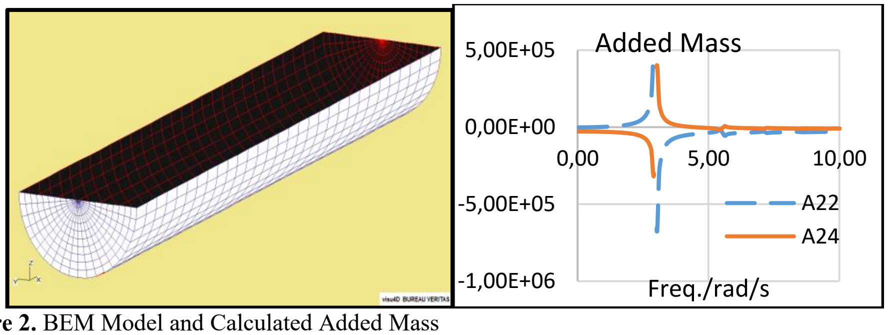


Figure 2. BEM Model and Calculated Added Mass

The results are shown below. The analytical solution to added mass in equation (17) and (18) suggests that added mass is dependent on excitation frequency. It will become infinite when the excitation frequency equals natural frequency, and will tend to a finite value at infinite excitation frequency. In numerical simulation, an infinite value is impossible to get. The added mass goes to a very large value near the natural frequency, causing a double peak in the diagram. The frequency at which the asymptotic line of this double peak lies is the natural frequency. It can be seen that in this case, the natural frequency is 2.97 rad/s, corresponding to a natural period of 2.12s, same as the CFD result in [3]. The mass is based on the 3D total mass of the liquid. For a 2D analysis, a sectional mass will be used. In addition, it can be seen that the first sloshing mode is the most prevailing mode. The second sloshing mode (near 5.5 rad/s) almost vanished. The prescribed motion of the vehicle is shown below. The transition period lasts for 3.07 s, and then the vehicle moves in full curve for 9.25 s. From t=9.25 s the vehicle decelerate linearly for 3.07 s and follows a straight trail after that. The acceleration can be described as:

$$a = \begin{cases} \frac{0.6}{3.07} * t, t < 3.07 \\ 0.6, 3.07 \leq t < 9.25 \\ -\frac{0.6}{3.07}(t - 12.32), 9.25 \leq t < 12.32 \end{cases} \quad (17)$$

In this case, we can see that the prescribed motion is a non-smooth function with a limited time length, which means that the motion cannot be treated as steady-state harmonic excitation. As a result, the added mass calculated by BEM is not directly applicable.

With motion known, the next requirement is hydrodynamic parameters to get sloshing force. But as shown above, the BEM software tends to solve coupled sloshing problem, in which the total sloshing potential is solved, without decomposing it. The sloshing force under prescribed motion expressed in section 1 does not directly relate to added mass. We still need to get the required parameters from the added mass results. The hydrodynamic parameters to be figured out are λ_{21}, and λ_{011}.

The added mass can be used to derive the relationship between these parameters. Let the excitation frequency go to infinity, we have:

$$A_{22}(\infty) = -\frac{\lambda_{21}^2}{\mu_1} \quad (18)$$

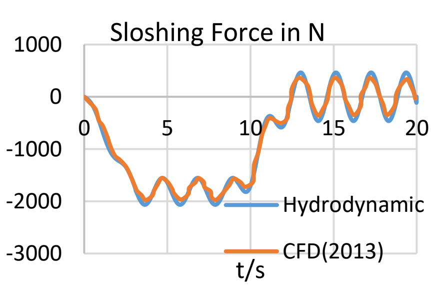
A_{24} can be expressed in a similar way. The hydrodynamic parameters are shown to be connected to added mass at infinite frequency. However, three unknowns exist when only two equations are provided by the calculation of added mass. The exact eigenfunction $f_{2,1}$ (also the modal shape of free surface during sloshing), does not have a complete analytical solution. Since one more equation is needed, we choose to use a trial function for free surface elevation $f_{2,1}$ to obtain an approximate solution.

The trial function we use for free surface elevation is cosine function, which is the eigenfunction of a rectangular tank:

$$f_{2,1} = \cos\left(\frac{\pi}{2r}(y + r)\right) \quad (19)$$

It is an odd function, which satisfies mass conservation boundary condition (6). Kinematic and dynamic free surface boundary conditions are not necessarily met. Together with the given added mass, we get an estimate on the three unknowns μ_2, λ_{21}, and λ_{011}. The modal equation and sloshing force can be calculated.

The calculated parameters, and the sloshing force and moment are plotted below. It can be seen that the force calculated is in good accordance with the CFD results presented in [3]. The periodical component of the moment has a slightly different amplitude. It is possibly because of the slow convergence of the added mass.



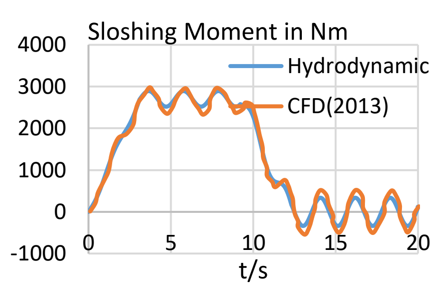


Figure 3. Comparison between Results from Hydrodynamics and CFD

Table 1. Calculated and Derived Parameters

Parameters	Value	
$A_{22\infty}$	-1835	kg
$A_{24\infty}$	-601	kg
λ_{21}	-1677	kg
μ_2	1538	kg
λ_{011}	-634	kgm
ω_1	2.97	rad/s

5. Conclusion

This paper aims at providing an interface between wave dynamics and vehicle dynamics, and confirmed the feasibility of the proposed method by a benchmark study. By applying linear potential flow theory, liquid sloshing is described by modal equations and hydrodynamic parameters. A trial function of free surface elevation is used in order to fill the gap between the results of steady-state sloshing calculation and the required inputs for the calculation of transient sloshing. The derived modal equation and hydrodynamic parameters can be solved together with the motion equation of freight vehicles to get the sloshing force and moment.

The advantage of this method is that it replaces the time-consuming CFD analysis required for calibration with a much more efficient BEM analysis, and can be easily expanded to 3D by adding one extra dimension to the boundary value problem. In addition, it is able to account for sloshing mechanisms including coupling effect between DoFs, additional moment induced by Stoke-Joukowski potential when there is external moment, and higher natural modes. However, it also inherits setbacks of linear potential flow theory: the viscosity effect of the liquid can only be simplified into a linear damping ratio, and nonlinear effect is not included. In the future, this method will be expanded for 6 DOF motion and applied for practical vehicle operations like braking, turning and travelling on uneven foundations.

References

[1] McIver, *Journal of Fluid Mechanics*, **201**, 243-257 (1989)

[2] O.M. Faltinsen, A.N. Timokha, *Sloshing*, Cambridge University Press, (2009)

[3] E.D. Gialleonardo, A. Premoli, S. Gallazzi, S. Bruni, *Vehicle System Dynamics*, **51**(10), 1640-1654 (2013)

[4] C. Vera, J. Paulin, B. Suarez, M. Gutierrez, Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, **219** (4), 245 – 259 (2005)

[5] S. Aliabadi, A. Johnson, J. Abedi, Computers & Fluids, **32**, 535-545 (2003)

[6] S. Malenica, M. Zalar, X.B. Chen, *Proceedings of the International Offshore and Polar Engineering Conference*, 1831-1837 (2003)