This world is full of vibrations

J. S. Rao

The Vibration Institute of India, Bangalore

Abstract. Man ventured out of the caves nearly 15000 years ago and since then tried to explain earth vibrations as mythology. Naturally occurring earthquakes were observed and described over the last two thousand years. Though the Universe is nearly 14 billion years old and the trillions of celestial bodies are continuously colliding, sending gravitational waves passing energy to bodies like micro-earth that are responsible for tectonic plate movement on this macro-earth and earthquakes, the effects are recorded over two years ago. Ever since scientific revolution in 17th century machines became ubiquitous and their vibrations are felt in every day's life. This paper summarizes these vibrations.

1 Introduction

With the close of the last ice age about 13,000 BC more clement weather patterns resulted and humans ventured out of caves and other natural shelters. Poséidon, Sea God, carries a trident in Greek Mythology, is called Earth-shaker. He is attributed to strike the ground with the trident, causing earthquakes, see Burkert (1987).

WordZz described in Ramayana "All of a sudden, there developed a crack in the earth and a divine light came to be emitted out of it. In no time, the goddess of the Earth came out of the crack. The divine light was coming out of the halo of the goddess who took Sita in her lap and gradually slipped into the crack" which probably alluded to an earthquake. Buck (1976) described Ramayana and Sharan (2004) dated the events of Ramayana to be 3355 BC based on the planetary positions described in the battle between Rama and Ravana, see also Rao (2017a).

These are mythological in nature as there was no clear historical evidence. However they were described and followed by generations and therefore there should be some meaning into them by inference.

Wallace-Hadrill (2011) describes the earthquake from Vesuvius Eruption of AD 79 in Pompeii. Though this volcano started forming and acting tens of thousands years ago, the first recorded observation is just about 2000 years ago when man has recognized volcanic earthquakes. The earth around shook and lot of vibrations were obviously felt.

The Great Lisbon earthquake followed by Tsunami occurred on 1 November, the holy day of All Saints' Day in the year 1755. This is the first time an earth quake was attempted to study scientifically by Michell (1760) in the Scientific Revolution period. It is reported that some 60 000 people died, either by collapsing buildings or in the Tsunami waves.

Up until this time, there had been a general religious belief that God would not allow such a calamity. The Lisbon quake triggered the thinkers of the time to try and explain earthquakes in a non-religious context. Michell showed that the focus of that earthquake

was underneath the Atlantic Ocean, and he proposed erroneously that the cause of earthquakes was high-pressure steam, created when water comes into contact with subterranean fires. A similar earthquake and Tsunami occurred in 2005, see Rao (2005). This was early morning on the Sunday of 26th December 2004 (just after Christmas) to hear the destruction caused by Tsunami waves hitting Chennai, just a little after two hours of the earth quake that struck in Indian Ocean off the Indonesian coast. The earth quake that struck off the Banda Aceh measured 9.1 on Richter scale, one of the deadliest.

Albert Wilhelm (1838) is attributed to be the first person to record observations of metal fatigue that are essentially caused by transient vibrations in the lower modes of vibration. His observations were in the Mining and Forestry Office in Clausthal, Germany on the failure of iron mine-hoist chains arising from repeated small loadings.

Jean-Victor Poncelet (1829) published his monograph on *Introduction à la mécanique industrielle*. He described in his classes around 1837-1839 for the first time that metals as being tired in his lectures at the military school at Metz. He introduced the notion of memory in materials under repeated variable loads; leading to the observation that all matter in the Universe has memory just like human brains consisting of this matter do. This is the Universal law about energy and transformation from one form to another like matter and energy once this Universe was created nearly 14 billion years ago. All the governing laws were borne then.

Rankine (1842) was one of the first engineers to recognize that fatigue failures of railway axles were caused by the initiation and growth of brittle cracks. Wöhler (1858 to 1870) summarized his work in several papers on railroad axles; see also Wöhler (1867). He concludes that cyclic stress range is more important than peak stress and introduces the concept of endurance limit. His work on fatigue marks the first systematic investigation of S-N Curves, also known as Wöhler curves, to characterize the fatigue behaviour of materials. He showed clearly that fatigue occurs by crack growth from surface defects until the product can no longer support the applied load. High Cycle Fatigue from vibrations and fracture mechanics was established then. At this time no component was allowed to go into plastic conditions in the regions of discontinuities and therefore no thought was given to low cycle fatigue until Ludwik (1909) defined the stress-strain relation beyond yield. For globally elastic and locally plastic structures, Neuber (1961) gave his hypothesis which is used for relating the nominal and local cyclic stresses and strains.

The golden era of rotating machines which are also responsible for major vibration problems began in the year of 1884 with the advent of steam turbines and dynamos see Rao (2011). Internal Combustion engines and aircraft with piston engines followed. The Second World War saw gas turbines for aviation followed by usage in stationary power plants. Rockets with solid propellants and liquid propulsion all became common in 20th century. Along came vibration problems hitherto unknown in bending, torsion and coupled motions of various flexible structural and machine members.

Whereas Newton (1786) brought out Science on firm ground during the Scientific Revolution Period in the 17th and 18th centuries, it is Einstein (1905, 1917, 1920) in the last century that brought out facts of the Universe regarding the Matter and Energy conversion, collision of trillions and trillions of bodies like black holes moving and colliding in the Universe at speed of light that would emanate gravitational waves, see Rao (2017a). His work led to nuclear fusion and design of the reactors in Cadarache France on one hand and detection of gravitational waves using Laser Interferometry Gravitational Observatories (LIGOs) three of them since 2015, two in USA and one in Italy.

The advances in digital computation with high speed computers handling huge data allowed multi-physics problems like flutter of an aircraft wing, modelling of earth subjected to displacements of the order of one hundredth of diameter of a Proton in Nucleus of an

atom and get the preliminary ideas of earthquakes from the movement of continental tectonic plates with the energy passed on to the earth from gravitational waves.

Thus the world is full of vibrations from machinery and so-called natural disasters of earthquakes and we will try and present some typical examples.

2 Structural vibrations

Just after Newton's II law, D'Alembert (1750) gave the first derivation of oscillations of a simple pendulum which was followed with a mass on spring discrete vibrating system. Columns and beams are ubiquitous in the structures of worship, e.g., Temple of Zeus built in 2500 years ago to Somnath temple 1500 years ago, see Rao (2017b). The curiosity was therefore on beams and columns and Leonardo da Vinci (1452-1519) was attracted to this, see Reti (1974), who correctly identified that in bending of beams due to transverse loads; plane cross-sections remain plane before and after bending and rotate. This idea was taken by Euler (1736-1770) who together with Bernoulli formed correct equations for simple bending in 1750.

It is in 1808 Ernst Florenz Friedrich Chladini (1756-1827) a German musician and scientist conducted experiments on vibrating plates, exhibiting the so-called Chladini figures, see Ullmann (2007). Chladini (1787) published the technique in his book. Sophie Germain derived the plate theory in 1815 - though it had some deficiencies corrected later by Kirchhoff in 1850 - see Kirchhoff (1876). It took us 65 years in understanding how to deal with a two dimensional structure. Besides an understanding on fatigue and life as mentioned earlier with the mines and railroad vehicles the vibration knowledge was relatively poor as there were no high speed rotating machinery.

About 130 years ago this world has seen in 1882 the birth of rotating machinery which was then hailed as vibration free engines in comparison to heavy slow speed reciprocating steam engines of James Watt era. However with that a host of vibration problems began to appear with multiple applications in marine, railroad, power plants, aero transport and gas turbines, aircraft structures and engines with fluid-structure interaction, space applications particularly high speed cryogenic pumps, fluid induced noise amongst others.

With Einstein's work on Energy and gravitational waves we are further poised for transient vibration problems in fusion reactors and earthquake understanding as in a fracture mechanics of the faults of tectonic plates. Some of these will be described in what follows.

3 Machine vibrations

Some of the earlier rotor failures belong to propeller shafts in torsion of steam driven war ships during I world war. When a propeller shaft failed its diameter was increased by 10%. The modified shaft however failed in less than half time of the previous shaft failure. For the first time dynamic design showed that the shaft the natural frequency became closer to the excitation harmonic resulting in an earlier failure. Holzer tabular method was used to determine the torsional natural frequencies of systems and perhaps it is the first application to the industry and so simple that it lasted into the computer age, see Rao (2011).

First attempts on understanding Rotor Dynamics began with Rankine (1869) study of a spinning shaft. He considered centrifugal whirling and shown the existence of critical speeds of a rotor and that the whirling speed was the same as the frequency at which the shaft vibrated transversely if struck and set out how to calculate this critical speed. He defined this as a limit of speed for centrifugal whirling. There were many doubts whether a rotor can cross this limit. It was presumed that it will be unstable after crossing the critical

speed. Rankine may have been responsible for setting back the science of rotor dynamics by nearly fifty years.

Rayleigh (1877) used the energy principle to determine the first critical speed of a rotor considering it as a stationary beam before Laval's steam turbine. Stodola (1910) presented a graphical method to determine the critical speeds of practical rotors. This method remained to be widely used for over five decades until digital computers became available.

The rotor dynamics understanding came into practice with the work of Jeffcott (1919) just about 100 years ago when the unbalance was understood and the difference between a stationary beam and rotor became clear. The rotor remained a beam until Simulation Based Engineering Science and solid models was understood, see Rao (2002).

Once the rotor and its whirl were understood rapid strides were made to improve rotor dynamic performance of rotating machinery, however, a vast number of problems cropped up; the modelling techniques of beams with disks as in centrifugal pumps using gyroscopic effects, bearing non-linearity, oil film supports and related instability problems, seals, internal friction and loosely mounted shafts, misalignment amongst others. These rotor-specific problems gave rise to a separate subject "Rotor Dynamics", see Rao (1983). The machinery being very expensive, costing billions of dollars, methods to monitor the condition and health of the machine to provide long life gave rise to another rotor-specific subject "Vibratory Condition Monitoring of Machines", see Rao (2000).

Frank Whittle, while testing the first Jet Engine W.2.B in 1942 experienced serious blade failure problems; in his own words – Frequency of Turbine Blade Failures was becoming the latest technological barrier to overcome, see Rao (1911). This has opened up today's air-transportation and the related rotor and blade problems. Gas turbine engines are high speed engines and they pose severe vibration problems and life issues. Their design also gets complicated. Some specific recent and advanced problems are presented here.

4 Vibration issues

We will briefly consider some recent advanced vibration problems faced in the industry:

1. An aircraft engine rotor dynamics design
2. Space Cryogenic Pump transient whirl with nonlinear bearings
3. Solid-Fluid Interaction of jet engines under bird hit
4. Flutter of an aircraft wing in time domain

4.1 Two spool aircraft engine rotor dynamics

The solid model of engines became a necessity with two spool engines and flexible casing where beam models fail. Rao (2003) described such a model using the solid model technique developed in 2002.

Rotor Dynamics plays an important role in engine design. The support characteristics are generally the main issue, from bearings, dampers and also seals all of which play significant roles in engine dynamics. The bearings in an engine are supported in flexible structures, the front frame of the fan, the middle frame ... The casing itself is 1.5 to 2 mm thick in advanced light engines and they add to considerable flexibility of the system. Therefore, it has become a necessity to be able to couple rotors and stators into a single model. Moreover, it is a complex process to determine the stiffness of the frames supporting the bearings for simulation of the rotor.

Making approximate beam models of the engine spools for analysis purposes has therefore become obsolete and one uses now solid elements directly in the model. Thus, one can use the same solid model and mesh adopted for structural and thermal analysis. A typical mode shape of an engine is shown in Fig. 1.

The whirl amplitudes at critical speeds will be quite large and the blade tips can rub at these speeds. Taking into account, the transient thermal and steady load growths of the casing and the radial displacements of different stages of the rotor under centrifugal and gas loads and whirling amplitudes at critical speeds, overall clearances required can be fixed.

Fig. 1 Mode Shape of a typical two spool aircraft engine

4.2 Cryogenic pump rotor dynamics

High speed cryogenic pumps are employed in Geo Synchronous Satellite Launch Vehicles. They use liquid hydrogen LH₂ and oxygen LOX as propellants. Space shuttle main engine (SSME) RS-25 had its first flight on April 12, 1981. A lumped parameter structural-dynamic model, consisting of 13 rigid bodies connected to each other by massless beam elements was used for rotor dynamic analysis by Childs (1975). Today Simulation Based Engineering Science Approach is used that begins with a FEA model shown in Fig. 2, see Rao (2014). Bearings and seals are simulated by 12 by 12 matrix elements accounting for all translational and rotational degrees of freedom between two nodes. Total number of elements is 379,779 and the total number of nodes is 534,115.

Fig. 2 FE Model of Turbo Pump

Fig. 3 Mode Shape at Peak Response

The first forward critical speed of the rotor is at 28000 rpm and the second forward critical speed is at 55000 rpm. The first and second backward critical speeds are observed at 20000 and 44000 rpm respectively. The critical speeds dropped significantly to 23500 rpm and 39000 rpm when the casing effect is included.

Macros are written to apply the unbalance load at five locations to determine the whirl response by using a platform approach and other commercial codes. There are six bearings and 8 seals. They are nonlinear in nature and linearized while using the commercial codes. Nonlinearities were taken into account by iteration between reaction force and whirl response until they matched with previous step. The first forward critical speed is now at 34,000 rpm and the second forward critical speed is above 55,000 rpm. Thus, the first forward critical speed increased from 28,000 rpm by 21.4%.

A transient study was made to consider high angular acceleration of the pump to reach full speed (50000 rpm) in three seconds. Peak response occurred at around 0.795 seconds and the mode shape is shown in Fig. 3, while the rotor is accelerating at 1800 rad/s^2

4.3 Bird hit on engines

The heat from combustion is carried by fluids to transfer the energy and convert it to kinetic form. Gas turbines employ axial compressors to compress sucked atmospheric to increase its density in the combustion chamber. The compressor blades act as air foils and from the aerodynamics subjected to pressures and therefore deflections and stresses. These stresses are alternating at nozzle passing frequency and harmonics and if excessive can lead to sudden failures.

On January 16, US Airways flight (2009) was hit by a flock of birds on take-off from LaGuardia airport and had to land in Hudson River because both the engines failed almost immediately after the bird hit. What was the life of the blades is the question answered by Rao and Rzadkowsky (2014).

Fig. 4 Blade Stage Interference and transient CFD solution showing shock

A 3D model of the first stage compressor is made as shown in Fig. 4. It consists of 44 blades in the Inlet Stator Cascade, 28 blades in the Rotor Cascade (only one of which is seen in the picture) and 34 blades in the Stator Cascade of the first stage. The reference rotor blade is divided into 10 cross-sections and the block was modelled ($125 \times 100 \text{ mm}$) in the inlet of the engine. The transient CFD solution was obtained using a commercial code, the contours of Mach number in the stage around the blocked area.

This was the shock force applied to the reference blade. An FFT was done to this pressure and it was found that the first two harmonics were 250.75 and 501.51 HZ. The Campbell diagram for this blade that a block simulated (as in a bird strike) produces resonance at 501 Hz 1st mode natural frequency. The average alternating pressure with four blade passage blocked at 15000 rpm was 7.6735 times the average steady pressure field for this condition. Considering hysteresis damping as a function of reference strain for the operating speed in the first mode the peak stress and its location was determined as shown in Fig. 5.

Using Paris law, crack propagation life at 15000 RPM for $2 \times \text{harmonic} = 3683$ cycles is estimated. The blade life in the present case of severe blockage is $\frac{3683}{501.51} = 7_{\text{sec}}$. SBES and Platform approach allows today an accurate determination of time for failure.

Fig. 5 Hysteresis damping and peak stress location of the blade

4.4 Aircraft wing flutter

Fluid-structure interaction could be contained in providing stable operation or can become unstable causing flutter or self-excited vibrations. Such fluid-structure interaction is observed in the galloping of transmission lines in the northern climates and oscillations to failure of Tacoma Narrows Bridge see Rao (1992, 2000). One of the most dreaded problems is an aircraft structure that goes into flutter under certain conditions of flight. Earlier in non-computational era this has been studied in closed form for two dimensional air foils, see Scanlan and Rosenbaum (1968) or one depended on tests in wind tunnels. With the advances in computers it is now possible to perform fluid structure interaction in time domain and find whether flutter occurs or not for a given aircraft wing or a turbomachine blade in specified conditions of operation, see Rao (2016). They considered a topology optimized wing as shown in Fig. 6 with its structural mesh.

Fig. 6. Topology optimised wing and the structural mesh

Fig. 7 shows the mesh generated for the CFD analysis which has 104742 nodes with 357048 elements. More care was taken towards refinement of the mesh to capture the flow physics exactly and also to avoid negative volume error occurring due to the dynamic mesh in the analysis. Mach number taken is 0.8 for three angles of attack, 0.05, 5 and 25 degrees.

Fig. 8 shows the time domain response of the wing tip upto 0.81 s for angle of attack 5°. Increasing the angle of attack the flutter condition is worsened as shown in Fig. 9.

One can increase the mesh by including the engines, fuselage etc. and adopt super element model to limit the mesh size as far as possible.

Fig. 7 Fluid Mesh of the Wing in Fig. 6

Fig. 8 Time Response curve of the wing tip upto 0.81 s for angle of attack 5 degree

Fig. 9 Response of the wing tip at 0.327 s for angle of attack 25 degree

5 Earthquakes from gravitational waves

Earthquakes were noticed by mankind since time memorial; volcanic earthquakes like that of Pompeii have their origins from volcanoes, see Wallace-Hadrill, A., (2011); tectonic plate movement earthquakes, e.g., Lisbon earthquake, see Michell (1760), are from continental plates jutting against another. Wegener (1912) proposed continental drift theory on the earth taking over hundreds of million years without identifying the source of energy.

Fig. 10 Afghanistan Pakistan Border (Himalayas) Earthquake on 16th April 2016

The source of energy is found from Einstein's (1917) theory of relativity which talks of collisions of trillions and trillions of black holes and creating gravitational waves that go out in the universe at speed of light and transferring some energy to particles or bodies like earth while doing so. Rao (2017-18) presented a finite element model of the earth subjected to minor strains of the order of 10^{-23} . He observed that there have been four instances of gravitational waves measurement and at all these times earthquakes have been reported, see <https://earthquake-report.com>.

This gives rise to speculation that it is the energy from gravitational waves that is responsible to generate the earthquakes. Rao (2017-18) studied one of these earthquakes Afghanistan Pakistan Border (Himalayas) Earthquake on 16th April 2016 as in Fig. 10. The epicentre or stress raiser reported is exactly similar to Mode I Crack opening of a turbine blade with almost infinite multiple of the small stress felt by the earth from gravitational waves. Opening of this crack causes failures or so-called earth-quakes.

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