Optimizing the Multi-Objective Deployment Problem of Mlat System

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Abstract. Multilateration (MLAT) systems are powerful means for air traffic surveillance. These systems aim to extract, and display to air traffic controllers identification of aircrafts or vehicles equipped with a transponder. They provide an accurate and real-time data without human intervention using a number of ground receiving stations, placed in some strategic locations around the coverage area, and they are connected with a Central Processing Subsystem (CPS) to compute the target (i.e., aircraft or vehicle) position. The MLAT performance strongly depends on system layout design which consists on deploying the minimum number of stations, in order to obtain the requested system coverage and performance, meeting all the regulatory standards with a minimum cost. In general, choosing the number of stations and their locations to cope with all the requirements is not an obvious task and the system designer has to make several attempts, by trial and error, before obtaining a satisfactory spatial distribution of the stations.
In this work we propose a new approach to solve the deployment of Mlat stations problem by focusing on the number of deployed stations and the coverage as the main objectives to optimize. The Non-dominated Sorting Genetic Algorithm II(NSGA-II) was used in order to minimize the total number of stations required to identify all targets in a given area, with the aim to minimize the deployment cost, accelerating processes, and achieve high availability and reliability. The proposed approach is more efficient and converge rapidly which makes it ideal for our research involving optimal deployment of Mlat station.

1 INTRODUCTION

Nowadays, multilateration (MLAT) systems are a feasible option to be used in the air traffic control (ATC) technological infrastructures. These systems were widely deployed over a high number of airports around the world, for the aircraft surveillance and control in all flight phases, and in many cases, they are replacing the classical secondary surveillance radar. A multilateration system aim to detect, locate, and identify cooperating targets (i.e., aircraft or vehicle) position by receiving and processing suitable signals emitted by onboard transponder devices, according to the Secondary Surveillance Radar (SSR) international standards (e.g., the Mode A/C and Mode S signals).

In these systems, a number of ground receiving stations, with capabilities to measure some physical characteristics of signals emitted by transponders, such as Time of Arrival (TOA), are placed in some strategic locations around the coverage area, and they are connected with a Central Processing Subsystem (CPS to compute the target (i.e., aircraft or vehicle) position as shown in figure 1.

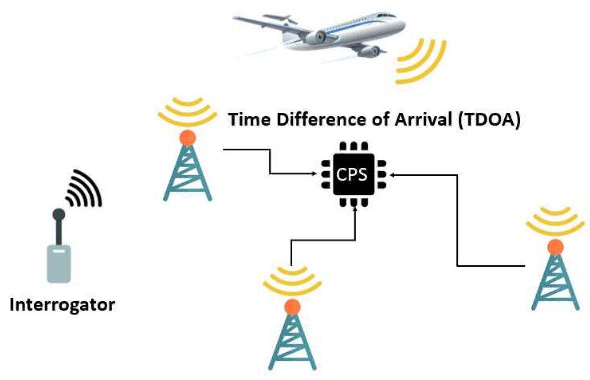

Figure 1: Principle of Mlat system

In the standard, and most widely extended, configuration of MLAT systems, the Mode S transmissions and the unsolicited transponder emissions (i.e., the squitter) as well as the responses to interrogations elicited by the MLAT system, can be used. These signals are received by the ground stations and their TOAs are measured and sent to the CPS, where the transponder position is calculated [1] [2]. This calculation is based on the Time Difference of Arrival (TDOA) measurements. They are the time of arrival (TOA) differences between the ith receiving station and a reference one (normally designated as the number 1). This is sketched in figure 2, which, for the sake of simplicity, refers to the two-dimensional case. It can be observed that the target is located at the intersection of the two hyperbolas.

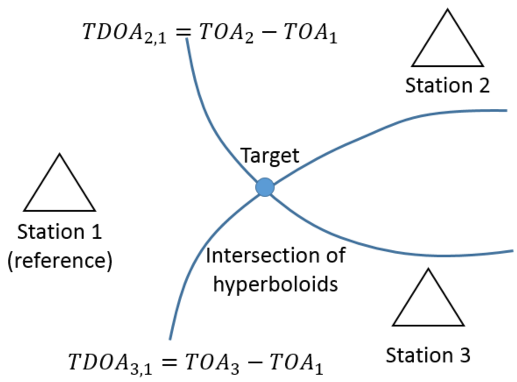

Figure 2: An illustrated (2D) view of TDOA Localization

The TDOA function geometrically represents an hyperboloid (or hyperbola), and it can be expressed as follows:

$$TDOA_{i,1} = \frac{1}{c}\Big(\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2} - \sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}\Big) + n_{i,1} \quad (1)$$

Where c is the speed of light, (x, y, z) the unknown target position (aircraft position), and (x_i, y_i, z_i) is the known position of the i^{th} station ($i = 1$ denotes the reference station). $n_{i,1}$ is a TDOA random noise term, which generally is assumed to be zero mean Gaussian distributed [3].

In some scenarios, it is common to find a numerical problem when solving the system of hyperbolic equations. This problem is defined in the literature as an ill-conditioned problem and the consequence of this is that, when the system of equations is solved, the solution is not correct or it has a big error. The mathematical interpretation of this problem goes back to the three conditions of Jacques Hadamard [4], namely, the solution exists, the solution is unique, and the solution depends continuously on the data. If at least one of these conditions is not satisfied the problem becomes ill-conditioned. On the other hand, the effects of this problem in the multilateration systems accuracy have been studied in [5] [6].

Several methods were adopted highlighting the challenges existing in target lacalization. Most of them aim to construct and solve the system of hyperbolic equations. Linearizing the system of hyperbolic equations by Taylor series expansion [7] [8] is the most accepted strategy to solve these hyperbolic equations, for estimating the target position [9]. Numerical approaches were also used to solve the localization problem [9] [10] [11]. These methods assume certain numerical approximations between the target position and its derived parameter in order to simplify the solution. The most common assumed approximation is that of mutual numerical independence between them. Contradictory to the numerical approaches, the algebraic approaches [12] [13] [14] do not use any statistical assumptions nor numerical approximations. They algebraically manipulate the hyperbolic equations until directly set an inverse problem that linearly relates the unknown target position with the known parameters. Some techniques add new measurement capabilities to the system, such as the angle of arrival [15] or the round-trip-delay [14] [16] in order to change the ill-conditioned Jacobian matrix into a well-conditioned Jacobian matrix. However all these solutions, although are efficient options, require in much cases significant economic investments.

The MLAT performance strongly depends on system layout design, which consists on deploying the minimum number of stations, in order to obtain the requested system coverage and performance, meeting all the regulatory standards with a minimum cost. In general, choosing the number of stations and their locations to cope with all the requirements is not an obvious task and the system designer has to make several attempts, by trial and error, before obtaining a satisfactory spatial distribution of the stations.

This paper introduces an original methodology to determine the number of stations that should be deployed and their position in order to obtain the requested system coverage and performance, meeting all the regulatory standards with a minimum cost. To the best of our knowledge, this article is the first to tackle the problem of Mlat system deployment with a multi-objective optimization approach.

The paper is organized as follows: Section 1 presents the general aspects and the classical solution for the MLAT localization problem. In Section 2 the mathematical formulation of the model and the optimization algorithm are defined. Section 3 presents some experimental results that validate the model and the optimisation process. Finally, Section 4 concludes the paper and states some open questions.

2 MATHEMATICAL MODEL

2.1 Multi-objective optimization

Multi-objective optimization involves the simultaneous optimization of multiple objective functions. In many cases, the objectives are defined in incomparable units, and they present some degree of conflict among them (i.e., one objective cannot be improved without deterioration of at least another objective).

In principle, multi-objective optimization is very different than the single-objective optimization. In single objective optimization, one tries to obtain the best design or decision, which is usually the global minimum or the global maximum depending on the optimization problem. In the case of multiple objectives, there may not exist one solution which is best (global minimum or maximum) regarding all objectives. In a typical multi objective optimization problem, there exists a set of solutions which are superior to the rest of solutions in the search space when all objectives are considered but are inferior to other solutions in the space in one or more objectives. These solutions are known as Pareto-optimal solutions or nondominated solutions. The rest of the solutions are known as dominated solutions.

One way to solve multi-objective optimization problems is to reformulate the problem as a single objective problem. The main advantage of this approach is that any generic optimization algorithm can be used to optimize this single objective function. However, this approach also suffers from several drawbacks: it requires a prior knowledge of the trade-off the decision maker is willing to make between the different objectives (the relative importance or weights of the objectives). The second approach requires that the decision maker interacts with the optimization procedure typically by specifying preferences between pairs of presented solutions. Interactive approaches consider only a small set of nondominated solutions due to the effort required.

An interesting approach based on game theory was proposed by the authors of [17] [18] [19] to solve multi-objective optimization problems. This approach considers the multi-objective optimization problem as a multi-player co-operative game where each objective function to be optimized is a player in the game. A game is said to be co-operative if the players are able to reach an agreement on strategies. The players are the objective functions, which are ultimately controlled by the decision maker and so can be expected to reach an 'agreement', meaning the game is co-operative. Using the fundamental text on co-operative games [20].

Pareto optimization, finds a representative set of non-dominated solutions approximating the Pareto front. Evolutionary Algorithms (EAs) [21] [22], bio-inspired optimization algorithms crudely mimicking natural evolution by implementing stochastic optimization through "natural selection" and "blind variations", can easily be turned into multi-objective optimizers by replacing the "natural selection", that favors the best value of the objective function, by some 'Pareto selection' based on the Pareto dominance relation. Some diversity criterion is generally used, ensuring a wide spread of the population over the Pareto front. The resulting algorithms, Multi-Objective Evolutionary Algorithms (MOEAs), have demonstrated their ability to do in a flexible and reliable way [23] [24].

Several MOEAs have been proposed in the literature, based on different implementation of the Pareto dominance selection and the diversity criterion [25] [26] [28]. In particular, many MOEAs use an archive of solutions, where they maintain the non-dominated solutions ever encountered during the search. Pareto optimization methods, such as evolutionary multi-objective optimization algorithms, allow decision maker to check the potential solutions without a priori judgements regarding the relative importance of objective functions.

In single-objective optimization, it is possible to determine between any given pair of solutions if one is better than another one. As a result, we usually obtain a single optimal solution. However, in multi-objective optimization there is not a straightforward method to determine if a solution is better than another one. The method most commonly adopted in multi-objective optimization to compare solutions is the one called Pareto dominance relation, which, instead of a single optimal solution, leads to a set of alternatives with different trade-offs among the objectives. Given two solutions **X** and **Y** of a multi-objective problems, **X** is obviously to be preferred to **Y** in the case where all objective values for **X** are better than those of **Y**, one at least being strictly better: in such case, **X** is said to Pareto-dominate **Y**. However, Pareto-dominance is not a total order, and most solutions are not comparable for this relationship. The set of interest when facing a multi-objective problem is the so-called Pareto set of all solutions of the search space that are not dominated by any other solution: such non-dominated solutions are the best possible trade-offs between the contradictory objectives, i.e., there is no way to improve any of them on one objective without degrading it on at least another objective. In multi-objective optimization, the search space is generally called the design space, by contrast with the objective space, where the Pareto front is the image of the set of non-dominated solutions. the Pareto set can contains a large number of solutions. At this stage, the decision-maker selects feasible solutions according to criteria which depend of explicit objective function preferences. In order to make the section of a feasible solution a manageable task, the set of Pareto solutions has to be filtered or reduced to a small number of representative solutions [28].

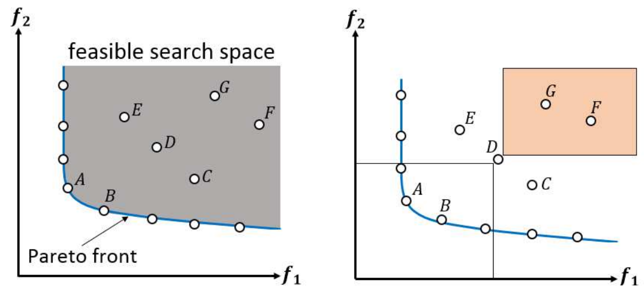

Figure 3: Pareto dominance

Figure 3 illustrates these ideas for a two-objective minimization problem. Solution D dominates F and G because their objective values for both f_1 and f_2 are greater than those for D. Meanwhile, the objective values for A and B are smaller than those for solution D, and so both A and B dominate D. However neither C nor E dominate, or are dominated by D.

2.2 Mathematical formulation

We consider a set of N_s stations that will be used to locate M targets, the stations are located at positions x_i with $i = 1,...,N_s$ and the targets are located at θ_j with $j = 1,...,M$. Each station is characterised by an interrogation range r_i which define the coverage area of the station i. The distance between the station i and the target j is given by :

$$d_{ij} = \|x_i - \theta_j\| \tag{2}$$

For each station we associate a binary variable.

$$a_i = \begin{cases} 1 & \text{if the station } i \text{ is deployed} \\ 0 & \text{otherwise.} \end{cases}$$

Then, the number of deployed stations can be stated as follows:

$$f_1 = \sum_{i=1}^{N_s} a_i \tag{3}$$

Now, we need to calculate the number of covered targets for a station i.

$$b_j^i = \begin{cases} 1 & \text{if } \exists! i \in \{1,...,N_s\} : d_{ij} \le r_i, j \in \{1,...,M\} \\ 0 & \text{otherwise.} \end{cases}$$

Then, the covered area for station i is expressed as follows:

$$f_2 = M - \sum_{j=1}^{M} b_j^i, \forall i \in \{1,...,N_s\} \tag{4}$$

The multi-objective optimization problem can be formally stated as follows:

$$\begin{cases} \min \sum_{i=1}^{N_s} a_i \\ \max M - \sum_{j=1}^{M} b_j^i \\ \left(x, b, a\right) \in \left\{\mathbf{R}_+^{N_s} \times \mathbf{R}_+^{N_s}, \{0,1\}^{N_s} \times \{0,1\}^M, \{0,1\}^{N_s}\right\}. \end{cases}$$

2.3 Algorithm

This section introduces the evolutionary optimization algorithm that has been used to validate numerically the mathematical model described above, namely the Non-dominated Sorting Genetic Algorithm (NSGA-II) [27].

The algorithm generates a first initial population and evaluates it. On this initial population, it performs crossover and mutation to obtain the offspring population. The offspring population is evaluated. The two populations are composed into a single one and sorted according to the domination relationship. The individuals, which are non-dominated with respect to one another, are further sorted by their crowding measurement (density computation). Then the best individuals are selected according to their fitness assigned after the sorting procedure. These selected individuals form the next population.

3 EXPERIMENTS

In this section, we present results from MATLAB simulation on the multi-objective optimization of the Mlat system design layout is described, the main goal is to choose the suitable number of deployed stations and where to set them up in order to obtain the requested system coverage and performance. The NSGA-II algorithm was used to solve an instance including $N_s = 20$ stations and $M = 50$ targets, all the stations have an interrogation range $r = 40m$. The targets are randomly distributed from (x, y) positions $(1,5)$ to $(100,120)$. NSGA-II has been chosen here for its long record of successes, demonstrating its robustness to find a good approximation of the Pareto front. The chosen variation operators for NSGA-II algorithms are shown in the table below:

NSGA-II operators

Encoding: Binary
Population size : 100
Maximal number of evaluations : 100
Selection : Binary tournament
Crossover : Intermediate Crossover , $P_c = 0.9$
Mutation : Gaussian, $P_m = 0.01$

All the results were computed on an Intel® Core™ i5-4200M CPU with 4× 2.5 GHz and 4Go of memory.

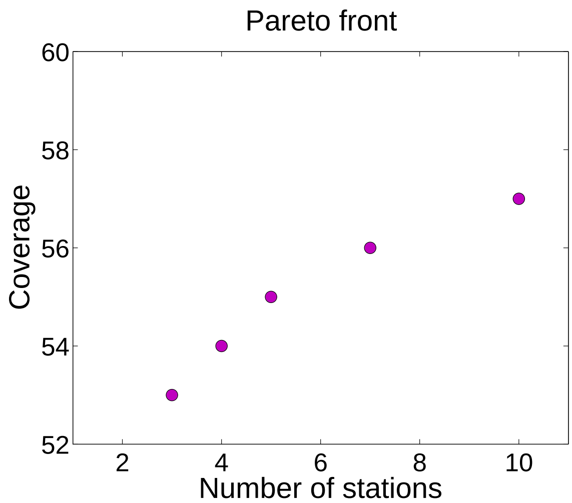

Figure 4: Pareto front obtained with NSGA-II

The Pareto front depicted in figure 4 shows the trade-off between minimizing the number of deployed stations and maximizing the number of lactated targets. If the decision maker wants to minimize the number of stations no matter how many targets are located she/he shall select a solution toward the lower left corner of the graph. Otherwise, if she/he thinks that the number of estimated targets matters, she/he can choose a compromise in the top right corner of the graph, resulting in higher number of located targets, at the price of a higher number of stations. Figure 5 shows a representation of a solution in the decision variable space. The solution corresponds to a number of 4 stations that were used to lactate 54 targets.

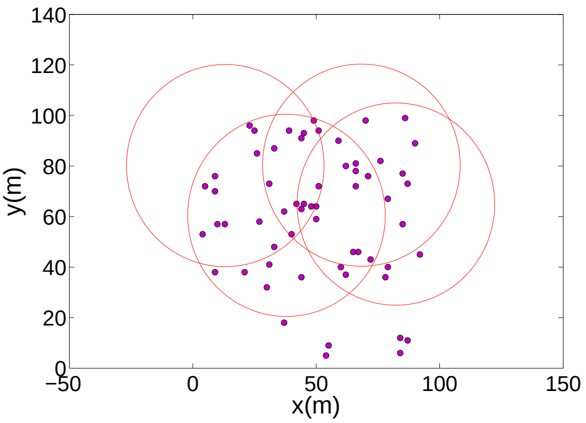

Figure 5: Representation of a solution in the decision variable space. A number of 4 stations were used to lactate 54 targets

4 Conclusion

Multilateration systems are a powerful means for the surveillance function of Air Traffic Control (ATC) operations. The MLAT performance strongly depends on system layout design which consists on deploying the minimum number of stations, in order to obtain the requested system coverage and performance, meeting all the regulatory standards with a minimum cost. In this paper we have presented a new approach for optimizing the system design layout by minimizing the number of stations while maximizing the number of lactated targets. The approach is based on multi-objective optimization and it is intended to find the optimal spacial distribution of the ground stations and also their number in order to have a maximum system coverage. Numerical results demonstrated the ability of the propose approach to obtain the most efficient localization strategy. The new technique can be applied to design, deploy and operate the MLAT systems for airport surface surveillance as well as for take-off-landing, approach and en-route control.

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