MATEC Web of Conferences **211**, 07001 (2018) https://doi.org/10.1051/matecconf/201821107001
*VETOMAC XIV*

# FE sloshing modelling in bidimensional cavity using wave equation

*Agnaldo Antônio Moreira T. da* Silva[1] and *Marcus Vinicius Girão de* Morais[1,2*]

[1] PPG, Graduate program in Engineering Material Integrity, University of Brasilia (UnB), Campus of Gama (FGA), 72444-240, Gama (Brasilia), Brazil.
[2] Department of Mechanical Engineering, University of Brasilia (UnB), Campus Darcy Ribeiro, 70910-900, Brasilia, Brazil.

**Abstract**. Tuned liquid column damper (TLCD) is a kind of passive damper that use water sloshing motion to control structural vibration in tall buildings, wind turbines and ships. This paper presents finite element analysis of sloshing in 2D water tank using a pressure based Eulerian approach. The fluid domain is discretised by an isoparametric quadrilateral finite element. The cavity was modelled with rigid contours and free surface in order to study the decoupled liquid reservoir. Next, the cavity was coupled to a flexible structure with free vibration and forced vibration. The numerical results obtained by the created program were validated using experimental results [1] presenting acceptable errors with less than 5k elements.

## 1 Introduction

Tuned liquid column damper (TLCD) is a kind of passive damper device that use de water sloshing motion to control structural vibration in tall buildings, wind turbines and ships [2]. The principle of a passive control device (secondary system) is to reduce the dynamic response of a structure (primary system) when its natural frequency is tuned very close to the dominate mode of primary system. TLCDs have been investigated extensively by many researches [3-7]. Masuda et al [8] show an application of semi-active TLCD using magnetic-rheological fluid controlled by magnetic fields. But in most of bibliography reference, TLCD is modelled as a reduced 1DoF non-linear (or stochastically linearized) differential equation. Lagrangian, Eulerian and mixed Eulerian-Lagrangian descriptions are used to modelling 2D/3D sloshing problems [9]. However, an Eulerian description [10] is an easy way to study sloshing and fluid-structure problems implemented in most of commercial platforms.

This paper presents finite element analysis of sloshing in 2D water tank using a pressure based Eulerian approach. The fluid domain is discretised by isoparametric quadrilateral finite elements. The cavity was modelled with rigid contours and free surface in order to study the decoupled liquid reservoir. Next, the cavity was coupled to a flexible structure with free vibration and forced vibration. The numerical results obtained by the created program were validated using experimental results [1] presenting acceptable errors with less than 5k elements.

* Corresponding author: mvmorais@unb.br

## 2 Governing equations and formulation

For discretization of the acoustic wave equation, the water is assumed as linear compressible, inviscid and irrotational fluid. The mean fluid flow was not considered. The acoustic pressure is described by a perturbation at the mean pressure. The density perturbation is small compared to the mean density.

In this way the linearized wave equation is described by:

$$\nabla^2 p = \frac{1}{c^2}\frac{d^2 p}{dt^2} \tag{1}$$

The fluid domain is limited by the following boundary conditions: (a) rigid wall $\Gamma_{PR}$, (b) free surface $\Gamma_{SL}$ and (c) fluid-structure interface $\Gamma_{FE}$ (Figure 1), where fluid boundary $\partial\Omega = \Gamma_{\mathrm{PR}} \cup \Gamma_{SL} \cup \Gamma_{FE}$. Being the free surface condition the most important of this work that aims to evaluate the sloshing effect.

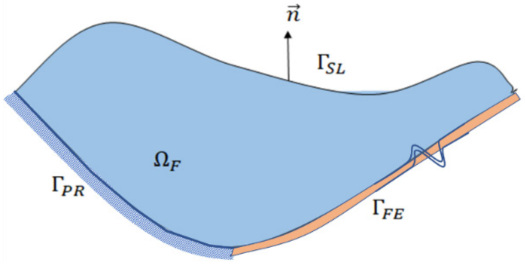



Figure 1. Fluid domain and boundary conditions of sloshing problem

The appropriate boundary conditions for these boundaries are as follows:

i. Free surface boundary condition ($\Gamma_{SL}$):

$$\frac{\partial p}{\partial z} = -\frac{1}{g}\frac{d^2 p}{dt^2} \tag{2}$$

ii. Fluid-structure interface boundary condition ($\Gamma_{FE}$):

$$\frac{\partial p}{\partial \vec{n}} = \nabla p \cdot \vec{n} = -\rho_f \ddot{u} \tag{3}$$

iii. Rigid wall condition (supposing $\ddot{u} = 0$) ($\Gamma_{PR}$):

$$\frac{\partial p}{\partial \vec{n}} = \nabla p \cdot \vec{n} = 0 \tag{4}$$

where, $\rho_f$, $\ddot{u}$ and $g$ are fluid density, normal structure acceleration at structure–fluid interface and acceleration due to gravity, respectively.

To discretise the wave equation, we adopted approximate function $\hat{p}$ that satisfy the boundary conditions. Substituting $\hat{p}$ in Eq. (1) we have:

$$\varepsilon(\hat{p}) = \nabla^2 \hat{p} - \frac{1}{c^2}\ddot{\hat{p}} \neq 0 \tag{5}$$

The weighted residues method defines the integral of residue function $\varepsilon(\hat{p})$ plus approximation function $\hat{p}$ as zero:

$$\int_{\Omega_f} \varepsilon(\hat{p})\,\hat{p}\,d\Omega = \int_{\Omega_f} \left(\nabla^2 \hat{p} - \frac{1}{c^2}\ddot{\hat{p}}\right)\hat{p}\,d\Omega = 0 \tag{6}$$

Using the Green theorem, Eq. (6) can be rewritten as:

$$-\int_{\Omega_f} \nabla\hat{p}\,\nabla\hat{p}\,d\Omega + \oint_{\partial\Omega} \hat{p}\,\frac{\partial p}{\partial \vec{n}}\,d\Gamma - \int_{\Omega_f} \frac{1}{c^2}\ddot{\hat{p}}\,\hat{p}\,d\Omega = 0 \tag{7}$$

Applying the boundary conditions (2)-(4), we obtain the following equation:

$$\int_{\Omega_f} \nabla\hat{p}\,\nabla\hat{p}\,d\Omega + \oint_{\Gamma_{FE}} \rho\,(\ddot{u}\cdot n)\,\hat{p}\,d\Gamma + \oint_{\Gamma_{SL}} \frac{1}{g}\,\ddot{\hat{p}}\,\hat{p}\,d\Gamma + \int_{\Omega_f} \frac{1}{c^2}\ddot{\hat{p}}\,\hat{p}\,d\Omega = 0 \tag{8}$$

Eq. (8) describes the weak form of the sloshing problem in acoustic fluid with mobile boundaries $\Gamma_{FE}$ . We have the weak form of the equation which is the starting point for discretizing the domain of the fluid by the finite element method. If we consider an incompressible fluid (sound velocity $c \to \infty$), Eq. (8) with only sloshing effect in 'mass' matrix results an ill-conditioned linear system. Then, as a penalty condition, Zienkiewicz and Taylor [11] propose compressibility inclusion to work around this difficulty.

### 2.2 Implementation

The weak form (7) was discretized using isoparametric finite elements. The elements adopted were: (a) isoparametric L2 (unidimensional element with two nodes) to discretize the free surface contour and fluid structure interface, and (b) isoparametric Q4 (two-dimensional four-node element) to discretize the fluid domain.

Assuming pressure in form of $\hat{p} \simeq \sum N_i \hat{p}_i$, where $\hat{p}_i$ is the nodal pressure in finite element and $N_i$ is interpolating shape function, the discretized form of Eq. (8) can be written as:

$$\boldsymbol{K}_f\,\hat{\boldsymbol{p}} + \left\{\frac{1}{c^2}\boldsymbol{M}_f + \frac{1}{g}\boldsymbol{M}_{SL}\right\}\ddot{\hat{\boldsymbol{p}}} = -\rho_f \boldsymbol{C}_{FS}^T\,\ddot{\boldsymbol{u}} \tag{9}$$

where, $\hat{\boldsymbol{p}}$ and $\ddot{\boldsymbol{u}}$ are the nodal acoustic pressure in fluid domain $\Omega_f$ and nodal solid acceleration in fluid-structure contour $\Gamma_{FS}$ , respectively; $\boldsymbol{K}_f$ and $\boldsymbol{M}_f$ is acoustic 'stiffness' matrix (first term of Eq. (8)) and acoustic 'mass' matrix (last term of Eq. (8)), respectively; $\boldsymbol{M}_{SL}$ is the free-surface boundary condition, and $\boldsymbol{C}_{FS}^T$ is the fluid-structure coupling matrix. In present work, we present results of free vibration ($\ddot{\boldsymbol{u}} = 0$).

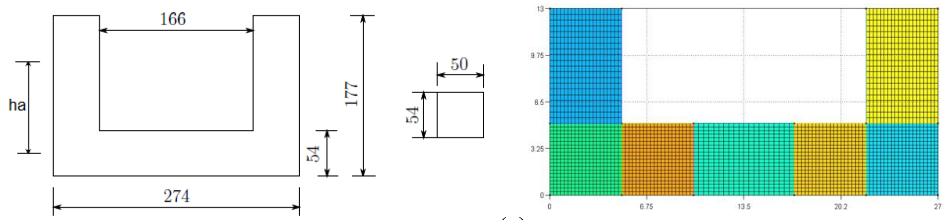
The shape functions are linear with continuity $C^0$ (infinitely continuous interpolation functions inside the elements and continuous at the element interface or first discontinuous derivative). The numerical implementation was developed in MATLAB (MatLab R2015b).

## 3 Results

The present study analyses the TLCD problem, as described in Figure 2, that is subjected to free and forced (harmonic) vibration. Initially, a free-vibration analysis was carried out varying the water column height $h_a$. TLCD's natural frequency was compared with experimental results [1]. Finally, TLCD was subjected to a harmonic vibration due to an acceleration of contained liquid. The frequency response function and operational modes of sloshing fluid are obtained.

### 3.1 TLCD with free vibration

Finite element meshes were building using mesh generator GMSH [12]. TLCD is divided into seven distinct parts, as described in Figure 2b. A mesh convergence study was done using a reference water level $h_a$. With an identical aspect ratio for all elements, we observe a good convergence from 20 elements (counting water level $h_a = 105mm$). We use the same element aspect ratio for all meshes.




(a) (b)

**Figure 2.** TLCD geometric description (a) and FE mesh example (b).

Using a convergent mesh, we carry out the numerical determination of TLCD's natural frequencies as function of the water height level $h_a$ (mm). Table 1 compare numerical solutions to experimental results [1]. The numerical solution presents a good agreement with experimental results. There was no discrepancy superior to 1.7%. Figure 3 shows the first four modal shapes of fluid contained. The fundamental modal form reproduces the expected U-tube behaviour. And the following three modal shapes behaves as two tanks with aspect ratio $h/L = (h_a + 27)/50$. We can also observe that the free surface modal shape (with low frequencies) is uncoupled to cavity modes (with higher frequencies).

**Table 1.** Fundamental natural frequency (Hz) of TLCD as function of water column height

| Column height $\boldsymbol{h_a}$ (mm) | Natural frequency (Hz) | | Relative Error (%) |
|---|---|---|---|
| | Numerical | Experimental [1] | |
| 35 | 1.39 | 1.38 | 0.72 |
| 45 | 1.34 | 1.33 | 0.75 |
| 55 | 1.29 | 1.27 | 1.57 |
| 65 | 1.25 | 1.23 | 1.63 |
| 75 | 1.21 | 1.20 | 0.83 |
| 85 | 1.18 | 1.17 | 0.85 |
| 95 | 1.14 | 1.15 | 0.87 |
| 105 | 1.11 | 1.12 | 0.89 |

### 3.2 TLCD with forced vibration

We analyse the TLCD subjected to a forced acceleration $(\ddot{u} = \ddot{u}_o \exp i\Omega t)$. Figure 4 shows frequency response function (FRF) of a node pressure on free surface of TLCD with water level height $h_a = 105mm$. We observe five peaks in frequency range $\Omega \in [0, 6]Hz$. This resonant frequency is near to the natural frequencies obtained in Section 3.1. Figure 5 shows the first four operational modal shapes of TLCD with water level height $h_a = 105mm$.

## 4 Conclusions

In the present work, we implement an isoparametric 2D finite element to analysis of sloshing problems. This numerical model is based in a pressure based Eulerian approach. The fluid domain is discretised by isoparametric quadrilateral finite elements.

The numerical formulation was applied to characterize the dynamic parameters (natural frequency and modal shape) of a TLCD containing liquid water. We analyse free and forced

 https://doi.org/10.1051/matecconf/201821107001

(harmonic) vibration of TLCD. For free-vibration analysis, we compare the numerical solution and experimental results of TLCD's fundamental frequency as a function of water column height $h_a$. The numerical solution shows a good agreement with experimental results. Finally, a harmonic analysis was done by acceleration container liquid of a TLCD with $h_a = 105mm$. The frequency response function captures the first two natural frequency quite precisely. A comparison with commercial platforms are necessary.

As perspectives, we intend the implementation of structural domain to perform fluid-structure interaction studies.

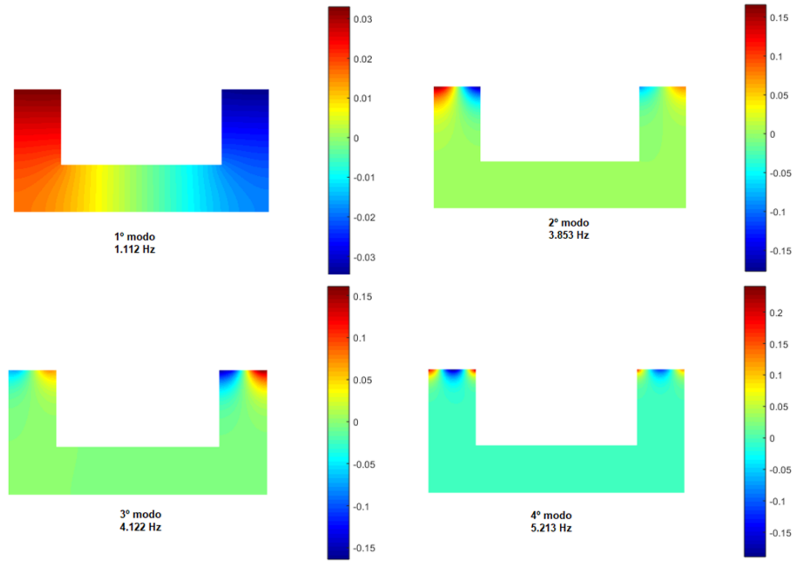



**Figure 3.** First four natural frequencies and modal shapes of TLCD with $h_a = 105mm$.

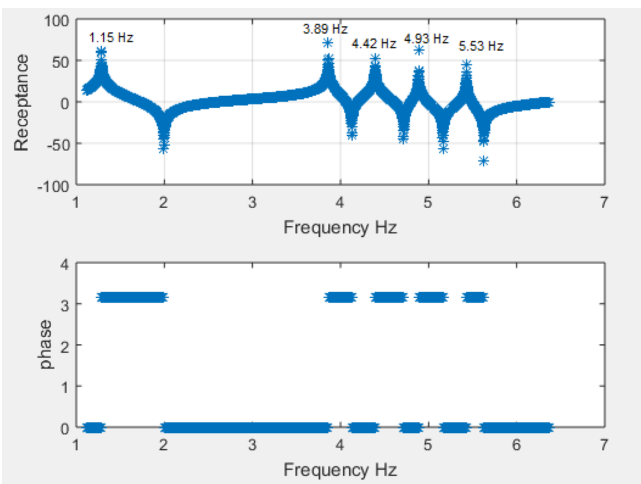



**Figure 4.** Pressure FRF on free surface node of TLCD with water level height $h_a = 105mm$.

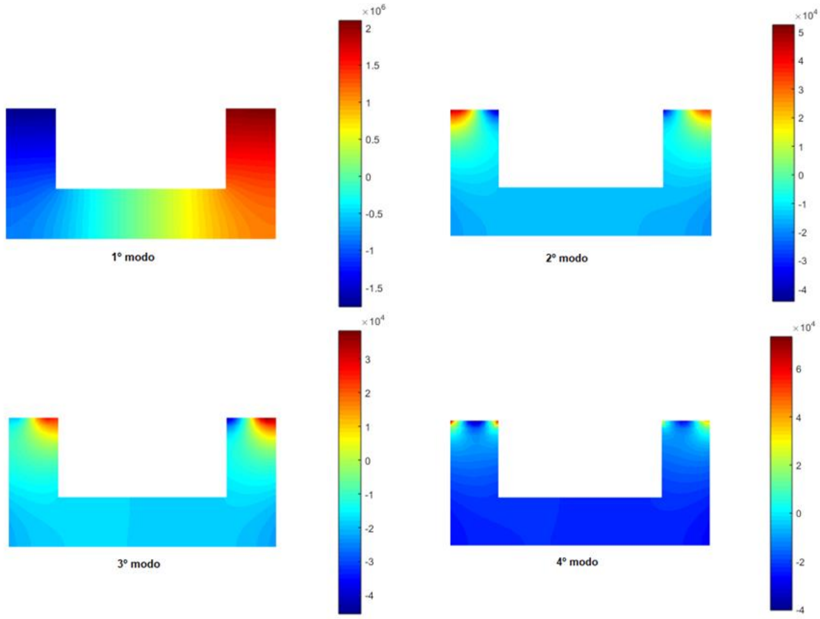



**Figure 5.** First four operational modal shapes of TLCD with water level height $h_a = 105mm$.

This work was supported by the Coordination for the Improvement of Higher Education Personnel (CAPES) and the Federal District Foundation for Research Support (FAPDF).

## References

[1] M.H. Alkmim, Master dissertation, Graduate program in Mechanical Science, University of Brasília (2017)

[2] P. Murtagh, A. Ghosh, B. Basu, B. Broderick, Wind Energy **11**(4), 305–317 (2008)

[3] H. Gao, K.C.S. Kwo, B. Samali, Engineering Structures **19**(6), 476-486 (1997)

[4] S.K. Yalla, A. Kareem, Journal of Structural Engineering, **126**(8), 906-915 (2000)

[5] V. Jaksic, C. Wright, A. Chanayil, S. Faruque Ali, J. Murphy, V. Pakrashi, Journal of Physics: Conference Series **628,** 012058 (2015)

[6] M.H. Alkmim, M.V.G. de Morais, A.T. Fabro, Journal of Physics: Conference Series, **744**(1), 012178 (2016)

[7] M.H. Alkmim, M.V.G. de Morais, A.T. Fabro, IOP Conf. Series: Materials Science and Engineering **280** (2017) 012007 doi:10.1088/1757-899X/280/1/012007 (2017)

[8] H. Masuda, T. Oyamada, T. Sawada, Journal of Physics: Conference Series **412** 012049 (2013)

[9] M.V.G. de Morais, Master dissertation, Graduate program in Structures and Civil Construction, University of Brasília (2000).

[10] K.K. Mandala, D. Maity. Procedia Engineering **144,** 1187-1194 (2016)

[11] O.C. Zienkiewicz, R.L. Taylor, *Finite Element Method*: Volume 1, Butterworth-Heinemann; 5nd edition (2000), O.C. Zienkiewicz, R.L. Taylor (1969).

[12] C. Geuzaine, J.-F. Remacle, International Journal for Numerical Methods in Engineering **79**(11), 1309-1331 (2009)