Finite Difference Method in Stress Analysis of Anchorage Zone of Highway Extradosed Cable Stayed Bridge

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Abstract. With the development and wide application of extradosed cable stayed bridge, an effective method is needed for simulation of the stress distribution in anchorage zone, which plays a vital role in transferring force from the cable to the pylon and diverting the cable direction. Based on the Finite Difference Method (FDM), an efficient and practical method of stress filed simulation was presented in the paper. First, a plane finite element model was established using ABAQUS, to determine the boundary condition for FDM. Based on this, FDM was used to simulate the stress distribution of the concrete in the anchorage zone. Finally, on the simulation result was calculated and in comparison with finite element model result. It has been found that: The vertical compressive stress of concrete in the anchorage zone gradually reduces from the middle to the two sides, and the stress assumes double peak type in transverse. Compared with the finite element solutions, the approximate solution simulated by FDM improve the computational efficiency with certain accuracy keeping, except the loading area. In vertical orientation, the concrete in the place of H (the width of the pylon) from cable force acting position is in a state of uniform stress.

1 Introduction

Finite difference method (FDM) is one of the simplest and oldest numerical simulation methods. The advent of finite difference techniques in numerical applications began in the early 1950s, and then it was widely used with constant development. The principle of FDM is close to numerical schemes used to solve ordinary differential equations. FDM usually included three steps: (1) the solution domain is divided into grid, and (2) the differential equation is approximated by the difference equation at the node of the grid, (3) the direct method or iterative method is used to solve the difference equations to get the approximate solution of the differential equation [1].

As a well-established numerical analysis method, FDM is widely used in the fields of civil engineering, material forming and other fields. Liao D M et al simulates the stress field, fluid flow field, temperature field and stress field simulation all based on FDM [2]; Chuang K P et al analyse cylindrical shell roofs with FDM [3]; Shen J B simulates the thermal behaviour of classical or composite Trombe solar walls with FDM[4]; Appelö D proposed a stable finite difference method for the elastic wave equation on complex geometries with free surfaces[5]; Griffin D S et al solved some plane elasticity problems [6].

2 Problem of stress distribution in anchorage zone of extradosed cable stayed bridge

As shown in Fig.1, Compared with the conventional Cable-Stayed Bridge, cables in extradosed cable stayed bridge could be anchored in the pylon directly, or through the pylon by sub-steel-pipe and double casing saddle and then anchored in the girder on the other side [7]. The anchorage zone in the pylon is a key part of the bridge design especially for saddle anchorage form, and its stress is so complex that researchers need to pay more attention to it.

Most recent results of the stress distribution researches were based on the general finite element software or measured in the model test [8-11]. Although the simulation results obtained possesses the advantages of high accuracy and stability, there are some defects. With the first approach, it needs proficient with the application of the software whose principle is very complex. For the second approach, researcher will spend more money and time to get the rule of the stress distribution. Therefore, a simplified the stress simulation method is needed to improve the efficiency of calculation, which will provide guidance for the design and construction of extradosed cable stayed bridge.

In this study, a new simulation method based on FDM is applied in the anchorage zone stress distribution simulation. This method could improve the efficiency of calculation with a few grids, and the results show good agreement with it finite element model results.

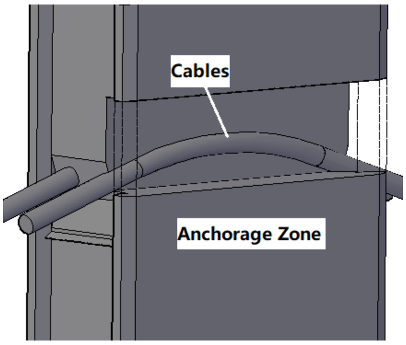

Figure 1. Anchorage zone

3 FDM simulation

According to the research results of Liu Z et al [12], the stress distribution in transverse direction could be simplified as a plane problem. Also, the FDM for simulating the stress distribution in this study was based on a plane model.

3.1 Stress boundary condition

In the analysis process, boundary condition has significant influence on result. Therefore, it is very important to determine it. The boundary condition includes the load boundary condition and the stress boundary condition. As the load boundary condition, according to Saint-Venant's Principle (the difference between the effects of two different but statically equivalent loads becomes very small at sufficiently large distances from load), the uniformly stress distribution of concrete is assumed in a position far away from the saddle, which means the boundary condition determination problem is transformed into distance determination problem. Although the stress distribution becomes more uniformity with the increase of distance, it could improve the simulation efficiency with a small domain.

In order to determine the nearest position with uniform pressure, a plane finite element model is established using ABAQUS as show in Figure 2. Besides, a new parameter called deviation rate was defined as follow:

$$DevR = \frac{(\sigma_{\max} - \sigma_{\min})}{\sigma_{AVG}} \times 100\% \quad (1)$$

Where DevR= deviation rate, the smaller of the DevR means the distribution of the stress is uniformity, the DevR of uniform load is 0; = maximum stress, minimum stress and average stress, respectively.

Figure 2 shows the relationship between DevR and distance. The stress of the concrete in line A, B, C, D and E were obtained by the FEM. It can be seen that the DevR is getting smaller as the distance increases. When the distance is increased to H, at line C, DevR equals 4.01%, which could believe that the stress at line C is uniform. So it is suggested that the depth of the anchorage zone using for analyzing equals the anchorage zone's width. In other words, the analysis domain is recommended as a square. Besides, the effect of the saddle on the concrete was assumed as concentrated force according to Saint-Venant's Principle. The grid spacing equals H/6 for the calculation of example, and the coordinate axis is shown in Figure 3.

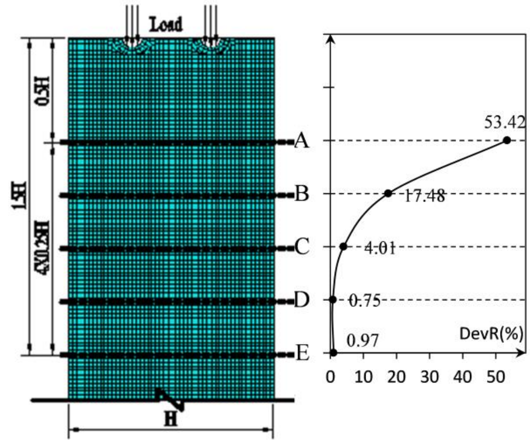

Figure 2. DevR

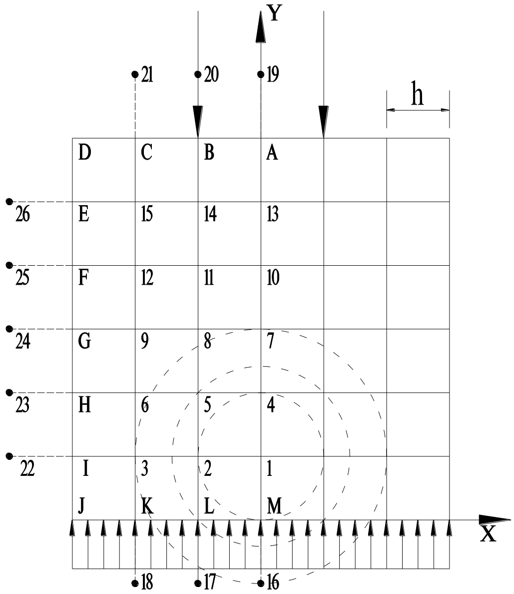

Figure 3. The grid

After the load boundary condition determined, the stress boundary condition is needed to be determined before establish the finite difference model. Only half domain is required to be calculated because of the symmetry. Take node A as the reference point, and let $\Phi_A = \left(\frac{\partial \Phi}{\partial x}\right)_A = \left(\frac{\partial \Phi}{\partial y}\right)_A = 0$.

In addition, some virtual nodes were added outside the boundary for calculation and the value of the values of those virtual nodes were calculated by the values of the nodes in the boundary and the nodes next to the boundary.

1) On the upper and lower sides, there is no surface force in X direction, which means $\frac{\partial \Phi}{\partial y} = 0$. So,

$$\Phi_{16}=\Phi_1,\ \Phi_{17}=\Phi_2,\ \Phi_{18}=\Phi_3,\\ \Phi_{19}=\Phi_4,\ \Phi_{20}=\Phi_{14},\ \Phi_{21}=\Phi_{15} \quad (2)$$

2) On the left side, $\left(\frac{\partial\Phi}{\partial\mathbf{x}}\right)_{I,H,G,F,E}=3\text{qh}$,

$\Phi_{3,6,9,12,15}=\Phi_{22,23,24,25,26}+2h\left(\frac{\partial\Phi}{\partial x}\right)_{I,H,G,F,E}$. So,

$$\Phi_{22,23,24,25,26}=\Phi_{3,6,9,12,15}-6qh^2 \quad (3)$$

Where $\frac{\partial\Phi}{\partial x}$, $\frac{\partial\Phi}{\partial y}$ = the sum of the surface forces in the X, Y direction, respectably.

Table 1. Stress boundary condition

Node	$\frac{\partial\Phi}{\partial x}$	$\frac{\partial\Phi}{\partial x}$	Φ
A	0	0	0
B	$3qh$	0	0
C	$3qh$	0	$-3qh^2$
D	$3qh$	0	$-6qh^2$
E,F,G,H,I	$3qh$	0	$-6qh^2$
J	$3qh$	0	$-6qh^2$
K	$2qh$	0	$-3.5qh^2$
L	qh	0	$-2qh^2$
M	0	0	$-1.5qh^2$

3.2 Difference formulas

By applying Taylor series, Difference formulas could be derived 5. Take node 1 as an example:

$$\left(\frac{\partial^4\Phi}{\partial x^4}\right)_1=\frac{1}{h^4}\left[6f_1-4\left(f_2+f_P\right)+\left(f_3+f_R\right)\right] \quad (4)$$

$$\left(\frac{\partial^4\Phi}{\partial x^2\partial y^2}\right)_1=\frac{1}{h^4}[4f_1-2\left(f_4+f_M+f_2+f_P\right)\\ +\left(f_5+f_L+f_O+f_Q\right)] \quad (5)$$

$$\left(\frac{\partial^4\Phi}{\partial y^4}\right)_1=\frac{1}{h^4}\left[6f_1-4\left(f_4+f_M\right)+\left(f_7+f_{16}\right)\right] \quad (6)$$

3.3 Difference equations

The stress differential equation is：

$$\frac{\partial^4\Phi}{\partial x^4}+2\frac{\partial^4\Phi}{\partial x^2\partial y^2}+\frac{\partial^4\Phi}{\partial y^4}=0 \quad (7)$$

From equation (4) - (7), the difference equations could be obtained at node 1 with the symmetry considering as,

$$20\Phi_1-8(2\Phi_2+\Phi_4+\Phi_M)+2\left(2\Phi_5+\Phi_L\right)\\ +\left(2\Phi_3+\Phi_7+\Phi_{16}\right)=0 \quad (8)$$

Substituting the known Φ, equation (8) is reduced to:

$$21\Phi_1-16\Phi_2+2\Phi_3-8\Phi_4+4\Phi_5+\Phi_7+4qh^2=0 \quad (9)$$

The same procedure may be adapted to obtain the difference equations of all nodes. Finally, 15 difference equations with 15 unknowns were got at node 1to 15. And Φ of the node1 to 15 was obtained by direct solution (unit: qh^2).

$\Phi_1=-1.44,\ \Phi_2=-1.95,\ \Phi_3=-3.49,\ \Phi_4=-1.27,$
$\Phi_5=-1.82,\ \Phi_6=-3.44,\ \Phi_7=-0.99,\ \Phi_8=-1.58,$
$\Phi_9=-3.34,\ \Phi_{10}=-0.58,\ \Phi_{11}=-1.19,\ \Phi_{12}=-3.19,$
$\Phi_{13}=-0.13,\ \Phi_{14}=-0.62,\ \Phi_{15}=-3.03.$

In addition, for the nodes at boundary and virtual nodes were calculated by equation (2)-(3):

$\Phi_{16}=-1.44,\ \Phi_{17}=-1.95,\ \Phi_{18}=-3.49,\ \Phi_{19}=-0.13,$
$\Phi_{20}=-0.62,\ \Phi_{21}=-3.03,\ \Phi_{22}=-9.49,\ \Phi_{23}=-9.44,$
$\Phi_{24}=-9.34,\ \Phi_{25}=-9.19,\ \Phi_{26}=-9.03.$

The vertical stress is much greater than the transverse stress, so the vertical stress is the key point what were concerned in this study. The vertical stress can be calculated:

$$\left(\sigma_y\right)_L=\frac{1}{h^2}\left[\left(\Phi_K+\Phi_M\right)-2\Phi_L\right]\\ =\left(-3.5-1.5+2\times2\right)q=-1.0q.$$

$\left(\sigma_y\right)_1=-1.02q,\ \left(\sigma_y\right)_2=-1.02q,\ \left(\sigma_y\right)_3=-0.99q,$
$\left(\sigma_y\right)_4=-1.10q,\ \left(\sigma_y\right)_5=-1.07q,\ \left(\sigma_y\right)_6=-0.94q,$
$\left(\sigma_y\right)_7=-1.18q,\ \left(\sigma_y\right)_8=-1.17q,\ \left(\sigma_y\right)_9=-0.90q,$
$\left(\sigma_y\right)_{10}=-1.22q,\ \left(\sigma_y\right)_{11}=-1.39q,\ \left(\sigma_y\right)_{12}=-0.81q,$
$\left(\sigma_y\right)_{13}=-0.98q,\ \left(\sigma_y\right)_{14}=-1.92q,\ \left(\sigma_y\right)_{15}=-0.56q.$
$\left(\sigma_y\right)_A=0,\ \left(\sigma_y\right)_B=-3.0q,\ \left(\sigma_y\right)_C=0,\ \left(\sigma_y\right)_D=0,$
$\left(\sigma_y\right)_E=-0.06q,\ \left(\sigma_y\right)_F=-0.38q,\ \left(\sigma_y\right)_G=-0.68q,$
$\left(\sigma_y\right)_H=-0.88q,\ \left(\sigma_y\right)_I=-0.96q,\ \left(\sigma_y\right)_J=-0.5q,$
$\left(\sigma_y\right)_K=-0.56q,\ \left(\sigma_y\right)_M=-0.98q.$

4 Verification

In order to verify the accuracy of the finite difference method, the solution was compared with the solution of the finite element model and in-place field test data.

In the finite element model, q takes as 100N/m for the convenience of comparison. The vertical stress distribution in anchorage zone is show in Figure 4 and Figure 5. It can be seen that the simulation results of stress distribution by FDM and Finite element model is very similar. The vertical compressive stress of concrete in the anchorage zone gradually reduces from the middle to the two sides. The stress assumes double peak type in transverse, and the peak is under load position. In the meanwhile, the stress distribution rules got in the finite element model is in good agreements with it.

Figure 6 shows the error between FDM results and finite element model results in node 1-15. Most of the errors are less than 5%, and the maximum error is 10%. That is to say, the FDM could simulate the internal stress

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field distribution with high efficiency, while maintaining good accuracy. But for the loading node, the stress error of B point is 70%.

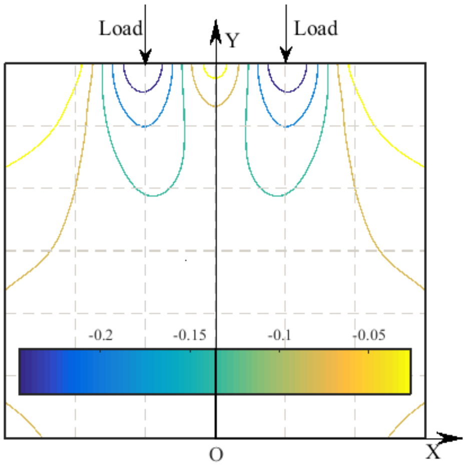

Figure 4. Vertical stress distribution simulated by FDM (unit: MPa)

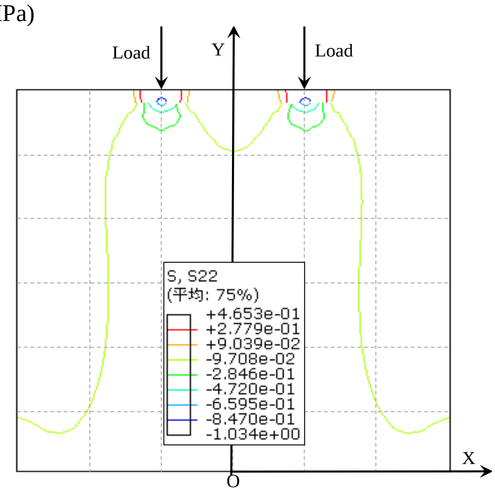

Figure 5. Vertical stress distribution simulated by simulated by Finite element model (unit: MPa)

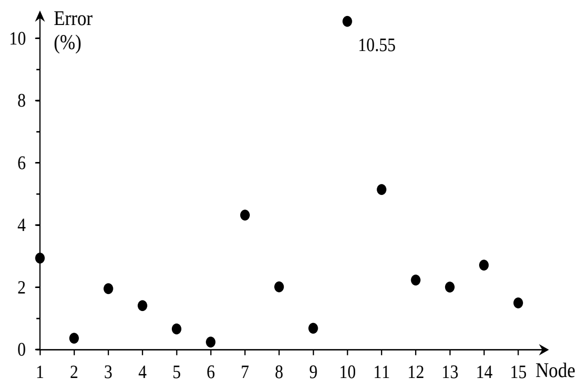

Figure 6. The stress error

The difference simulations of different grid spacing (H/4, H/8, H/10 and H/12) were established respectively to improve the simulation accuracy and analyze the effect of the grid density used for FDM. Figure 7 shows the relationship between the simulation error and the grid spacing. The decrease of internal nodes' stress error was not obvious because of the good accuracy at the beginning. Although the loading point is reduced quickly, it is still larger. So, it needs to further reduce the grids spacing to obtain a more accurate stress in the load position. But it will really affect the work efficiency to add enough grids. Therefore, it is suggested that when the difference method is applied to the simulation, the stress of the loading position should be properly enlarged before interpolating.

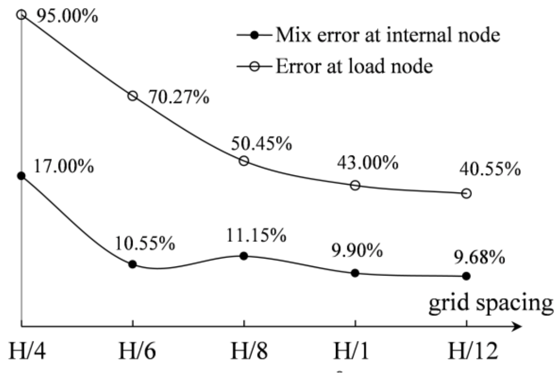

Figure 7. Relationship the simulation error and the grid spacing

A field test was done based on Nanpanjiang Bridge (108+180+108m) to validate the feasibility and effectiveness of the proposed method. Limited by the article space, the simulation results were only validated by correlation with in-place field test at the middle section of the anchorage zone, the comparison result was shown in Figure 8.

The stress distribution simulated by the FDM show a good agreement with the field test data. It is also noted that the simulated stress (Horizontal and Vertical stress) less than the test data. It was analyzed that the changes on stress attenuate quickly with distances and the grid was too little to keep good precision. Increasing the number of the grid could improve the precision at the cost of the equations number increasing, which will .reducing the efficiency of manual calculation. The future study will focus on this problem, and make the difference equation automatically generate with the help of computer. Then it will not only improve the precision by using more grids, but also improve the calculate efficiency.

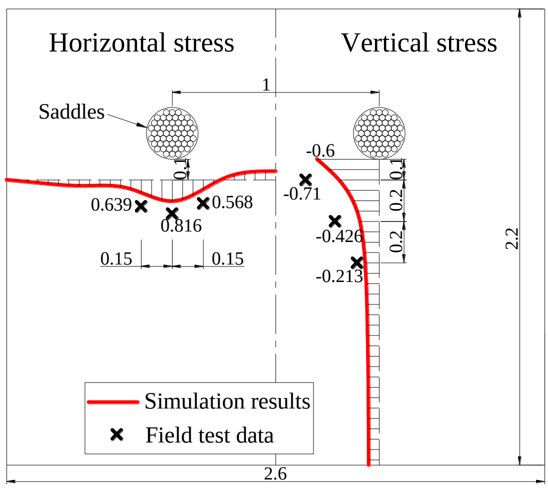

Figure 8. Simulation results and field test data (unit: m; MPa)

5 Conclusion

In this study, FDM was adopted to simulate the concrete in anchorage zone stress field. Based on the comparison of the FEA results, the following conclusions can be drawn: (1) The stress distribution in anchorage zone of extradosed cable stayed bridge simulated by finite difference method is in good agreements with the one simulated by the finite element model, the vertical compressive stress of concrete in the anchorage zone gradually reduces from the middle to the two sides.; (2) The difference method could increase the efficiency with keeping the accuracy of the simulation at most nodes except the load node; (3) The stress distribution becomes uniformity with a distance which equals the anchorage zone's width.

Acknowledgments

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