An optimized 2D-Robust Adaptive Beamforming algorithm based on Matrix Completion in sparse array

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Abstract. The sparse arrays can reduce the cost of beamforming, it greatly reduces the number of actual array elements. However, it also brings about the problem of information loss. A 2D-robust adaptive beamforming algorithm in sparse array based on Singular Value Thresholding algorithm is proposed. At first, a signal model of planar array is established based on Matrix Completion, which can be proved to meet Null Space Property. Then the Genetic Algorithm is used to optimize the sparse array, which is determined to reduce the Spectral Norm Error of Matrix Completion and make the array recovered closer to the full array. In the case of sparse array, the missing information is restored by using the theory of Singular Value Thresholding, and then the restored signal is used to design the digital beamformer weights. This algorithm significantly reduces the Spectral Norm Error and forms robust adaptive beam.

1 Introduction

Digital Beam Forming (DBF) technology uses advanced digital signal processing techniques to process signals received by an array antenna. It can greatly improve the anti-jamming capability of radar system, and is one of the key technologies for radar to improve the performance of target detection. In order to implement 2D-robust adaptive beamforming effectively, a large number of antennas and sampling devices are required. There is a problem of low utilization of array elements, which means increased hardware complexity and cost increase in engineering. Since 1960s, sparse arrays have been extensively studied because of their high target resolution and low cost[1]. Sparse arrays can reduce the number of antennas, but the average sidelobe increases with the number of units removed, therefore, the application of it is limited. For low-rank matrix, the Matrix Completion (MC) theory [2],[3],[4],[5] can recover the complete matrix by partial matrix elements, it is a generalization of the theory of compressed sensing on matrix, it is widely used in image processing and pattern recognition. The Singular Value Thresholding (SVT) algorithm is a fast and effective algorithm for matrix completion[6], it brings great convenience to solve the problem above.

Spectral Norm Error(SNE) is an important index to measure the quality of MC. The smaller the SNE, the more accurate the restored matrix. In order to reduce the SNE of MC, we have to find an optimum set of array elements. The Genetic Algorithm(GA) is applied to optimize the set of array elements[7],[8]. It seeks the optimal solution by imitating the mechanism of selection and inheritance of nature. A new 2D robust adaptive beamforming[9] MC model is constructed, the signal matrix is proved to satisfy the Null Space Property(NSP). Thus, an algorithm is proposed to realize adaptive two dimensional robust beamforming[10],[11],[12].

2 Signal model

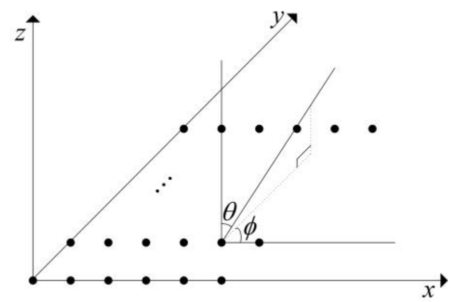

Fig. 1. URA

Now we consider a uniform rectangular array[13] (URA). Fig.1 shows the distribution of the full array elements, where the number of elements in x-direction and y-direction is M_x and M_y. The distance between the neighboring elements in x-direction and y-direction is d_x and d_y. (ϕ,θ) means the pitch and the azimuth angle of the target.

Assume K targets are contained in received signals, , and $s_i(t)$ refers to the waveform of the i th target, (ϕ_i,θ_i) means the elevation and azimuth angle of the i th target and N is the number of snapshot. Suppose the uncorrelated narrow-band signals are received in far field, thus the received signal of M_x elements in x-direction can be written as

$$\mathbf{Y}_x(t) = \mathbf{A}_x\mathbf{S}(t) + \mathbf{N}_x(t) \quad (1)$$

where $S(t) = diag(s_i(t))$ is the diagonal matrix of $K \times K$ dimensions, $\mathbf{N}_x(t)$ is the received noise, $\mathbf{A}_x$ is the steering vector of $K \times K$ dimensions, which can be given by

$$\mathbf{A}_x = [\mathbf{a}_x(\phi_1,\theta_1), \mathbf{a}_x(\phi_2,\theta_2), \cdots, \mathbf{a}_x(\phi_K,\theta_K)] \quad (2)$$

Suppose the wavelength of received signal is λ, then $\mathbf{a}_x(\phi_i,\theta_i)$ is

$$\left[\mathbf{a}_x(\phi_i,\theta_i)\right]_{m_x} = e^{j\frac{2\pi}{\lambda}d_x\sin(\theta_i)\cos(\phi_i)(m_x-1)} \quad (3)$$

where $m_x = 1, 2, \cdots, M_x$. Likewise, the received signal of M_y elements in y-direction can be represented as

$$\mathbf{Y}_y(t) = \mathbf{A}_y\mathbf{S}(t) + \mathbf{N}_y(t) \quad (4)$$

where $\mathbf{N}_y(t)$ is the received noise, $\mathbf{A}_y$ is the steering vector of $K \times K$ dimensions, which can be denoted as

$$\mathbf{A}_y = [\mathbf{a}_y(\phi_1,\theta_1), \mathbf{a}_y(\phi_2,\theta_2), \cdots, \mathbf{a}_y(\phi_K,\theta_K)] \quad (5)$$

where $\mathbf{a}_y(\phi_i,\theta_i)$ is defined as

$$\left[\mathbf{a}_y(\phi_i,\theta_i)\right]_{m_y} = e^{j\frac{2\pi}{\lambda}d_y\sin(\theta_i)\cos(\phi_i)(m_y-1)} \quad (6)$$

where $m_y = 1, 2, \cdots, M_y$. Therefore, the received signal model of URA can be demonstrated as

$$\mathbf{Y}(t) = \mathbf{A}_x\mathbf{S}(t)\mathbf{A}_y^T + \mathbf{N}(t), t = 1, 2, \cdots, N \quad (7)$$

When the noise power is far smaller than the signal powers, we can get the $rank(\mathbf{Y}(t)) \le rank(\mathbf{S}(t)) = \eta$, namely, matrix $\mathbf{Y}(t)$ is low rank[14]. And matrix in (7) meets strong incoherence property, thus it can be recovered with high probability via MC[15].

3 Beaming forming algorithm

3.1 GA sampling

The original matrix cannot be recovered from the sampled matrix by MC when $m < r(n_1 + n_2 - r)$ [16]. Where the n_1, n_2 are the number of rows and columns for the target matrix and r is the rank of it. The smaller the SNE is, the closer the restored matrix is to the full array, and the higher accuracy of the system. The influence of sparse array elements on SNE can not be ignored. GA is used to optimize the sparse array in this paper to reduce the SNE.

$$\mathbf{Y}_S(t) = (d_1, d_2, \cdots, d_m) \quad (8)$$

It is assumed that $\mathbf{Y}_S(t)$ is the sparse array consisting of m array elements $d_1, d_2, \cdots, d_m$. The optimization of spatial sampling array selection based on GA can be expressed as follows:

$$\min SNE = f(d_1, d_2, \cdots, d_m) \quad (9)$$

Take the sparse array $\mathbf{Y}_S(t)$ as the decision variable. A sparse array is chosen as an individual, the fitness function of GA is used to evaluate the merits of each individual. In order to reduce the optimization objective SNE, the fitness function is

$$f(d_1, d_2, \cdots, d_m) = SNE = \|\mathbf{M} - \mathbf{Y}\|_2 / \|\mathbf{M}\|_2 \quad (10)$$

where $\mathbf{Y}_S(t)$ is the full array, $\mathbf{M}$ is the matrix recovered by MC.

The objective function can be written as

$$f(d_1, d_2, \cdots, d_m) = \min\left\{fit\left(d_1, d_2, \cdots, d_m\right)\right\} \quad (11)$$

The optimized spatial sampling array selection algorithm based on GA is as follows:

Step 1 Initial population establishment.
Step 2 The fitness function of individuals in a population is calculated.
Step 3 Determine whether the optimization criteria are met and, if satisfied, turn to step 8, or continue.
Step 4 Select dominant individuals.
Step 5 Generalized cross operation.
Step 6 Generalized mutation operation.
Step 7 Turn to step 2.
Step 8 The end of the output is the best individual.

Full rectangular antenna array can be represented as $\mathbf{Y}_R(t)$, P_Ω is the sparse sampling operator. Thus, a signal model based on MC can be represented as

$$\begin{cases} \min \left\|\mathbf{Y}_R(t)\right\|_* \\ s.t. \quad P_\Omega\left(\mathbf{Y}_R(t)\right) = P_\Omega\left(\mathbf{Y}_S(t)\right) \end{cases} \quad (12)$$

Null space of sampling operator is $Null\left(P_\Omega\right) = \left\{\mathbf{M} \in \mathbf{R}^{n_1 \times n_2} : P\left(\mathbf{M}\right) = 0\right\}$. According to the NSP, the matrix in the null space is hard to recover. As we analyze the array signal, on account of vector $\mathbf{a}_x\left(\phi_i,\theta_i\right) \ne 0$, hence, elements in $\mathbf{A}_x$ and $\mathbf{A}_y$ are all non-zero. The elements in $\mathbf{S}(t)$ are non-zero, so the elements in $\mathbf{Y}_R(t) = \mathbf{A}_x\mathbf{S}(t)\mathbf{A}_y^* + \mathbf{N}(t)$ are non-zero. Thus, we can draw the conclusion that $P_\Omega\left(\mathbf{Y}_R(t)\right) \ne 0$, which means the matrix meets NSP.

3.2 SVT-EXP-LCMV algorithm

After the establishment of the sparse array, the SVT algorithm is applied to recover the full signal, and the EXP-LCMV algorithm is used to form the robust beam. SVT algorithm combines Bregman iteration with fixed point iteration[17], the algorithm is simple and fast. It is suitable for solving the signal model in this paper.

In SVT algorithm, the signal model of MC is transformed as (11). The corresponding iterative process can be expressed as the following formula, which can be solved by the corresponding iterative algorithm

$$\begin{cases} \mathbf{Y}_R^k(t) = D_\tau\left(\mathbf{Z}^{k-1}\right) \\ \mathbf{Z}^k = \mathbf{Z}^{k-1} + \delta_k P_\Omega\left(\mathbf{Y}_S(t) - \mathbf{Y}_R^k(t)\right) \end{cases} \quad (13)$$

where δ_k is the step size of each iteration. The initial value of the iteration step is

$$\delta_0 = 1.2\frac{m_1 \times m_2}{m} \quad (14)$$

where m_1, m_2 are the number of rows and columns of a matrix, m is the number of data in $\mathbf{Y}_S(t)$, and 1.2 is down sample rate multiple.

Firstly, singular value decomposition of the matrix $\mathbf{Z} \in \mathbf{R}^{m_1 \times m_2}$ is

$$\mathbf{Z} = \mathbf{U}_Z diag(\sigma)\mathbf{V}_Z \quad (15)$$

For each given positive number τ, $D_\tau(\mathbf{Z})$ can be represented as

$$D_\tau(\mathbf{Z}) = \mathbf{U}_Z diag(\{\sigma - \tau\}_+)\mathbf{V}_Z^T \quad (16)$$

After the iterative process, the corresponding left and right singular value vectors $\mathbf{U}_S$ and $\mathbf{V}_S$ of the sparse array. Without the consideration of noise, the sparse array can be written as

$$\mathbf{Y}_R(t) = \mathbf{A}_x\mathbf{S}(t)\mathbf{A}_y^* = \mathbf{U}_S\mathbf{\Sigma}\mathbf{V}_S^* \quad (17)$$

Vectorize(17), it holds that

$$vec\left(\mathbf{Y}_R(t)\right) = vec\left(\mathbf{A}_x\mathbf{S}(t)\mathbf{A}_y^T\right) = vec\left(\mathbf{U}_S\mathbf{\Sigma}\mathbf{V}_S^*\right) \quad (18)$$

Based on Kronecker product property, (17) can be transformed in vector form as

$$\mathbf{y}_R(t) = \mathbf{A}_y \otimes \mathbf{A}_x vec\left(\mathbf{\Sigma}(t)\right) = \bar{\mathbf{V}}_S \otimes \mathbf{U}_S vec\left(\mathbf{\Sigma}\right) \quad (19)$$

Solving the auto-correlation matrix $\mathbf{R}_Y$ of $\mathbf{y}_R(t)$, we get that

$$\mathbf{R}_Y = \mathbf{U}_S\mathbf{\Sigma}_S\mathbf{V}_S^H + \mathbf{U}_n\mathbf{\Sigma}_n\mathbf{V}_n^H \quad (20)$$

where $\mathbf{\Sigma}_S$ is the diagonal matrix of large eigenvalues, $\mathbf{U}_S$ is the interference subspace, $\mathbf{\Sigma}_n$ is the diagonal matrix of small eigenvalues, and $\mathbf{U}_n$ is the noise subspace.

In robust adaptive beamforming, static weight vector of Linear Constrained Minimum Variance(LCMV)[16] is

$$w_{LCMV} = \mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{f} \quad (21)$$

where $\mathbf{C}$ is the constraint matrix of dimension $m_x m_y \times R$, $\mathbf{f}$ is the constraint response vector of dimension $R \times 1$, R is the rank of correlation matrix $\mathbf{R}_Y$. For the purpose of forming a two-dimensional robust adaptive beam, the covariance matrix of input signal of LCMV algorithm is corrected by exponential method to improve the degree of divergence of small eigenvalue in the noise subspace and improve the robustness of beamforming at low snapshot.

We can get $\mathbf{R}_Y^\alpha = \mathbf{U}_S\Lambda_S^\alpha\mathbf{U}_S^H + \mathbf{U}_n\Lambda_n^\alpha\mathbf{U}_n^H$ by correcting $\mathbf{R}_Y$, where α is correction index, and $\alpha \in [0,1]$.

The diagonal matrix of the small eigenvalues of the modified noise subspace is

$$\Lambda_n^\alpha = diag\left\{\sigma_1^\alpha, \sigma_2^\alpha, \cdots, \sigma_n^\alpha\right\} \quad (22)$$

Hence, the small eigenvalues of the noise subspace are updated to $\sigma_i^\alpha, i = 1, 2, \cdots, n$

Since $\alpha \in [0,1]$, the degree of divergence $\sigma_i^\alpha, i = 1, 2, \cdots, n$ is improved.

The small eigenvalue divergence in the noise phonon space decreases with the decrease of the correction exponent α. The weighted vector of the modified EXP-LCMV algorithm is

$$w_{EXP-LCMV} = \mathbf{R}_X^{-\alpha}\mathbf{C}\left(\mathbf{C}^H\mathbf{R}_X^{-\alpha}\mathbf{C}\right)^{-1}\mathbf{f} \quad (23)$$

4 Simulations

The effectiveness of the proposed SVT-EXP-LCMV algorithm is verified by simulations in this section. Let the full array be a 64×64 URA where the total amount of array elements is $M = 4096$ and the array element spacing in x-direction and y-direction are equal. We sample array elements of the URA by GA to build a sparse array. Assume the pitch angle of main gain direction is 45°, and the azimuth angle is 90°. The corresponding pitch angles and azimuth angles of the three incoherent interference signals in the space are $(50°, -120°)$, $(55°, -70°)$, $(45°, 90°)$. Set Interference Noise Ratio (INR) as 20dB, and the number of snapshots in the following experiments remains 50.

4.1 The comparison under different arrays

We select 1200 elements from the URA. In Fig.2-Fig.4, the 2D robust beamforming in different arrays are shown.

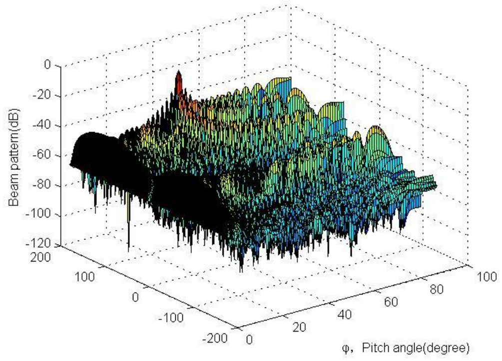

Fig. 2. Beamforming in Full Array

The 2D beamforming in full array is shown in Fig.2. It shows that the main peak value is 0dB and average sidelobes level is -65.49dB . The 2D beamforming in sparse array is shown in Fig.3. It shows that the main peak value is 0dB and average sidelobes level is -32.85dB. In the third experiment, 2D beamforming by SVT-EXP-LCMV algorithm is examined in the same sparse array.

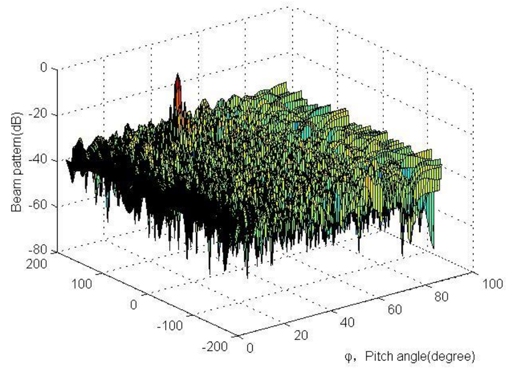

Fig. 3. Beamforming in Sparse Array

From Fig.4, it can be noted that peak value differs by -15.76dB from main peak and main sidelobe, and average sidelobes level is -62.69dB. Compared to the full array, the number of sparse array elements is reduced by 70%. The beam can be effectively formed, and the problem of proportional deterioration of the average sidelobe as the number of units removed is effectively solved at the same time.

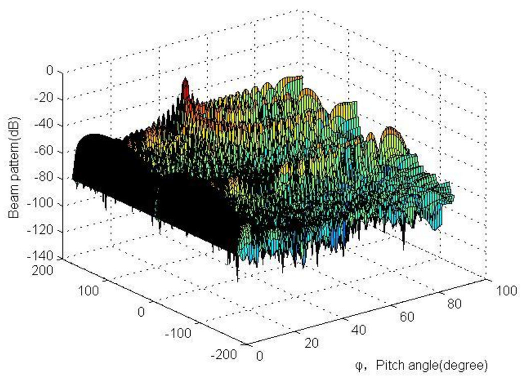

Fig. 4. Beamforming in MC

4.2 The comparison of SNE under different sampling ways

Table 1 is the SNE under the random matrix selection method and the method of element selection based on genetic algorithm.

It can be seen from the table that the number of sparse array elements is the same at the same time, and the SNE of matrix filling under the method of element selection based on Genetic Algorithm is smaller than that of the average matrix selection method. The advantage of the array element selection method based on GA is explained. algorithm. It can be seen from the table that the number of sparse array elements is the same at the same time, and the SNE of matrix filling under the method of element selection based on Genetic Algorithm is smaller than that of the average matrix selection method. The advantage of the array element selection method based on GA is explained.

Table 1. SNE under GA and random sampling

	The number of the sparse elements		
	900	1200	1500
SNE under GA sampling	2.75×10^{-2}	6.68×10^{-3}	4.53×10^{-3}
SNE under random sampling	4.62×10^{-2}	3.55×10^{-2}	6.87×10^{-3}

5 Conclusion

In this paper, an exponential correction beamforming (SVT-EXP-LCMV) algorithm based on singular value threshold (SVT) is proposed, and the genetic algorithm (GA) is used to optimize the sparse array elements. The restoration error of the matrix filling is reduced, and a robust two-dimensional beam forming direction map is generated, which has a good ratio of main lobe to side lobe, small number of array elements and low cost.

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