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# Fundamental study on countermeasures against subharmonic vibration of order 1/2 in automatic transmissions for cars

*Fumiya* Takino[1], *Takahiro* Ryu[2,*], *Takashi* Nakae[2], *Kenichiro* Matsuzaki[3], and *Risa* Ueno[1]

[1] Graduate School of Engineering, Oita University, 700 Dannoharu, Oita-shi, Oita, Japan
[2] Department of Mechanical and Energy Systems Engineering, Oita University, 700 Dannoharu, Oita-shi, Oita, Japan
[3] Department of Mechanical Engineering, Kagoshima University, 1-21-40 Korimoto, Kagoshima-shi, Kagoshima, Japan

**Abstract.** In automatic transmissions for cars, a damper is installed in the lock-up clutch to absorb torsional vibrations caused by combustion in the engine. Although a damper with low stiffness reduces the torsional vibration, low-stiffness springs are difficult to use because of space limitations. To address this problem, dampers have been designed using a piecewise-linear spring having three different stages of stiffness. However, a nonlinear subharmonic vibration of order 1/2 occurs because of the nonlinearity of the piecewise-linear spring in the damper. In this study, we experimentally and analytically examined a countermeasure against the subharmonic vibration by increasing the stages of the piecewise-linear spring using the one-degree-of-freedom system model. We found that the gap between the switching points of the piecewise-linear spring was the key to vibration reduction. The experimental results agreed with results of the numerical analyses.

## 1 Introduction

Engines used in automobiles generate strong torsional vibrations in the powertrain. To address this problem, a torsional damper is installed in the automatic transmission lock-up clutch. Although a damper with low stiffness reduces the torsional vibration, low-stiffness springs are difficult to use because of space limitations. Dampers with piecewise-linear spring characteristics having three different stages of stiffness are used to overcome this problem. This type of spring can realize a wide range of restoring torque characteristics in a small space. However, a nonlinear subharmonic vibration of order 1/2 (referred to hereafter as the subharmonic vibration) occurs because of the nonlinearity of the piecewise-linear damper spring. In this vibration, the main frequency component is half the excitation frequency. In previous studies, we analytically clarified the mechanism that produces subharmonic vibration in automatic transmissions and performed experiments and

* Corresponding author: ryu@oita-u.ac.jp

numerical analyses using a simple one-degree-of-freedom system with the piecewise-linear spring [1-3]. Although a number of studies [4-6] have examined nonlinear vibrations in mechanical systems having piecewise-linear spring properties, countermeasures against the subharmonic vibration have not yet been studied. In this study, we experimentally and analytically examined a countermeasure against the subharmonic vibration by increasing the stages of the piecewise-linear spring using the one-degree-of-freedom system model. The effects of the gap between the switching points of the piecewise-linear spring were evaluated. The results of the experiments and the numerical analyses were found to be in good agreement.

## 2 Subharmonic vibration in vehicles

Figure 1 shows a schematic diagram of a torque converter. The thick line shows the torque flow through the converter when the lock-up clutch is engaged. During lock-up, torque from the engine is transmitted to the gear train through the converter cover, the lock-up clutch, and the damper. A damper with a three-stage piecewise-linear spring is used in the actual vehicle. The subharmonic vibration occurs only near the second switching point of the piecewise-linear spring. Figure 2 shows the waveform at the pump impeller and turbine runner when the subharmonic vibration occurs. The frequency of the turbine runner is half of the pump impeller frequency, which characterizes the subharmonic vibration.

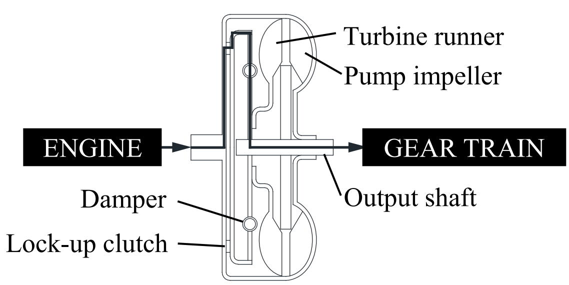


**Fig. 1.** Schematic diagram of torque converter

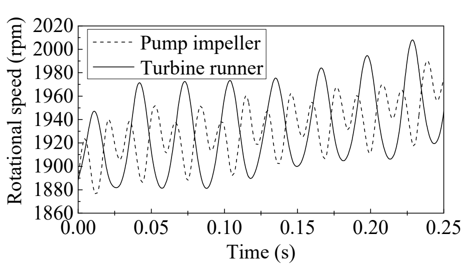


**Fig. 2.** Waveform of subharmonic vibration during vehicle operation

## 3 Laboratory experiments

### 3.1 Experimental setup

The subharmonic vibration occurred only near the second switching point in the actual vehicle. The suppression of the subharmonic vibration in one-degree-of-freedom system with two-stage spring which simulate the second switching point in the actual vehicle is the target of the experiment. We performed experiments to evaluate the countermeasure for suppressing the subharmonic vibration by increasing the number of stages at the switching point of the spring. Figures 3 and 4 show the experimental setup and its analytical model, respectively. The mass $m$ was supported by leaf spring A with spring constant $K_1$. A damper with damping coefficient $C$ was added by attaching a sponge to leaf spring A. As a countermeasure against the subharmonic vibration, the number of stages of the spring was increased from two to three. In the experiment with the three-stage spring, leaf springs B and C with spring constants $K_2$ and $K_3$, respectively, were placed to provide the second and third stages of the spring restoring force characteristics. The mass was also connected to leaf spring D with spring constant $k$, which was connected to the exciter. The vibration amplitude of the exciter was $u = a\cos\omega t$ . Figure 5 shows the restoring force characteristics for two- and three-stage springs, which simulate the second switching point in the actual vehicle. In the experiment with the two-stage spring, leaf spring C was removed, the stiffness constant of leaf spring B was changed, and origin of displacement $x$ was set to P. In the experiment with the three-stage spring, the two switching points were P and Q. The initial gap between leaf springs B and C was $D$, and the attachment positions of leaf springs B and C were moved to change the equilibrium point for the piecewise-linear spring. The equilibrium point for the three-stage spring is R, and the gap between leaf spring C and the

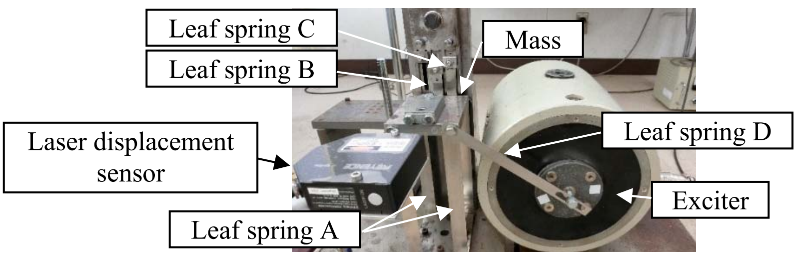


**Fig. 3.** Experimental setup

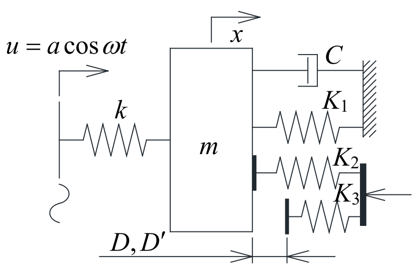


**Fig. 4.** Analytical model of experimental setup

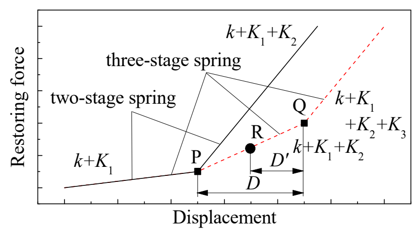


**Fig. 5.** Restoring force characteristics

**Table 1.** Standard parameters and values

| Parameter | Two-stage spring | Three-stage spring |
|---|---|---|
| $m$ (kg) | 0.75 | |
| $k$ (N/m) | 440 | |
| $K_1$ (N/m) | 850 | |
| $K_2$ (N/m) | 1,290 | 534 |
| $K_3$ (N/m) | 0 | 756 |
| $C$ (N·s/m) | 4.5 | |

mass is $D'$. Table 1 shows the values for these standard parameters. The excitation frequency $f$ (= $\omega/2\pi$) was varied from 14 to 18 Hz. Because of the limited performance of the exciter amplifier, the amplitude of excitation $a$ varied almost linearly from 2.8 mm at 14 Hz to 2.4 mm at 18 Hz. Displacement of the mass was measured by a laser displacement sensor.

### 3.2 Occurrence of subharmonic vibration

A two-stage spring was used to reproduce the subharmonic vibration in the experimental setup. The stiffness ratio of the first to second stage $\gamma = (K_1 + K_2 + k)/(K_1 + k)$ was 2. Generally, subharmonic vibrations occur easily when the stiffness ratio $\gamma$ is larger than 2 in an actual machine [1]. The natural frequencies of the first and the second stages were 6.6 Hz and 9.3 Hz, respectively.

Figure 6 shows the frequency response curve for the two-stage system. The abscissa represents the excitation frequency of the exciter and the ordinate represents the peak-to-peak (P-P) amplitude of the mass displacement. Figure 7 shows the frequency analysis at point X in Fig. 6. The excitation frequency $f$ was 15.3 Hz, and the fundamental frequency was half of the excitation frequency. This is the characteristic of the subharmonic vibration of order 1/2. The fundamental frequency of the subharmonic vibration was located between the first and second natural frequencies.

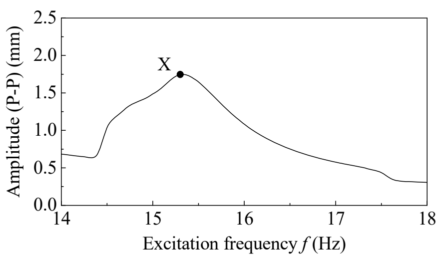


**Fig. 6.** Experimental frequency response curve of two-stage spring

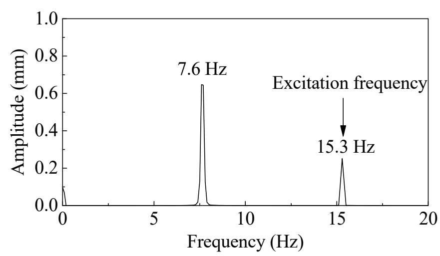


**Fig. 7.** Frequency analysis at point X in Fig. 6

### 3.3 Countermeasure against subharmonic vibration by multi-stage spring

A larger spring constant ratio can contribute to the occurrence of subharmonic vibration. As a countermeasure against this, a three-stage spring was used. The stiffness ratios at points P and Q were $\gamma_1 = (K_1 + K_2 + k)/(K_1 + k)$ and $\gamma_2 = (K_1 + K_2 + K_3 + k)/(K_1 + K_2 + k)$,

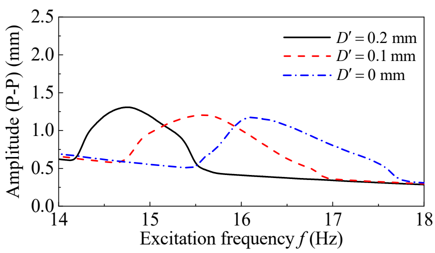


**Fig. 8.** Experimental frequency response curves for $D = 0.2$ mm

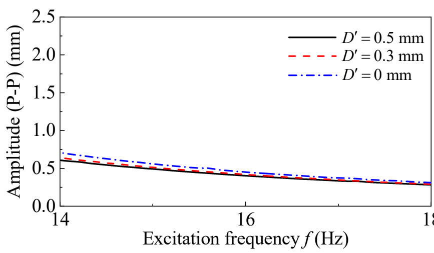


**Fig. 9.** Experimental frequency response curves for $D = 0.5$ mm

respectively, and they were set to the same smaller value $\gamma_1 = \gamma_2 = \sqrt{2}$ rather than the $\gamma$ value used for the two-stage spring. The stiffness ratio between the third and first stages, $\gamma_1\gamma_2$, was equal to $\gamma$ for the two-stage spring. In the experiments, the effect of the initial gap $D$ on subharmonic vibration was examined.

Figure 8 shows the frequency response curves for $D$ = 0.2 mm. The gap $D'$ was changed to 0.2 mm, 0.1 mm, and 0 mm. In every case, subharmonic vibrations occurred. Figure 9 shows the frequency response curves for $D$ = 0.5 mm. The gap $D'$ was changed to 0.5 mm, 0.3 mm, and 0 mm. The subharmonic vibration did not occur in any of these cases. Thus, we found that the subharmonic vibrations can be suppressed by slightly increasing the space between the switching points.

## 4 Numerical analysis

### 4.1 Analytical model

We conducted numerical analysis using the analytical model shown in Fig. 4. The equation of motion for this model is

$$m\ddot{x} + C\dot{x} + \left(K_1 + k + K\right)x = ka\cos\omega t\ , \tag{1}$$

where $x$ is the displacement from the equilibrium point. The spring coefficient $K$ for the two-stage spring is given by

$$\begin{cases} K = 0 : x < 0 \\ K = K_2 : x \geq 0 \end{cases}, \tag{2}$$

and that for the three-stage spring is given by

$$\begin{cases} K = 0 : x < -\left(D - D'\right) \\ K = K_2 : -\left(D - D'\right) \leq x \leq D' \\ K = K_2 + K_3 : D' < x \end{cases}. \tag{3}$$

Equations (1), (2), and (3) were used to calculate the nonlinear vibration with the shooting method. In the numerical integration used in the shooting method, the time when the displacement is at the switching point of the piecewise-linear spring was calculated precisely using the Newton-Raphson method.

### 4.2 Result of numerical analysis

The same parameters from the experiments were used in the numerical analysis. For simplicity, the amplitude of the exciter $a$ was set to a constant 2.7 mm, which is equal to the amplitude at point X in Fig. 6. Figure 10 shows frequency response curves for the two-stage spring. Thick and thin lines show the stable and the unstable solutions, respectively. The subharmonic vibration occurred around $f$ = 15 Hz. This result agrees well with the experimental result shown in Fig. 6.

Figure 11 shows the frequency response curves for $D$ = 0.2 mm for the three-stage spring. The thick solid, dashed, and dashed-dotted lines show the results when the gap $D'$ was changed to 0.2 mm, 0.1 mm, and 0 mm, respectively. The thin lines show the unstable solutions. In this case, the amplitude became smaller than that of the two-stage spring (Fig. 10), but the subharmonic vibration was not suppressed perfectly.

Figure 12 shows the frequency response curves for $D$ = 0.5 mm. The gap $D'$ was changed to 0.5 mm, 0.3 mm, and 0 mm. The subharmonic vibrations were suppressed perfectly in every case. Thus, the effectiveness of the countermeasure using a multi-stage spring was confirmed in both the experiments and the numerical analysis. Normally, multiple springs are used to realize the spring constant for each stage. If the timing of contact for each spring is changed, subharmonic vibration will be suppressed in the actual vehicle.

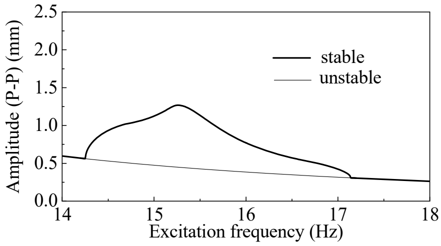


**Fig. 10.** Analytical frequency response curves for two-stage spring

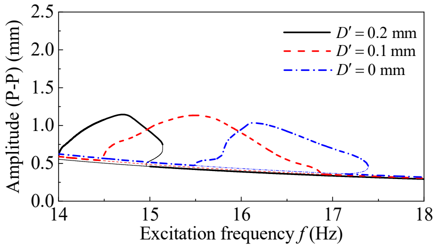


**Fig 11.** Analytical frequency response curves for $D$ = 0.2 mm

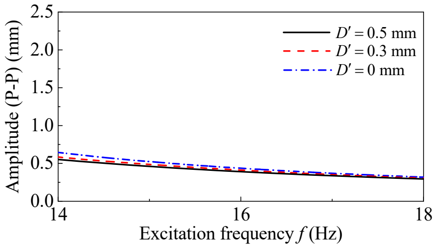


**Fig 12.** Analytical frequency response curves for $D$ = 0.5 mm

## 5 Conclusions

In this paper, a piecewise-linear spring was used as a countermeasure against subharmonic vibration in an automotive drive train. Its configuration was evaluated using a one-degree-of-freedom system numerically and experimentally. We found that (1) by increasing the gap of the switching point between stages, the subharmonic vibration could be completely suppressed, and (2) the results of the numerical analysis agreed well with the experimental results.

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