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Seismic vibration control using a novel inerto-elastic damper

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Abstract. The use of advanced structural control devices is an effective engineering solution to reduce earthquake induced damages to structures. Owing to rapid advancement in technology and persistent research efforts, a variety of control devices have been developed and successfully implemented. Quite recently, a new passive damper, called inerter has been introduced, which is capable of developing a fictive mass. This study presents a novel inerto-elastic damper, which combines the inerter devices with classical elastic springs, and demonstrates the effectiveness of these devices in achieving seismic response reduction. The inerto-elastic device employs the inerter and elastic spring in parallel to control the seismic structural response. The effectiveness of the inerto-elastic dampers has been demonstrated through the response of a multi-degree of freedom system subjected to seismic excitations. The results of the analysis show a significant reduction in the response of the structure with novel inerto-elastic damper, as compared to those of structures with normal elastic spring as well as no dampers. The response quantities of interest, considered for this study are top floor displacement, inter-storey drift and base shear. The study also underlines optimal parameters for the inerter fictive mass and the elastic spring stiffness on the basis of the results obtained.

1 Introduction

One of the primary challenges for civil engineers is the safety of structures against dynamic lateral loading, such as wind, blast or earthquake. Significant efforts have been made to enhance the dynamic performance of structures using control devices. The area of structural control can be summarised in the conception, design, development and implementation of vibration suppression devices. These control devices may be passive such viscous fluid dampers [1], active such us active tuned mass damper [2, 3] or semi-active such us magneto-rheological damper [4-6].

One of the most recent developments in the area of structural control is a type of rotational inertia damper, called *Inerter*. This new type of damper can be described as a two-terminal massless device that generates resisting force proportional to the acceleration between its nodes [7]. In civil engineering, the inerter was introduced as a fictive mass that

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can be added to a classical tuned mass damper to enhance its mass, thereby improving its performance; or that can replace the tuned mass damper mass by a tuned inerter damper (TID). The performances of these systems were studied by Lazar, Neild [8] for multi-degrees of freedom systems subjected to both, sinusoidal and seismic excitations. It was concluded that the optimal location for an inerter based tuned mass damper (TMDI) or a tuned inerter damper (TID) is at the bottom of the structure. Marian and Giaralis [9] presented an optimal design for inerter based tuned mass damper (TMDI), that aims at suppressing the oscillatory motion of stochastically excited support. The damping system was designed for a single degree of freedom system equipped with a TMDI,

Beside the use of a particular control device, a combination between two or more control devices can be accomplished to form a hybrid control strategy. This combination approach may be used to compensate the insufficiency of one control device by combining it to another [10]. Literature suggests that several hybrid systems have been developed to enhance the performance of controlled systems. One of the most widespread hybrid systems is the combination of two passive systems (passive-passive). These control devices may be connected in series, as well as in parallel to each other, to yield efficient control results.

In this research, a hybrid control device is formed by combining an inertial rotational damper and an elastic damper. The effectiveness of the proposed hybrid device in seismic response reduction is investigated. Beside the seismic response reduction the optimal parameters of the inerter and the elastic spring are presented.

2 Mathematical model of the novel control device

2.1 Mathematical formulation of an inerter

A schematic representation of the most basic model of the inerter is shown in the Fig. 1 below. The inerter comprises of a rack and pinion arrangement rotating about a shaft, represented by the point O. The two extremities of the inerter, represented by a and b in Fig. 1, are connected to the pinion (of radius ρ) through racks and are responsible for reversing the direction of reactions (represented by $F(t)$) at both ends. A flywheel having a radius R and mass m is mounted on the other end of the shaft, to reduce the impact through its higher inertia.

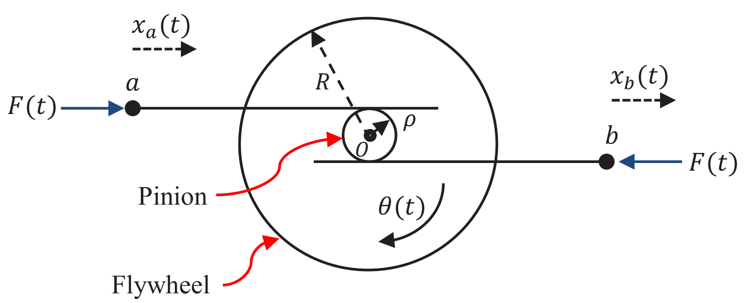

Fig. 1. Simplified Rheological model of an inerter device

The rotation of the flywheel can be written as:

$$\theta(t) = \frac{x_a(t) - x_b(t)}{\rho} \tag{1}$$

The displacements at the extremities of the inerter element are expressed by x_a and x_b. The dynamic moment equilibrium about the point O can be represented by

$$I_O \ddot{\theta}(t) = F(t)\rho \tag{2}$$

Thus, the moment of inertia of the flywheel rotating about point O will be $I_O = \frac{1}{2}.m.R^2$
Substitution of Eq. (1) and Eq. (2) results in the mechanical linear equation of the inerter

$$F(t) = \frac{1}{2} m \frac{R^2}{\rho^2} [\ddot{x}_a(t) - \ddot{x}_b(t)] \tag{3}$$

It can be seen from Eq. (3) that the force generated by the inerter is related to the relative acceleration between it extremities. The term $\frac{1}{2} m \frac{R^2}{\rho^2}$ is denoted as the inertence b of the device, and is expressed in unit of mass (kg). The inertence of the device can be enhanced by serially increasing the number of pinions. Hence, the general equation of inerter become

$$F(t) = b[\ddot{x}_a(t) - \ddot{x}_b(t)] \tag{4}$$

For instance, if a single pinion inerter can produce an inertence of 1440 kg for a device that weights 10 kg, then adding three pinions to the same device will results in an inertence of 116640 kg.

2.2 Mathematical formulation of the inerto-elastic damper

The proposed inerto-elastic device is based on a combination of an inerter and an elastic spring assembled in parallel as shown in Fig. 2. This assembly aims at combining the effects of the fictive mass, induced by the inerter and an additional stiffness, induced by the elastic damper. The resultant hybrid device modifies multiple parameters of the controlled system.

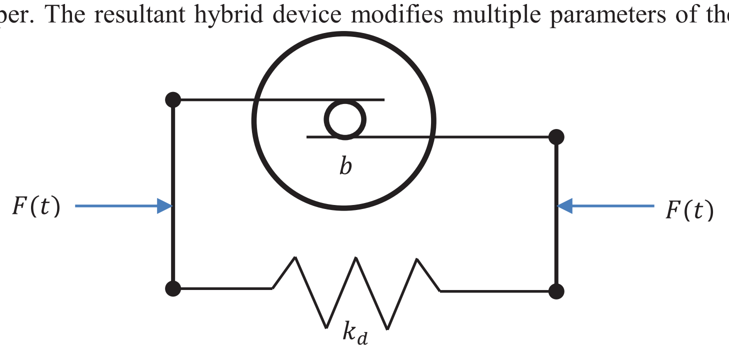

Fig. 2. Rheological model of an inerto-elastic device

The equation of a linear elastic spring which has a constant k_d can be written as

$$F(t) = k_d[x_a(t) - x_b(t)] \tag{5}$$

As it can be seen for Fig.2 the proposed device is formed by superposing an inerter and a spring in parallel. This configuration will lead to the addition of the right hand side terms of Eq. (4) and Eq. (5) which results in the expression of the total force generated by the inerto-elastic device expressed in Eq. (6)

$$F(t) = b[\ddot{x}_a(t) - \ddot{x}_b(t)] + k_d[x_a(t) - x_b(t)] \tag{6}$$

3 Equation of motion of the controlled system

A three-story benchmark building [11], configured with an inerto-elastic damper fixed at the first level, is considered for this study (Fig. 3). The system uses a simple benchmark model of the scaled three-story test structure, which has been used previously at Structural

Dynamics and Control Earthquake Engineering Laboratory (SDC/EEL) at the University of Notre Dame. The inerto-elastic damper is rigidly connected between the base and the first floor of the structure.

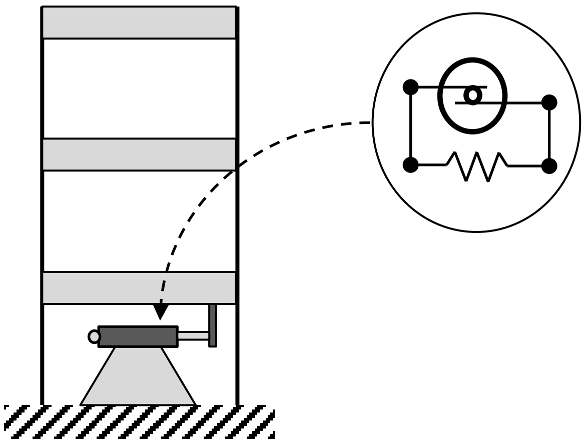

Fig.3. Three storey structure model equipped with an inerto-elastic damper

The equation of motion of the structure equipped with a control device is

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \Gamma\{F\} - [M]\{\Lambda\}\{\ddot{x}_g\} \quad (7)$$

where, $[M], [C]$ and $[K]$ are the overall mass, damping and stiffness matrices, while $\{\ddot{x}\}, \{\dot{x}\}$ and $\{x\}$ are the system acceleration, velocity and displacement vectors, respectively. The damper location vector Γ is as follows

$$\Gamma = \begin{bmatrix}1 & 0 & 0\end{bmatrix}^T \quad (8)$$

while, the ground acceleration force distribution vector is expressed as

$$\Lambda = \begin{bmatrix}1 & 1 & 1\end{bmatrix}^T \quad (9)$$

The state space representation of Eq. (7) can thus be written as:

$$\dot{z} = Az + Bu \quad (10)$$

$$y = \nabla z + Du \quad (11)$$

where,

$$A = \begin{bmatrix}-M^{-1}C & -M^{-1}K \\ E & 0\end{bmatrix}, \qquad B = \begin{bmatrix}M^{-1}\Gamma & -E \\ 0 & 0\end{bmatrix}, \qquad \nabla = [E], \qquad D = [0]$$

and, $[E]$ and $[0]$ are identity and zeros matrices of convenient sizes, respectively. Also, the vectors z and u in this case are:

$$z = \begin{bmatrix}\dot{x} \\ x\end{bmatrix}, \qquad u = \begin{bmatrix}f \\ \ddot{x}_g\end{bmatrix}$$

4 Numerical study and results

The benchmark structure with inerto-elastic damper has been subjected to under El Centro earthquake. However, the earthquake excitation data has been enhanced by five times to increase the effectiveness of the results, as the considered structure is a scaled down model of actual structure. The response parameters of interest, considered for this study are the top floor displacement (D_{max}), inter-storey drift (Δ_{max}) and maximum base shear (BS_{max}).

The damper parameters considered for this study are the inerter fictive mass b, and the elastic spring stiffness coefficient, k_d. In order to determine the optimal parameters, the inertence and damper stiffness are varied simultaneously, and the variance is expressed in percentage of the total building mass for the inertence and the floor stiffness for the elastic spring coefficient. The variance covers a range of percentages, from 2.5% to 50%. The optimal response as observed for top floor maximum displacement is shown in Fig. 4. From Fig. 4 it can be observed that the maximum response reduction is obtained for relatively high values of inertence and stiffness. Out of these results, optimal parameters for the inertence and stiffness are selected which are 50% of the total structure mass for the inertence b and 47.5% of the storey stiffness for the elastic spring parameter k_d .

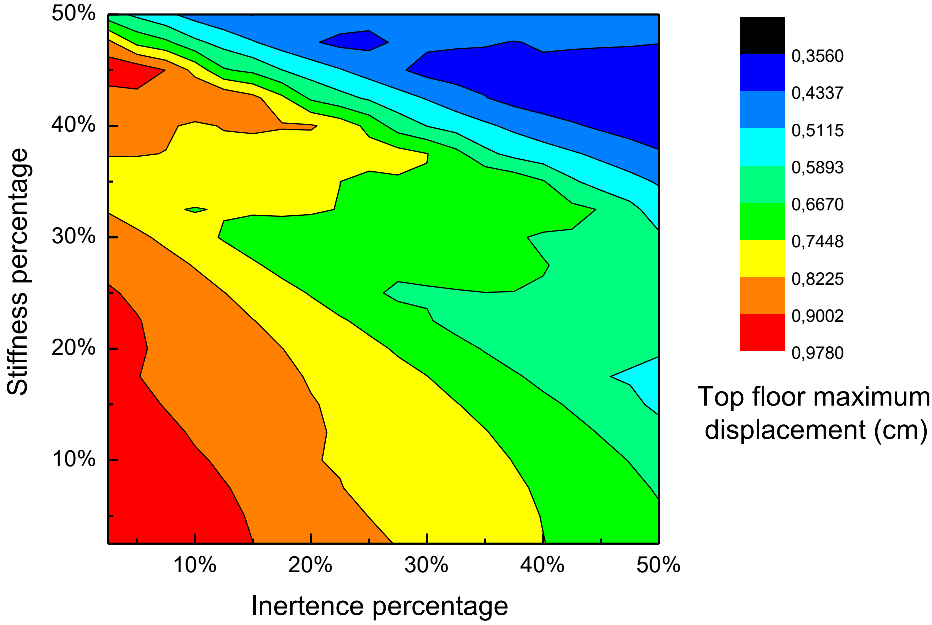

Fig.4. Top floor maximum displacement for different values of inertence and damper stiffness coefficient

The Table 1 below indicates the maximum top floor displacement of the uncontrolled structure, and the same is compared with the respective responses using different control strategies, namely, structure with an elastic spring only, structure with an inerter only and structure with an inerto-elastic damper with aforementioned optimal parameters.

Table 1. Maximum top floor displacement under different control strategies

Dynamic parameter	Uncontrolled	Controlled		
		Elastic spring $k_d = 0.5 \times k_{floor}$	*Inerter damper* $b = 0.5 \times m_{total}$	*Inerto-elastic Optimal coeiff.*
D_{max} (cm)	0.9712	0.6937	0.7048	0.3566

Form Table 1, it is clear that the performance of inerto-elastic damper, in terms of response reduction, surpasses the classical elastic spring and inerter. The combination of the two devices offers a substantial reduction (62.3%) in top floor displacement.

Table 2 presents a comparison of the maximum inter-storey drift for the uncontrolled and the controlled structure.

Table 2. Maximum inter-storey drift under different control strategies

Dynamic parameter	Storey N°	Uncontrolled	Controlled		
			Elastic damper $k_d = 0.5\times k_{floor}$	*Inerter damper* $b = 0.5\times m_{total}$	*Inerto-elastic optimal coeiff.*
Δ_{max} (cm)	1	0.5469	0.5264	0.4312	0.2847
	2	0.3167	0.1205	0.1780	0.0796
	3	0.3167	0.0651	0.0996	0.0626

The results show that maximum inter-storey drift is significantly reduced in case of the structure controlled using an inerto-elastic damper, as compared to other control strategies, the percentage reductions for the 1st, 2nd and 3rd storey are 47.9%, 74.8% and 80.3% respectively with respect to the uncontrolled structure.
The maximum base shear of the structure for various control cases are presented in Table 3.

Table 3. Maximum base shear under different control strategies

Dynamic parameter	Uncontrolled	Controlled		
		Elastic spring $k_d = 0.5\times k_{floor}$	*Inerter damper* $b = 0.5\times m_{total}$	*Inerto-elastic optimal coeiff.*
BS_{max} (N)	8.35×10^5	3.38×10^5	5.35×10^5	1.95×10^5

Results indicate that maximum base shear reduction (76.6% as compared to uncontrolled structure) is obtained when the inerto-elastic damper is used to control the response.

5 Conclusions

A novel inerto-elastic damper used for seismic response reduction is presented, the device is based on the combination of an inerter and an elastic spring connected in parallel. The effectiveness of the proposed damper is tested on a three storey benchmark building model. The obtained responses are compared with those using other classical control devices (i.e. Elastic spring and Inerter). Following are the broad conclusions from the study:

- Combining an inerter with an elastic spring in parallel results in very efficient seismic control device, herein called inerto-elastic damper.
- In general, the efficiency of the inerto-elastic damper is closely related to high inertence ratio and stiffness coefficient.
- A comparison between inerto-elastic damper and classical passive damper (inerter, elastic spring) indicates that inerto-elastic damper is more effective.
- The efficiency of the inerto-elastic damper is demonstrated for three dynamic parameters, namely top floor displacement, inter-storey drift and base shear.
- As perspective, the work accomplished in this study can be completed by experimental testing on scaled or real models.

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