

The P_{ij} matrix and flux calculation of one-dimensional neutron transport in the slab geometry of nuclear fuel cell using collision probability method

Mohammad Ali Shafii^{1,*}, Jakaria Usman¹, Seni H. J. Tongkukut², and Ade Gafar Abdullah³

¹Andalas University, Department of Physics, Padang, West Sumatera, Indonesia

²Sam Ratulangi University, Department of Physics, Manado, North Sulawesi, Indonesia

³Universitas Pendidikan Indonesia, Department of Electrical Engineering Education, Bandung, Indonesia

Abstract. Calculation of P_{ij} matrix of one-dimensional neutron transport in the slab geometry of the nuclear fuel cell using Collision Probability (CP) method has been done. P_{ij} matrix is one of important parameters within the distribution of neutron flux in the nuclear fuel cell. The CP method is the most efficient methods to solve the neutron transport equation in the reactor core. The study is focused on neutron interaction with nuclear fuel cell of U-235 and U-238 for homogeneous condition. The parameters to calculate the P_{ij} matrix are the cross section of nuclear fuel, width of the region and number of regions. A lattice of slabs have been constructed using void boundary conditions for model of finite system to calculate the collision probabilities. If the P_{ij} matrix has been calculated then neutron flux can be determined. The results show that total value of P_{ij} matrix using CP method for U-235 and U-238 is less than one, respectively. This is in accordance with the definition of void boundary conditions for finite slab geometry. Along with P_{ij} matrix, neutron flux is also appropriate with the reference.

1 Introduction

The fundamental problem in the nuclear reactor analysis is solving the neutron transport equation. The neutron distribution system is described by the integral transport equation [1]. The integral transport equation is obtained from the integro-differential equation by using some basic assuming. The solution of the neutron transport equation determines the distribution of neutrons in the reactor to predict neutron flux at all points directions and energies for any given geometry and material distribution [2]. The numerical solution of neutron transport equation solving should be able to explain the neutron behavior in the reactor [3]. For small geometries, the neutron transport theory is more appropriate to calculate the neutron distribution than the diffusion theory [4].

There are two classification methods that used to solve the neutron transport equation: stochastic and deterministic methods. Stochastic method is the random movement of neutrons in phase space governing the probability of neutron interaction with material. Deterministic methods are based on solving the neutron transport equation which governs the average behavior of neutrons. There are several methods in the class of deterministic methods such as Method of Characteristic (MOC), Collision Probability (CP) method, Discrete Ordinates method (S_N) and the spherical harmonics (P_N) method. The most popular method in deterministic class is CP method. The Collision Probability method is

frequently used for the construction of relevant matrices. It can handle complex geometry. It is often employed to solve small sized problems such as those over pin cell or fuel assembly in a reactor. This method is, however, unsuitable for large-sized problems such as whole core, because the matrices are large as well as dense and relevant equations are difficult to solve [5].

In previous research [1,6,7], the CP method was used to solve the transport of neutrons in a 1D cylindrical nuclear fuel cell. The integral transport is solved using CP method with non flat flux approach which is usually solved by the flat flux approach. In this research, CP method was implemented to simple geometry, with flat flux approach. The research is focused on the calculation of P_{ij} matrix using CP method for U-235 and U-238 of 1D slab of nuclear fuel cell with void boundary condition. If the P_{ij} matrix has been calculated then neutron flux can be determined.

2 Theoretical backgrounds

The CP method is one of the solutions the integral form of the neutron transport equation. The main idea behind the integral transport method is to integrate out the angular dependence and to solve the neutron transport equation for the scalar flux directly [5]. The neutron transport equation is difficult to resolve analytically, unless it is simplified according to the assumptions made. An independent of time neutron transport equation is [8],

* Corresponding author: mashafii@fmipa.unand.ac.id

$$\begin{aligned} & \hat{\Omega} \cdot \nabla \phi(\vec{r}, E, \hat{\Omega}) + \Sigma_s(\vec{r}, E) \phi(\vec{r}, E, \hat{\Omega}) \\ & = \int_{4\pi} d\hat{\Omega}' \int_0^\infty dE' \Sigma_s(E' \rightarrow E, \hat{\Omega}' \rightarrow \hat{\Omega}) \phi(\vec{r}, E', \hat{\Omega}') \\ & + S(\vec{r}, E, \hat{\Omega}) \end{aligned} \quad (1)$$

The basic assumptions of the CP method are the flat flux approximation and the isotropic scattering. By defining $\hat{\Omega} \cdot \nabla = d/dR$, the angular neutron flux as a solution of Eq. (1) at position \vec{r} in the angle of space $\hat{\Omega}$ and energy E follows the form of the integral transport equation

$$\begin{aligned} \phi(\vec{r}, \hat{\Omega}, E) & = \int_0^\infty dR \exp(-\overline{\Sigma}R) \\ & \times \left[\int_0^\infty dE' \int_{4\pi} d\hat{\Omega}' \Sigma_s(\vec{r}, \hat{\Omega}' \rightarrow \hat{\Omega}, E' \rightarrow E) \phi(\vec{r}', \hat{\Omega}', E') \right. \\ & \left. + S(\vec{r}', \hat{\Omega}, E) \right] \end{aligned} \quad (2)$$

where R is the distance between \vec{r} and \vec{r}' , vector space angle $\hat{\Omega} = (\vec{r} - \vec{r}')/R$, $\overline{\Sigma}R = \int_0^R \Sigma_s(E)$ is optical path length, $\Sigma_s(\vec{r}, \hat{\Omega}' \rightarrow \hat{\Omega}, E' \rightarrow E)$ is the scattering cross section at the point \vec{r}' from $\hat{\Omega}'$ with the energy E' to $\hat{\Omega}$ with the energy E , and $S(\vec{r}', \hat{\Omega}, E)$ is a neutron source. If the neutron scattering and neutron sources are considered isotropic, then Eq. (2) becomes to

$$\begin{aligned} \phi(\vec{r}, E) & = \int_{4\pi} d\hat{\Omega} \frac{1}{4\pi} \int_0^\infty dR \exp(-\overline{\Sigma}R) \\ & \times \left[\int_0^\infty dE' \Sigma_s(\vec{r}', E' \rightarrow E) \phi(\vec{r}', E') + S(\vec{r}', E) \right] \end{aligned} \quad (3)$$

Equation (3) can be rewritten by using a relation $d\vec{r}' = R^2 dR d\hat{\Omega}$, so it follows [8]

$$\begin{aligned} \Sigma(\vec{r}, E) \phi(\vec{r}, E) & = \int_{4\pi} dr P(\vec{r}' \rightarrow \vec{r}, E) \\ & \times \left[\int_0^\infty dE' \Sigma_s(\vec{r}', E' \rightarrow E) \phi(\vec{r}', E') + S(\vec{r}', E) \right] \end{aligned} \quad (4)$$

where $P(\vec{r}' \rightarrow \vec{r}, E) = \frac{\Sigma(\vec{r}')}{4\pi R^2} \exp\left[-\int_0^R \Sigma_s(s) ds\right]$

is the probability of a neutron having energy E coming from a point \vec{r}' will experience a collision at a point \vec{r} in the volume element dr . Equation (4) can be simplified form as follows

$$\begin{aligned} \Sigma_j(E) V_j \phi_j(E) & = \sum_i P_{ij}(E) V_i \\ & \times \left[\int_0^\infty dE' \Sigma_{si}(E' \rightarrow E) \phi_i(E') + S_i(E) \right] \end{aligned} \quad (5)$$

with the CP matrix of Eq. (5) is defined by

$$P_{ij}(E) = \frac{\Sigma_j(E)}{4\pi V_i V_j} \int_{V_j} dr_j \int_{V_i} dr_i \frac{\exp(-\overline{\Sigma}R)}{R^2} \quad (6)$$

The collision probabilities matrix P_{ij} given by Eq. (6) are the first flight collision probabilities from region i to region j as they give the probability for a neutron born isotropically and uniformly distributed within region i to have its first collision in region j [5].

To simplify computing calculations, Eq. (5) can be rewritten as multi group energy [1,6]

$$\Sigma_{jg} V_j \phi_{jg} = \frac{1}{k_{eff}} \sum_i V_i P_{ijg} S_{ig} \quad (7)$$

where index i or j indicates the spatial region i or j and index g denotes the energy group, Σ_{jg} is the total macroscopic cross section in region j , S_{ig} is source, P_{ij} is CP matrix and $V_i = \pi(r_i^2 - r_{i-1}^2)$. The special treatment that only one group one energy, Eq.(7) can be modified into

$$\Sigma_j V_j \phi_j = \frac{1}{k_{eff}} \sum_i V_i P_{ij} S_i \quad (8)$$

In the case of one dimensional of slab geometry for one speed one energy neutron in x axis, the neutron born in the point having its first collision is shown in the Figure 1.

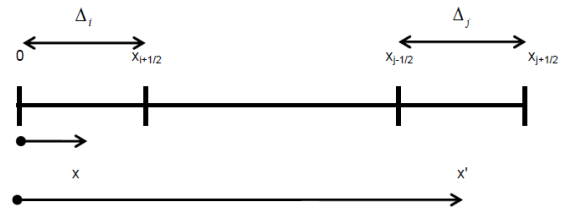


Fig. 1. Discretization of slabs i and j [5].

Based on the discretization of slab, equation (6) can be written as [5]

$$P_{ij} = \frac{\Sigma_j}{2\Delta_i \Delta_j} \int_{\Delta_i} dx_i \int_{\Delta_j} dx_j \int_{|x-x'|}^\infty \frac{d\gamma}{\gamma} \exp(-\gamma\tau(x, x')) \quad (9)$$

where $\gamma = \frac{1}{\cos \theta} = \frac{R}{|x-x'|}$. By using the definition of exponential integral,

$$E_1(\tau(x, x')) = \int_{|x-x'|}^\infty d\gamma \frac{1}{\gamma} e^{-\gamma\tau(x, x')} \quad (10)$$

Eq.(9) can be expressed as

$$P_{ij} = \frac{\Sigma_j}{2\Delta_i \Delta_j} \int_{\Delta_i} dx_i \int_{\Delta_j} dx_j E_1[\tau(x, x')] \quad (11)$$

The optical distance between the two points x and x' is [5]

$$\tau(x, x') = \Sigma_i(x_{i+1/2} - x) + \Sigma_j(x' - x_{j-1/2}) + \Sigma_{ij} \Delta_{ij} \quad (12)$$

where $\tau_{ij} = \Sigma_{ij} \Delta_{ij}$. If Eq. (12) is substituted into Eq. (11), then it is obtained

$$P_{ij} = \frac{1}{2\Sigma_i \Delta_i} \left(E_3(\tau_{ij}) - E_3(\tau_{ij} + \tau_i) - E_3(\tau_{ij} + \tau_j) + E_3(\tau_{ij} + \tau_i \tau_j) \right) \quad (13)$$

Equation (13) is the expression of the first flight collision probabilities that a neutron introduced in cell i has its next collision in cell j on a slab geometry for $i \neq j$ [5]. On the other hand, for $i = j$, an expression for self-collision probability, the probability that a neutron introduced in cell i has its next collision in cell i , is [9]

$$P_{ii} = \Sigma_i \Delta_i \left[1 - \frac{1}{2\Sigma_i \Delta_i} (1 - 2E_3(\Sigma_i \Delta_i)) \right] \quad (14)$$

The neutron flux in the slab geometry can be derived from Eq. (3)[9]

$$\phi(x) = \int_{-\infty}^{\infty} dx' \frac{1}{2} E_1(\tau(x, x')) S(x') \quad (15)$$

For isotropical neutron source.

3 Computational Method

For limitation case, the research have the following assumptions: the cross sections of U-235 and U-238 are constant in each region, the flux is constant in each volume and the source is constant. The boundary conditions that used is finite system that composed of meshes with void boundary. Void boundary conditions is implemented on both sides of slab as shown in the Figure 2.

void	1	2	3	4	5	6	void
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Fig. 2. Slab with void boundary conditions that discretized into six regions

To evaluate the first flight collision probabilities in this system is assumed that there are no contributions from outside. The consequence of this boundary condition is the sum of P_{ij} matrix must be less than one

$$\left(\sum_j P_{ij} < 1 \right) \text{ for all mesh due to leakages from the system.}$$

This is different with an infinite system modelled as having N meshes surrounded by reflective boundary conditions that the sum of P_{ij} matrix must be equal to one

$$\left(\sum_j P_{ij} = 1 \right). \text{ The slab is discretized into 6 meshes.}$$

It follows that for a domain with 6 meshes, the resulting collision probabilities matrix is of size 6×6 .

In general, a computational procedure to calculate the collision probability using MATLAB R2013b follows :

- Determine the void boundary conditions. Lattice of slabs are constructed using void boundary conditions

to an finite model system to calculate the collision probabilities.

- Determine the total and scattering macroscopic cross section of U-235 and U-238, source of neutron, width and number of mesh as an input of the program.
- Calculates the optical path length using Eq.(12).
- Calculates the exponential integral using Eq.(10).
- The collision probabilities matrix is executed by using Eq. (13) simultaneously with Eq.(14).
- After that, neutron flux is calculated using Eq. (15).

4 Results and discussion

In the case of fuel cells in a homogeneous slab geometry composed of U-235 and U-238, the value of the P_{ij} matrix is presented in Table 1 and Table 2. It appears that diagonal of P_{ij} values ($P_{11}, P_{22}, P_{33}, P_{44}, P_{55}$ and P_{66}) are equal and greater than the other. Beside the P_{ij} located on the diagonal, there are a pair of P_{ij} that have the same value (e.g. P_{12} with P_{21}, P_{13} with P_{31} , ect.), provided that $i \neq j$.

Table 1. P_{ij} matrix for a finite slab composed of U-235.

Pij	1	2	3	4	5	6
1	0,6730	0,1410	0,0182	0,0034	0,0007	0,0002
2	0,1410	0,6730	0,1410	0,0182	0,0034	0,0007
3	0,0182	0,1410	0,6730	0,1410	0,0182	0,0034
4	0,0034	0,0182	0,1410	0,6730	0,1410	0,0182
5	0,0007	0,0034	0,0182	0,1410	0,6730	0,1410
6	0,0002	0,0007	0,0034	0,0182	0,1410	0,6730
Total	0,8365	0,9773	0,9948	0,9948	0,9773	0,8365

Table 2. P_{ij} matrix for a finite slab composed of U-238.

Pij	1	2	3	4	5	6
1	0,6730	0,1410	0,0182	0,0034	0,0007	0,0002
2	0,1410	0,6730	0,1410	0,0182	0,0034	0,0007
3	0,0182	0,1410	0,6730	0,1410	0,0182	0,0034
4	0,0034	0,0182	0,1410	0,6730	0,1410	0,0182
5	0,0007	0,0034	0,0182	0,1410	0,6730	0,1410
6	0,0002	0,0007	0,0034	0,0182	0,1410	0,6730
Total	0,8365	0,9773	0,9948	0,9948	0,9773	0,8365

Table 3 is a data from a study of Mtsetfwa [5] for a homogeneous slab geometry of fuel cell composed of U-235 for 4 regions as a comparison. It shows that there are no significant difference between Table 1 and Table 2 with Table 3.

Table 3. P_{ij} matrix for a finite slab composed of U-235 [5].

P_{ij}	1	2	3	4
1	0.59954	0.23412	0.10163	0.06471
2	0.23412	0.46707	0.19718	0.10163
3	0.10163	0.19718	0.46707	0.23412
4	0.06471	0.10163	0.23412	0.59954
Total	1.00000	1.00000	1.00000	1.00000

The main difference is only seen in the total number of the P_{ij} matrix. It occurs because of the results in Table 1 and Table 2 use the void boundary conditions, whereas in Table 3 it uses the reflective boundary condition. In the void boundary conditions, the total number of the P_{ij} matrix must be less than 1 in each

regions, on the other hand the total number of the P_{ij} matrix must be equal to 1 in each regions. In other words, the calculation of the CP matrix has been in accordance with the theory.

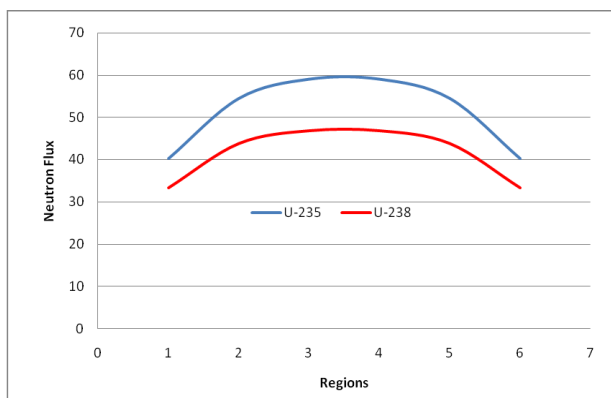


Fig. 3. Neutron Flux in the 1 D slab of fuel cell.

Neutron flux distribution in the slab geometry nuclear fuel cells composed of U-235 and U-238 is shown in the Figure 3. It appears that the scalar neutron flux of U-235 is greater than U-238. This event occurs due to the scattering and removal cross section of U-235 are also greater than U-238.

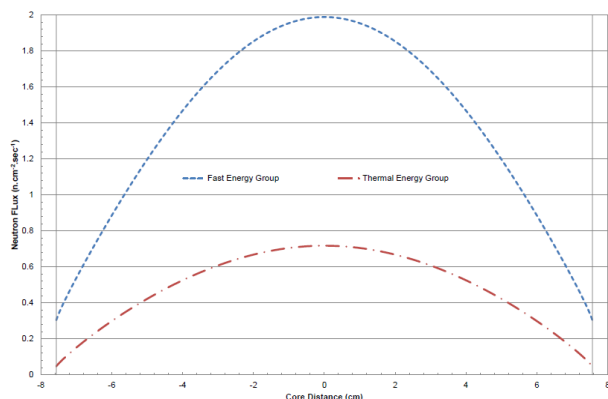


Fig. 4. Neutron Flux in the 1 D slab geomtry for multigroup energy [5].

The greater the removal cross sectional, the smaller the scalar flux in the region, since the removal cross section indicates the probability that neutrons will be transferred to the energy other due to a collision. While scattering cross section shows the amount of neutrons scattered without being absorbed or out of the slab. The greater the scattering cross section value, the greater also the scalar neutron flux. The profile of the neutron distribution of slab fuel U-235 and U-238 is simetry that have maximum value occurred in middle of slab region.

Figure 4 is the neutron flux distribution data from a study of Mtsetfwa [5] for a homogeneous slab geometry of fuel cell composed of U-235 for multigroup enrgy. There is no different pattern result of one dimensional flux neutron distribution using one group and multigroup energy. This indicates that the results of this study have been in accordance with the reference.

5 Conclusions

A one dimensional neutron transport theory based on CP method is developed for slab geometry. The calculation requires the cross sections U-235 and U-238 and void boundary condition as input. Total value of P_{ij} matrix using CP method for U-235 and U-238 is less than one, respectively. This is in accordance with the definition of void boundary conditions for finite slab geometry. Along with P_{ij} matrix, neutron flux is also appropriate with the reference. In general, implementation of neutron transport using CP method with applied to the geometry slab of fuel cell yields fairly good results comparing with reference.

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