Vibration analysis of footbridges: an overview of the current practice

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Abstract. The design of a footbridge is typically controlled by serviceability criteria. The dynamic forces induced by pedestrians may result in high-amplitude vibrations that the users may feel as uncomfortable, or unsafe. As a consequence of the several cases of footbridge vibration problems that occurred over the last two decades, research has been conducted to improve design criteria that account for pedestrian loading, traffic density, and comfort levels. To date, there are no accepted design guidelines in the United States to assess vibration levels in footbridges. This paper offers a general overview of the current practice for the vibration analysis of footbridges.

1 Background

According to the AASHTO LRFD Guide Specifications for the Design of Pedestrian Bridges [1], pedestrian-induced vertical vibrations are negligible when the fundamental frequency in a vertical mode of the bridge is greater than 3.0 Hz. Transverse vibrations are considered negligible when the fundamental frequency in a transverse mode is greater than 1.3 Hz. When these limits are not satisfied, AASHTO recommends performing a dynamic analysis following European Standards (namely, SETRA [2]) for analysis methods, loading scenarios, and acceleration acceptance criteria. In this paper, the authors highlights the critical steps to follow for assessing the performance of a footbridge during the design phase.

2 Footbridge usage and traffic classes

The first item to be addressed in the design of footbridges is the definition of its usage and the relative expected traffic levels. In fact, footbridges in highly populated areas are crossed more often and have a higher probability of experiencing sparse crowd loading. The definition of the pedestrian traffic levels is generally a decision of the Owner/Developer with input from the Design Team. The UK National Annex to Eurocode 1 [3] defines four traffic classes as a function of the footbridge usage, as shown in Table 1.

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Table 1. Footbridge usage and traffic classes

Traffic Class	Footbridge Usage
I	Rural locations seldom used and in sparsely populated areas
II	Suburban locations likely to experience slight variations in pedestrian loading intensity on an occasional basis
III	Urban routes subject to significant variation in daily usage (e.g. structures serving as access to offices or schools)
IV	Primary access to major public assembly facilities such as sports stadia or major public transportation facilities

3 Dynamic analysis

It is necessary to know the dynamic properties of the bridge deck to evaluate its response to pedestrian excitations. The dynamic properties of the structure depend on its mass, its stiffness, and the structural damping. Considerations should be made to determine whether the stiffness of the foundation system has any impact on the global dynamic behavior of the structure. The structure’s dynamics may be determined with numerical simulation using commercially available software. However, in many cases, the structure may be simplified as a beam with uniformly distributed mass. In these cases, close-form solutions are available to determine the theoretical dynamic properties. The natural frequencies and the characteristic mode shapes of simple beams are summarized in Table 2 for typical boundary conditions. The reader should refer to Biggs [4] for a detailed discussion. The pinned-pinned condition applies to the vast majority of footbridge design. However, the fixed-fixed or fixed-pinned solutions may be applicable to approximate spans of continuous decks on multiple supports.

Table 2. Natural frequencies and modes shapes of simple beams with various end conditions

End restraint conditions	Natural angular frequency (rad/sec)	Mode shape
Pinned-Pinned	$\omega_n = \frac{n^2 \pi^2}{l^2} \sqrt{\frac{EI}{m}}$	$\varphi_n(x) = \sin\left(\frac{n\pi x}{l}\right)$
Fixed-Fixed	$\omega_n = \frac{(n + 1/2)^2 \pi^2}{l^2} \sqrt{\frac{EI}{m}}$	$\varphi_n(x) = C_n(\sinh(a_n x) - \sin(a_n x)) + \cosh(a_n x) - \cos(a_n x)$ with $C_n = \frac{\cos(a_n x) - \cosh(a_n x)}{\sinh(a_n x) - \sin(a_n x)}$
Fixed-Pinned	$\omega_n = \frac{(n + 1/4)^2 \pi^2}{l^2} \sqrt{\frac{EI}{m}}$	

3.1 Mass

It is important to evaluate the mass of the footbridge components as accurately as possible. The weight of the bare structure, connection hardware, and architectural components on the bridge span should not be overestimated. On one end, overestimating the mass results in

mode frequencies that are smaller than the actual frequencies. On the other end, it underestimates the acceleration response of the bridge span due to pedestrian excitations.

The mass of the live load may be disregarded in the computation of the modal mass if it represents less than approximately 5% of the total bridge mass. For crowds analysis, when the crowd mass is a significant percent of the bridge mass, the live load mass should be included in the bridge modal mass.

3.2 Damping

Damping is a measure of the inherent capacity of a structure to dissipate energy. Damping helps reducing the structural response to a dynamic excitation near resonance and, therefore, should be estimated as accurately as possible. Structural damping can only be evaluated experimentally. For crowds analysis, it is recommended that the crowd-structure interaction is accounted for to determine the effects of crowd on structural damping.

There are several damping models in literature. The viscous damping model is the simplest one and, therefore, the one that is most often used. The usual way to express viscous damping is by defining a critical damping ratio. The recommended values of critical damping ratio for different deck systems that are listed in Table 3 are based on the recommendations found in SETRA [2]. The values shown are for bridges with no services, fixtures, or fittings. Bridges with many non-structural fixtures, or with full or partial enclosures will have significantly higher damping.

Table 3. Critical damping ratio values for various deck types

Type of Deck	Critical Damping Ratio	
	Minimum Value	Average Value
Reinforced Concrete	0.8%	1.3%
Prestressed Concrete	0.5%	1.0%
Steel	0.2%	0.4%
Composite	0.3%	0.6%
Timber	1.5%	3.0%

4 Loading scenarios

The effects of group walking, group running, and crowds should be evaluated. The group size and crowd density to be used for the dynamic evaluation of footbridges depend on the design traffic class. Recommended group sizes and crowd densities are listed in Table 4. They are based on the recommendations in the UK National Annex to Eurocode 1 [3]. The values shown are for a 25% chance of exceedance.

5 Pedestrian Loading

For dynamic analysis, the pedestrian loading is simplified as a Fourier series of component sine waves. Only the first four harmonics are considered. The amplitude of each harmonic component is proportional to the walker body weight that participates to the footfall motion

and accounts for the dynamic effects. The pedestrian weight is commonly assumed ranging between 156 lbf and 167 lbf. Other values may be used where appropriate.

Table 4. Loading scenarios

Traffic Class	Group Size, Walking N_{group} (persons)	Group Size, Running N_{group} (persons)	Crowd Density, D (persons/ft ²)
I	2	0	0
II	4	1	0.04
III	8	2	0.08
IV	16	4	0.10

5.1 Walking loads

Typical footfall frequencies may vary between 1 Hz and 3 Hz, with average values falling between 1.6 Hz and 2.2 Hz as recommended in AISC DG11. Walking loads may be based on the recommendations of SCI P354 and CCIP-016. In Table 5, values per Willford and Young [5] are listed.

Table 5. Walking loads

Harmonic Number, h	Harmonic Forcing Frequency, hf_{step} (Hz)	Design Dynamic Coefficient, α
1	1.6 - 2.2 Hz	$0.41(hf_{step}-0.95) \leq 0.56$
2	3.2 - 4.4 Hz	$.0056(hf_{step}+12.3)$
3	4.8 - 6.6 Hz	$.0064(hf_{step}+5.16)$
4	6.4 - 8.8 Hz	$.0065(hf_{step}+2.0)$
$h > 4$	> 8.8 Hz	0

For individual or group walking events, the resonant build-up duration is limited to the time required for the individual or group to cross the bridge. As such, the walking speed must also be known to calculate the response. It has been shown that the speed of a walking individual is dependent on the step frequency, and an expression for walking speed based on step frequency is given in SCI P354 [6]:

$$v_w = 1.672f_{step}^2 - 4.834f_{step} + 4.502$$

(1)

where v_w is the walking speed in m/s and f_{step} is the walking step frequency Hz.

5.2 Running loads

The range of running step frequencies ranges between 2.0 Hz and 4.0 Hz based on published data [7, 8, 9, 10, 11]. AISC Design Guide 11 [9] recommends the running loads to be defined

as summarized in Table 6Error! Reference source not found.. Statistical information related to running loads is limited, but a review of available literature suggests the recommended design loads are consistent with the level of reliability provided by the walking design loads [7, 9, 10, 11].

Table 6. Running loads

Harmonic Number, h	Harmonic Forcing Frequency, hf_{step} (Hz)	Design Dynamic Coefficient, α
1	2 - 4 Hz	1.4
2	4 - 8 Hz	0.4
3	6 - 12 Hz	0.2
4	8 - 16 Hz	0.1
$h > 4$	> 16 Hz	0

6 Interaction

There are two types of interactions that should be considered. The interaction among walkers and the interaction between walkers and the structure. The inter-walker interaction is related to the fact that each walker perceives other nearby walkers and is affected by them. The main parameter that affects this type of interaction is the walker density. Limited studies are available in literature. The study by Butz et al. [12], for example, correlates the mean and standard deviation of the walking frequency to the walker density. It was found that the mean and standard deviation of the walking frequency reduce almost linearly with the increase in crowd density.

The walker-structure interaction is related to the fact that the walker dynamics are affected by the structure’s vibrational performance. This can be accounted for by modeling the walker as a mass-spring-damper system [13]. There are several biomechanics models available in literature that can be used to calibrate the mass-stiffness-damper walker model [14]. The other option is to revise the modal mass and the elastic damping of the structure to account for the presence of the crowd loading.

6.1 Lock-in effect

Walkers shift their weight to the side at each step, thus exerting a lateral force to the bridge deck. If the bridge deck lateral frequency is close to the lateral stepping frequency of the walker, the bridge sways laterally. When the sway exceeds a certain threshold level, each walker has to adjust its gait to maintain its balance. As multiple walker adjust their gait, they tend to synchronize to the motion of the structure, amplifying the structural response. The authors believe that the likelihood of lock-in occurrence should be reduced during the design phase by assuring that the first transverse mode frequency of the bridge deck is significantly higher than the lateral walking frequency.

The lateral walking frequency is half the vertical footfall frequency because the walker’s body weight shifts to opposite sides for two consecutive steps. With reference to the values provided above, the range of lateral walking frequencies varies between 0.5 Hz and 1.5 Hz.

It must be noted that there are several studies in literature aiming at determining the critical number of users to cause lock-in instabilities. These studies were developed following the failure of the Millennium Pedestrian Bridge in London.

6.2 Synchronization

Pedestrian walking in group at random frequencies and phases tend to synchronize. A number of studies that discuss the synchronization of groups of walkers are available in literature. FIB [15] suggests to use an equivalent number of walkers $N_e = 3$ for $N \leq 10$, $N_e = 0.225 \cdot N$ for a low-density stream, and $N_e = 0.27 \cdot 0.46 \cdot N$ to account for synchronization in higher-density streams. SETRA [2] gives two equations, depending on the flow density; the first applies to low densities and depends on damping, $N_e = 10.8 \cdot (\xi N)^{0.5}$, while the second applies to large densities and is independent of damping, $N_e = 1.85 \cdot N^{0.5}$. The study done by Matsumoto et al. [16] found that the acceleration response of a footbridge crossed by a group of N uncorrelated walkers is proportional to the square root of N . The walker group is considered uncorrelated because the walkers have the same frequency, but phases uniformly distributed between 0 and 2π . In the case of correlated walkers (i.e., same step frequency and same phase), the response can conservatively be taken proportional to N , although, in reality, it is impossible for walkers to be perfectly in step and in phase.

7 Comfort levels

Different studies aiming at defining acceptable levels of vertical vibrations are available in literature. Typically, the acceptable acceleration varies with the frequency of the excitation. A different approach is presented in SETRA [2], where three levels of comfort for vertical vibrations are identified as summarized in Table 7. These comfort levels are defined based on the maximum theoretical accelerations (in percent of gravity) that the bridge deck experiences due to the pedestrian loading. Accelerations that exceed 25%g are considered not acceptable. The values in Table 7 are converted from peak accelerations to root mean square (RMS) values.

Table 7. Comfort levels

Comfort Level	Definition	Acceleration Range
Maximum	Accelerations practically imperceptible to users.	$\leq 3.535\%g$
Average	Accelerations merely perceptible to users.	$\geq 3.535\%g$ but $\leq 7.07\%g$
Minimum	Accelerations are perceived by the users, but are not intolerable.	$\geq 7.07\%g$ but $\leq 17.675\%g$

When acceptable levels of accelerations cannot be reached, the structural dynamic performance should be improved. This may be achieved by increasing the natural frequencies to avoid range of footfall frequencies, or by increasing the structural damping. This second option is generally the preferred and most economical one. In fact, increasing the natural frequencies generally translates into increasing structural stiffness, which goes along with an increase in mass and, therefore, produces a counteractive effect. The increase in structural damping is typically achieved with the addition of damping devices. The most widely used damping device is the tuned-mass damper, which consists in adding mass tuned to the mode frequency to be damped.

8 Analysis methodology

To assess the response of a footbridge to pedestrian-induced vibrations, moving, harmonic loads should be considered. The frequencies of the harmonics should equal, in turn, each of the vertical mode frequencies that do not exceed 15 Hz. This corresponds to exciting each one of the fundamental vertical modes with a frequency that falls within the footfall frequency range. The theoretical RMS acceleration values should be computed and compared against the selected acceptance criteria. Peak accelerations are difficult to predict and to measure properly. RMS accelerations should be preferred when assessing vibrations. The same procedure can be followed for lateral vibrations. However, as discussed above, it is generally recommended that the lateral stiffness of the bridge deck is high enough for reducing the risk for lock-in effects. An alternative situation is that of a footbridge crossed by more than one walker. In that case, the characteristics of the walkers have to be described on a probabilistic basis, and the response can be evaluated either in the frequency domain through random vibration theory, or in the time domain through Monte Carlo simulations.

A simple, approximate way to assess the response of a simply supported footbridge span for vertical excitation is to use a stationary, harmonic load applied at the midspan. The frequency of the harmonic load should vary and, each time, match each one of the frequencies of the first fundamental modes, for all of the modes with frequencies not exceeding 15 Hz. Based on principals of structural dynamics, the steady-state acceleration when the mode J is excited due to a stationary, harmonic load in full quadrature with the mode J is the following:

$$a_{steadystate_modeJ} = \delta_{static} \omega_{hJ}^2 / (2\xi_J) \quad (2)$$

where δ_{static} is the static displacement due to a static load applied at the mid-span of the footbridge, ω_{hJ} is the natural frequency of the mode J, and ξ_J is the critical damping ratio of the mode J. To obtain the peak acceleration, the walking speed and the span length should be taken into consideration. There are several correction factors available in literature. The one from Willford and Young [5], for example, is reported here as follow:

$$\psi(n, \xi_I) = 1 - \exp(-2\pi 0.257 n \xi_I) \quad (3)$$

where n is the number of effective load cycles and is equal to

$$n = 0.55 \cdot h \cdot L / l \quad (4)$$

where h is the harmonic of walking (i.e., 1, 2, 3, or 4), L is the bridge span, and l is the stride length. The theoretical RMS acceleration can be then written as:

$$a_{RMS} = 0.707 a_{steadystate_modeJ} \psi(n, \xi_I) \quad (5)$$

9 Reliability analysis

A reliability analysis of the response acceleration of a footbridge can be undertaken when all variables are treated as stochastic variables. The simplest approach is to assume that these variables are normally distributed. Data published in literature can be used to quantify mean and standard deviation of the stochastic variables. As an example of how this approach can be used to determine the likelihood of occurrence of lock-in effects in a simply supported footbridge, Figure 1 shows the probability density functions of the first lateral mode frequency of the footbridge and the lock-in frequency. The shaded area is the probability of lock-in frequency exceeding the lateral mode frequency of the bridge. A target probability of failure can be used to guide the design.

Figure 1. Reliability analysis

10 Conclusions

The paper provides an overview of the analysis methods, loading scenarios, and acceptance criteria for the vibration assessment of footbridges. A brief discussion of iteration phenomena is presented. Finally, an example of how a reliability study may be implemented to determine the likelihood of occurrence of lock-in effects.

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