

Linear Optimal Fusion of Local Unbiased FIR Filters

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Abstract. This paper presents a multi-sensor decentralized fusion unbiased finite impulse response (UFIR) filter for discrete time-invariant state-space models. Fusion is provided in the minimum variance sense. By calculating the cross covariance between any of two local filters for the extended state-space model, linear optimal weights are derived to fuse local UFIR estimates. Simulation conducted for a two-state polynomial model shows that the proposed fusion UFIR filter has higher robustness than the fusion Kalman filter against errors in the noise statistics and temporary model uncertainties.

1 Introduction

State estimation plays as an important role in many engineering areas such as tracking [1], multi degrees-of-freedom mechanical systems [2], timekeeping and clock synchronization [3], and positioning. Generally, estimates can be provided employing either the finite impulse response (FIR) structures having limited memory or infinite impulse response (IIR) ones requiring all data [4].

The Kalman filter (KF) [5], which has an IIR structure, has been applied widely owing to simple programming, small required memory, and fast recursive computation. However, the KF-based estimators often face problems in industrial applications when the noise environment is augmented with outliers and become non-Gaussian and/or both the process and its measurement undergo temporary uncertainties [6]. Under such conditions, the KF produces extra errors and may become redundantly noisy or biased, while the extended KF (EKF) may even diverge. Therefore, many efforts were made during decades to improve the performance and satisfy the required conditions [7–9].

Unlike the KF, linear FIR estimators can be represented in discrete time index n on an averaging horizon $[m, n]$ of N points, from $m = n - N + 1$ to n , in a batch form of [10]

$$\hat{x}_n = \sum_{i=m}^n H_i y_i + \sum_{i=m}^n L_i u_i, \quad (1)$$

where \hat{x}_n is the estimate of state x_n , y_i is the observation vector, and u_i is the control input. Matrices H_i and L_i are the filter gains, which must be determined to obey some cost function. Since data beyond the horizon $[m, n]$ do not affect estimate \hat{x}_n , the FIR structure has higher robustness than the KF. It is also known that the FIR structure has the inherent bounded input/bounded output (BIBO) stability [11, 12] and produces smaller round-off errors [11].

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Among different kinds of FIR estimators, the unbiased FIR (UFIR) filter [13] is blind on given horizons. It is also a most robust linear estimator owing to an ability to ignore the noise statistics and initial values. Unlike the KF, which relies on accurate noise statistics, initial state, and initial errors, the UFIR filter requires only an optimal averaging horizon N_{opt} in order to minimize the mean square error [14]. Because N_{opt} can be determined in a way much easier than for the noise statistics, the UFIR filter often demonstrates better performance than the KF [15, 16].

To meet the requirements for accuracy in wireless sensor networks (WSN), the information fusion filtering theory has been developed for multi-sensor systems [17]. Common strategies of the WSN include the centralized and decentralized fusion [18]. The former can minimize information losses, but with poor flexibility and large computational burden. The latter employs estimates provided by local filters, which can overcome the above shortcomings and improve accuracy of local estimators [19]. However, existing fusion filtering algorithms were mostly constructed based on the KF, while the FIR-type estimators are still under the development for fusion in WSN. Motivated by above, this paper proposes a linear optimal decentralized state fusion strategy based on discrete-time state-space UFIR filtering.

The notations used in this paper are summarized as follows: I_k denotes the $k \times k$ identity matrix, $\mathbb{E}\{x\}$ denotes the expectation of x , x^T denotes the transpose of x , and 0 denotes the zero matrix of proper dimensions.

2 Problem Formulation and Preliminaries

Consider the following linear discrete time-invariant multi-sensor system

$$x_{n+1} = Ax_n + Eu_n + Bw_n, \quad (2)$$

$$y_n^{[i]} = C^{[i]}x_n + v_n^{[i]}, \quad (3)$$

where $i = 1, 2, \dots, r$ is the sensor index, $x_n \in R^k$ is the state vector, $y_n^{[i]} \in R^{m_i}$ is the measurements provided by i th sensor, $u_n \in R^p$ is a known control signal, and A, E, B , and $C^{[i]}$ are known constant matrices of proper dimensions. Here, $w_n \in R^q$ and $v_n^{[i]} \in R^{m_i}$ are the process and measurement noise vectors, which are assumed to be zero mean, $\mathbb{E}\{w_n\} = 0$, $\mathbb{E}\{v_n^{[i]}\} = 0$, uncorrelated and white Gaussian with variances Q and R_i , respectively.

The aim now is to design a decentralized UFIR filter \hat{x}_n^o for time-invariant systems represented with (2)-(3) in frames of the optimal information fusion strategy.

2.1 Extended State Space Model

On a horizon $[m, n]$, equations (2) and (3) can be expanded as shown in [13],

$$\begin{aligned} X_{n,m} &= A_{n,m}x_m + E_{n,m}U_{n,m} + B_{n,m}W_{n,m}, \\ Y_{n,m}^{[i]} &= C_{n,m}^{[i]}x_m + G_{n,m}^{[i]}U_{n,m} + F_{n,m}^{[i]}W_{n,m} + V_{n,m}^{[i]}, \end{aligned} \quad (4)$$

where the extended vectors are specified by

$$\begin{aligned} X_{n,m} &= [x_n^T, x_{n-1}^T, \dots, x_m^T]_{kN \times 1}^T, \\ U_{n,m} &= [u_n^T, u_{n-1}^T, \dots, u_m^T]_{pN \times 1}^T, \\ Y_{n,m}^{[i]} &= [(y_n^{[i]})^T, (y_{n-1}^{[i]})^T, \dots, (y_m^{[i]})^T]_{m_i N \times 1}^T, \\ W_{n,m} &= [w_n^T, w_{n-1}^T, \dots, w_m^T]_{qN \times 1}^T, \\ V_{n,m}^{[i]} &= [(v_n^{[i]})^T, (v_{n-1}^{[i]})^T, \dots, (v_m^{[i]})^T]_{m_i N \times 1}^T. \end{aligned}$$

Extended matrices $A_{n,m} \in \mathbb{R}^{kN \times k}$, $B_{n,m} \in \mathbb{R}^{kN \times qN}$, $E_{n,m} \in \mathbb{R}^{kN \times pN}$, $C_{n,m}^{[i]} \in \mathbb{R}^{m_i N \times k}$, $G_{n,m}^{[i]} \in \mathbb{R}^{m_i N \times pN}$, and $F_{n,m}^{[i]} \in \mathbb{R}^{m_i N \times qN}$ are time-invariant and N -dependent,

$$A_{n,m} = [(A^{N-1})^T, (A^{N-2})^T, \dots, A^T, I_k]^T, \quad (6)$$

$$B_{n,m} = \begin{bmatrix} B & AB & \dots & A^{N-2}B & A^{N-1}B \\ 0 & B & \dots & A^{N-3}B & A^{N-2}B \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & B & AB \\ 0 & 0 & \dots & 0 & B \end{bmatrix}, \quad (7)$$

$$C_{n,m}^{[i]} = \bar{C}_{n,m}^{[i]}A_{n,m}, \quad (8)$$

$$G_{n,m}^{[i]} = \bar{C}_{n,m}^{[i]}E_{n,m}, \quad (9)$$

$$F_{n,m}^{[i]} = \bar{C}_{n,m}^{[i]}B_{n,m}, \quad (10)$$

where

$$\bar{C}_{n,m}^{[i]} = \text{diag}(\underbrace{C^{[i]} \quad C^{[i]} \quad \dots \quad C^{[i]}}_N), \quad (11)$$

and $E_{n,m}$ can be obtained as (7) by substituting B with E .

2.2 Local UFIR Filter

A local i th subsystem can also be modeled with (4) and (5) and designed as the UFIR filter employing measurements on $[m, n]$ to provide state estimation at n .

In the discrete convolution form, the i th FIR state estimate $\hat{x}_n^{[i]}$ can be obtained in the batch form as

$$\hat{x}_n^{[i]} = H_n^{[i]}Y_{n,m}^{[i]} + L_n^{[i]}U_{n,m}, \quad (12)$$

where $H_n^{[i]}$ and $L_n^{[i]}$ still needs to be determined.

To guarantee unbiasedness in the local estimator (12), the unbiasedness condition $\mathbb{E}\{\hat{x}_n^{[i]}\} = \mathbb{E}\{x_n\}$ must be satisfied. Using the first row vector of (4) and substituting (5) into (12) yield

$$x_n = A^{n-m}x_m + \bar{E}_{n,m}U_{n,m} + \bar{B}_{n,m}W_{n,m}, \quad (13)$$

$$\begin{aligned} \hat{x}_n^{[i]} &= H_n^{[i]}(C_{n,m}^{[i]}x_m + G_{n,m}^{[i]}U_{n,m} + F_{n,m}^{[i]}W_{n,m} + V_{n,m}^{[i]}) \\ &\quad + L_n^{[i]}U_{n,m}, \end{aligned} \quad (14)$$

where $\bar{E}_{n,m}$ and $\bar{B}_{n,m}$ are the first vector rows of $E_{n,m}$ and $B_{n,m}$. By averaging (13) and (14) and referring to the zero-mean assumption of the noise sources, we obtain

$$\begin{aligned} H_n^{[i]}C_{n,m}^{[i]}x_m + H_n^{[i]}G_{n,m}^{[i]}U_{n,m} + L_n^{[i]}U_{n,m} \\ = A^{n-m}x_m + \bar{E}_{n,m}U_{n,m}. \end{aligned} \quad (15)$$

To solve (15), the unbiasedness constraints $H_n^{[i]}C_{n,m}^{[i]} = A^{n-m}$ and $H_n^{[i]}G_{n,m}^{[i]} + L_n^{[i]} = \bar{E}_{n,m}$ must be satisfied that yields

$$H_n^{[i]} = A^{n-m}[(C_{n,m}^{[i]})^T C_{n,m}^{[i]}]^{-1}(C_{n,m}^{[i]})^T, \quad (16)$$

$$L_n^{[i]} = \bar{E}_{n,m} - H_n^{[i]}G_{n,m}^{[i]}. \quad (17)$$

Substituting (16) and (17) into (12) gives

$$\hat{x}_n = H_n^{[i]}(Y_{n,m}^{[i]} - G_{n,m}^{[i]}U_{n,m}) + \bar{E}_{n,m}U_{n,m}, \quad (18)$$

where $H_n^{[i]}$ is specified by (16).

Following the lines developed in [20], the i th local batch UFIR filter (18) can be equivalently realized through an iterative procedure of

$$\hat{x}_l^{[i]*} = A\hat{x}_{l-1}^{[i]} + Eu_l, \quad (19)$$

$$M_l^{[i]} = [(C^{[i]})^T C^{[i]} + (AM_{l-1}^{[i]}A^T)^{-1}]^{-1}, \quad (20)$$

$$\hat{x}_l^{[i]} = \hat{x}_l^{[i]*} + M_l^{[i]}(C^{[i]})^T(y_l^{[i]} - C^{[i]}\hat{x}_l^{[i]*}), \quad (21)$$

where l is an iterative index ranging from $s = m + k - 1$ to n and initial values $M_s^{[i]}$ and $\hat{x}_s^{[i]}$ are computed in short batch forms as

$$M_s^{[i]} = A^{s-m}[(C_{s,m}^{[i]})^T C_{s,m}^{[i]}]^{-1}(A^{s-m})^T, \quad (22)$$

$$\hat{x}_s^{[i]} = H_s^{[i]}(Y_{s,m}^{[i]} - G_{s,m}^{[i]}U_{s,m}) + \bar{E}_{s,m}U_{s,m}. \quad (23)$$

The true filtering result corresponds to $l = n$.

Provided local estimates $\hat{x}_n^{[i]}$, $i \in [1, r]$, the linear optimal fusion strategy can be implemented in the minimum mean square error (MSE) sense as will be shown next.

3 Optimal Decentralized State Fusion

In this section, we first derive an optimal state fusion weighted matrix for the FIR structure and next discuss fusion errors.

Employing local UFIR filters, fusion of filtering estimates can be organized as

$$\hat{x}_n = \alpha_1 \hat{x}_n^{[1]} + \alpha_2 \hat{x}_n^{[2]} + \dots + \alpha_r \hat{x}_n^{[r]}, \quad (24)$$

where α_i , $i \in [1, r]$, are weights, which are required to be optimal. By applying the unbiasedness conditions

$\mathbb{E}\{\hat{x}_n\} = \mathbb{E}\{x_n\}$ and $\mathbb{E}\{\hat{x}_n^{[i]}\} = \mathbb{E}\{x_n\}$ to (24) and taking the expectation on both sides yield

$$I_k = \alpha_1 + \alpha_2 + \dots + \alpha_r. \quad (25)$$

Next, a new variable $\Lambda = [\alpha_1, \alpha_2, \dots, \alpha_r]$ produces the unbiasedness constraint

$$\Lambda \bar{I} = I_k, \quad (26)$$

where $\bar{I} = [I_k, I_k, \dots, I_k]_{kr \times k}^T$. Using (24), we can now calculate the fusion estimation error e_n by

$$\begin{aligned} e_n &= x_n - (\alpha_1 \hat{x}_n^{[1]} + \alpha_2 \hat{x}_n^{[2]} + \dots + \alpha_r \hat{x}_n^{[r]}) \\ &= \alpha_1 (x_n - \hat{x}_n^{[1]}) + \dots + \alpha_r (x_n - \hat{x}_n^{[r]}) \\ &= \Lambda [(x_n - \hat{x}_n^{[1]})^T, \dots, (x_n - \hat{x}_n^{[r]})^T]^T. \end{aligned} \quad (27)$$

Since all of the estimates are unbiased, the error covariance matrix of the fusion estimate can be computed by

$$P = \mathbb{E}\{e_n e_n^T\} = \Lambda \Sigma \Lambda^T, \quad (28)$$

where $\Sigma = [P_{ij}]_{i,j=1,2,\dots,r}$ with $P_{ij} \triangleq \mathbb{E}\{(x_n - \hat{x}_n^{[i]})(x_n - \hat{x}_n^{[j]})^T\}$.

Taking the trace operator, we next define $J = \text{tr}(\Lambda \Sigma \Lambda^T)$ and minimize J subject to the constrain (26). Applying the Lagrange multiplier approach, an auxiliary cost function J^* is constructed as

$$J^* = \text{tr}(\Lambda \Sigma \Lambda^T) + 2\Gamma(\Lambda \bar{I} - I_k), \quad (29)$$

where $\Gamma = (\lambda_{ij})_{k \times k}$ is a $k \times k$ matrix. Setting $\partial J^* / \partial \Lambda = 0$ and considering $\Sigma^T = \Sigma$ allows writing

$$2\Lambda \Sigma + 2\Gamma \bar{I}^T = 0. \quad (30)$$

Combining (30) with the constraint (26) and using the formulation of block matrix inversion, a solution can be found as

$$\bar{\Lambda} = (\bar{I}^T \Sigma^{-1} \bar{I})^{-1} \bar{I}^T \Sigma^{-1}. \quad (31)$$

In the minimum variance sense [17], the optimal linear fusion UFIR filter becomes

$$\hat{x}_n^o = \bar{\alpha}_1 \hat{x}_n^{[1]} + \bar{\alpha}_2 \hat{x}_n^{[2]} + \dots + \bar{\alpha}_r \hat{x}_n^{[r]}, \quad (32)$$

where weights α_i are given by (31), $\bar{\Lambda} = [\bar{\alpha}_1, \bar{\alpha}_2, \dots, \bar{\alpha}_r]_{kr \times k}$ collects the particular weights, and $\Sigma = [P_{ij}]_{i,j=1,2,\dots,r}$ is an $kr \times kr$ symmetric positive definite matrix, which will be considered below.

3.1 Error Cross Covariance P_{ij}

As can be seen, to compute optimal weights $\bar{\alpha}_i$, matrix $\Sigma = [P_{ij}]_{i,j=1,2,\dots,r}$ is required, because it contains the error cross covariances P_{ij} for the i th and j th subsystems.

For the i th sensor, we write the estimation error as

$$\begin{aligned} e_n^{[i]} &= x_n - \hat{x}_n^{[i]} \\ &= [A^{n-m} - H_n^{[i]} C_{n,m}^{[i]}] x_m + \bar{B}_{n,m} W_{n,m} \\ &\quad - H_n^{[i]} (F_{n,m}^{[i]} W_{n,m} + V_{n,m}^{[i]}), \end{aligned} \quad (33)$$

and further reduce it to

$$e_n^{[i]} = (\bar{B}_{n,m} - H_n^{[i]} F_{n,m}^{[i]}) W_{n,m} - H_n^{[i]} V_{n,m}^{[i]} \quad (34)$$

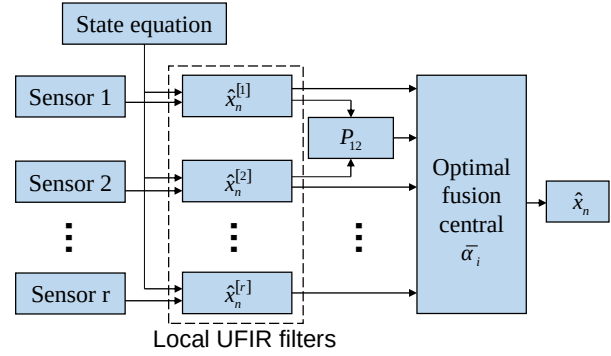


Figure 1. Block diagram of the proposed fusion UFIR filter.

referring to $A^{n-m} = H_n^{[i]} C_{n,m}^{[i]}$. Since $W_{n,m}$ and $V_{n,m}^{[i]}$ are uncorrelated, we have $\mathbb{E}\{W_{n,m} (V_{n,m}^{[i]})^T\} = 0$, and the error cross covariances between the i th and j th sensors can be found as

$$\begin{aligned} P_{ij} &= \mathbb{E}\{e_n^{[i]} (e_n^{[j]})^T\} \\ &= (\bar{B}_{n,m} - H_n^{[i]} F_{n,m}^{[i]}) \mathbb{E}\{W_{n,m} W_{n,m}^T\} \\ &\quad \times (\bar{B}_{n,m} - H_n^{[j]} F_{n,m}^{[j]})^T \\ &\quad + H_n^{[i]} \mathbb{E}\{V_{n,m}^{[i]} (V_{n,m}^{[j]})^T\} (H_n^{[j]})^T \end{aligned} \quad (35)$$

that allows specifying the following covariances

$$\begin{aligned} \bar{Q} &= \mathbb{E}\{W_{n,m} W_{n,m}^T\} \\ &= \mathbb{E}\{[w_n^T, w_{n-1}^T, \dots, w_m^T]^T [\dots]^T\} \\ &= \text{diag}(Q, Q, \dots, Q), \end{aligned} \quad (36)$$

$$\begin{aligned} R_{ij} &= \mathbb{E}\{V_{n,m}^{[i]} (V_{n,m}^{[j]})^T\} \\ &= \mathbb{E}\{[(v_n^{[i]})^T, (v_{n-1}^{[i]})^T, \dots, (v_m^{[i]})^T]^T [\dots]^T\} \\ &= \text{diag}(R_{ij}, R_{ij}, \dots, R_{ij}), \end{aligned} \quad (37)$$

where $R_{ij} = \mathbb{E}\{v_n^{[i]} (v_n^{[j]})^T\}$. Substituting (36) and (37) into (35), the cross error covariance P_{ij} between the i th and j th sensor subsystems is finally given by

$$\begin{aligned} P_{ij} &= (\bar{B}_{n,m} - H_n^{[i]} F_{n,m}^{[i]}) \bar{Q} (\bar{B}_{n,m} - H_n^{[j]} F_{n,m}^{[j]})^T \\ &\quad + H_n^{[i]} R_{ij} (H_n^{[j]})^T. \end{aligned} \quad (38)$$

3.2 Fusion UFIR Filtering Algorithm

At this point, the optimal decentralized state fusion UFIR estimate can be summarized as

$$\hat{x}_n^o = \bar{\alpha}_1 \hat{x}_n^{[1]} + \bar{\alpha}_2 \hat{x}_n^{[2]} + \dots + \bar{\alpha}_r \hat{x}_n^{[r]}, \quad (39)$$

where estimates $\hat{x}_n^{[i]}$, $i \in [1, r]$, are given by (19)-(23), optimal matrix weights α_i by (31), and the error cross covariance P_{ij} by (38). A structure of the proposed algorithm is shown in Fig. 1. As can be seen, each subsystem estimates states independently at n and the estimation errors of any two sensors are fused to determine the cross covariance matrix $[P_{ij}]$, $\{i, j\} \in [1, r]$.

4 Simulations

In this section, we provide simulations employing a two-state polynomial model to demonstrate better performance of the proposed algorithm (denoted as FUF) against the fusion KF (denoted as FKF) in different noise environments. To this end, we consider a time-invariant system combined with three sensors

$$x_{n+1} = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix} x_n + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_n + \begin{bmatrix} 0.1 \\ 1 \end{bmatrix} w_n, \quad (40)$$

$$y_n^{[i]} = C^{[i]} x_n + v_n^{[i]}, \quad (41)$$

where $i \in [1, 3]$, $\tau = 0.1$, and $u_n = 0.1$. The covariances of process noise w_n and measurement noise $v_n^{[i]}$ are $Q = 0.1$, $R^{[1]} = 5$, $R^{[2]} = 8$, and $R^{[3]} = 10$.

4.1 Comparing Local and Fusion Performances

To show that FUF outperforms any of the local UFIR filters, we assume that the noise statistics and modeling parameters are known accurately. We then simulate the process at 500 points and provide filtering with $N = 20$. The average root MSEs (RMSEs) produced by the FUF and three sub-filters are shown in Fig.2 based on 100 Monte-Carlo runs. Even a quick look shows that the FUF has better accuracy than any of the local filters.

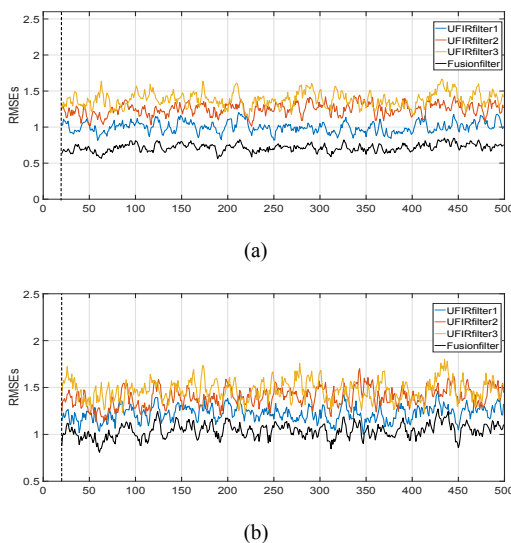


Figure 2. RMSEs produced by local and fusion filters: (a) first state and (b) second state.

4.2 Robustness Against Incorrectness in Noise Covariances

Information of noise not always completely available that may severely affect estimates. To investigate effect of errors in the noise covariances, in this part we introduce a scaling coefficient β and substitute Q with $Q^* = \beta Q$.

By defining $J_n = E \{ (x_n - \hat{x}_n^o)(x_n - \hat{x}_n^o)^T \}$, the average RMSEs computed by $\sqrt{\text{tr}(J_n)}$ over 100 Monte-Carlo

Table 1. Average RMSEs $\sqrt{\text{tr}(J_n)}$ produced by FUF and FKF for different β

β	0.01	0.1	1	10	100
FKF	14.8	5.10	1.23	1.39	3.10
FUF	1.25	1.23	1.25	1.26	1.32

runs for the FKF and FUF with $\beta = 0.01, 0.1, 1, 10, 100$ are listed in Table 1. For each β , the smallest values are bolded for clarity. The results are supported with Fig. 3, in which three sub-figures sketch the RMSEs for $\beta = 0.1$, $\beta = 1$, and $\beta = 100$. It follows from Fig. 3 that the FUF performance is less vulnerable to β than that of FKF. Specifically, when noise is exactly known by $\beta = 1$, the difference between the FUF and FKF outputs is indistinguishable. When $\beta \neq 1$, errors in FKF grow more rapidly and may become unacceptable, while the FUF still provides good estimates even when $\beta \gg 1$ and $\beta \ll 1$.

4.3 Robustness Against Temporary Model Uncertainty

It is known that temporary uncertainties such as jumps in frequency and velocity [6] may severely affect estimates on a short time span. To learn effects associated with such phenomena, we set

$$A = \begin{bmatrix} 1 & \tau + d \\ 0 & 1 \end{bmatrix},$$

where $d = 0.1$ acts on $100 \leq n \leq 150$ and $d = 0$ otherwise. Because d is unknown, all algorithms are run with $d = 0$ over all time.

Figure 4a and Fig. 4b illustrate typical responses of the FKF and FUF to model mismatch at $100 \leq n \leq 150$ for the two-state polynomial model. As can be seen, the FKF estimates of the first state are poor and get worse than in the FUF. The difference between the FUF and FKF estimates of the second state is less pronounced, but the observations are generally the same.

5 Conclusion

Under the linear unbiased minimum variance rule, this paper derives an optimal fusion UFIR filter for linear discrete time-invariant system with multiple sensors. The proposed algorithm has several useful characteristics. Its accuracy is fundamentally better than in any of local UFIR filters. Under the idea conditions, the FUF provides accuracy, which is closely related to FKF. When the underlying model is uncertain, the FUF outperforms the FKF as being a more robust estimator. Although better performance of the FUF has been confirmed, the determination of optimal horizons for local UFIR filters remains an open problem.

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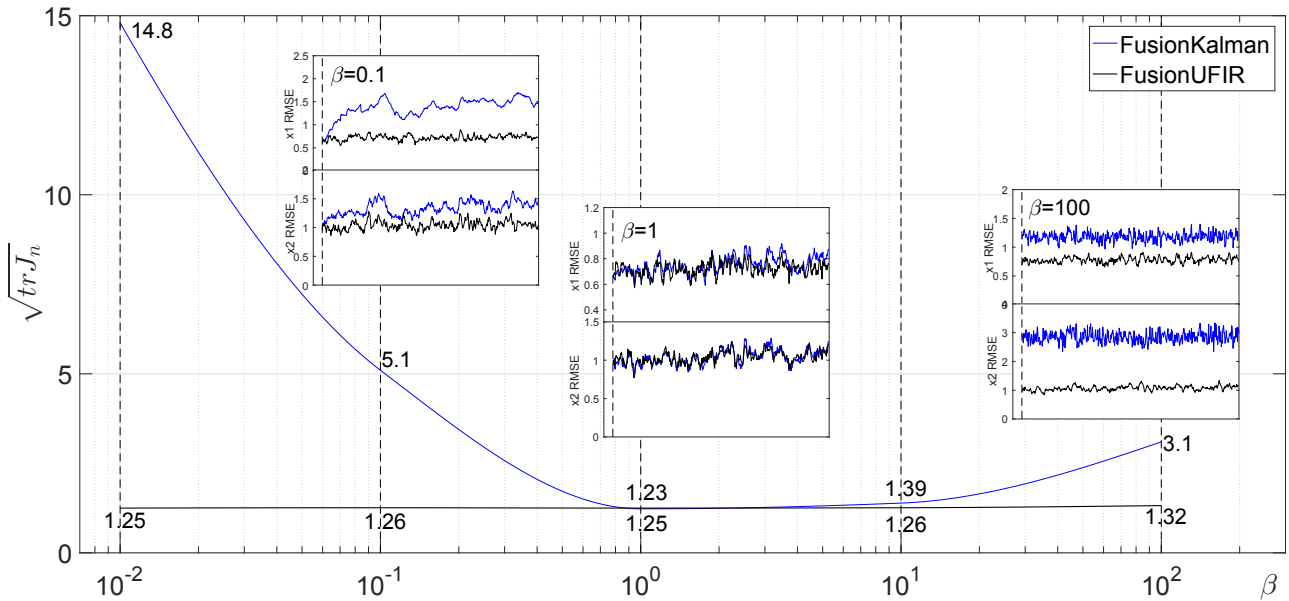


Figure 3. Averaged $\sqrt{\text{tr}(J_n)}$ caused by system noises uncertainties with $\beta = 0.01, 0.1, 1, 10, 100$ for FKF and FUF.

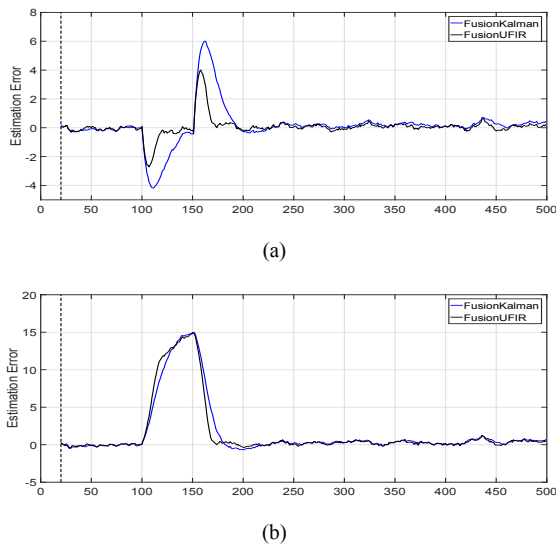


Figure 4. Estimation errors produced by FKF and FUF in response to temporary model uncertainty in a gap of $100 \leq n \leq 150$: (a) first state and (b) second state.

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