

Reduction of the bearing capacity of a thin-walled steel beam-column as a result of uniform corrosion

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Abstract. The paper presents the analysis of the load-bearing capacity of thin-walled steel bars such as beam-column. It was assumed that the rods are subject to uniformly distributed surface corrosion in the atmosphere over their entire length. As a result of corrosion, the mass loss of these rods, i.e. the thickness of the cross-sectional walls of the rod are evenly reduced. Therefore, the dependence of the critical force - the eccentricity changes. The theory of stability of thin-walled bars was used to calculate the load capacity of the rod. To calculate changes in the load capacity of the rod, an interactive relationship was used that combines compression with bending. A calculation example of the load capacity of an eccentrically compressed rod with a “C” section which has been corroded has been presented.

1 Introduction

Thin-walled steel bars with an open cross-section that are subjected to eccentric compression load easily lose their global stability. The form of buckling of bar can be flexural or warping. It depends on the shape of the cross-section, on the thickness of the cross-sectional walls, on the length of the rod under the given support conditions and on the material of the rod. The walls of a thin-walled steel bar that evenly corrodes in the atmosphere become thinner, so all the geometric characteristics needed to calculate the load capacity change. In the literature, the loss of corrosion in the profile is modeled as a corresponding reduction in wall thickness [1, 2, 3]. In general atmospheric corrosion [4], occurring over time, weakens the ability of the profile to carry loads. In order to obtain an answer to the question about how much and how the load-bearing capacity of the corrosive profile will be reduced, the issue of stability of thin-walled bars should be analyzed. In the case of a beam-column bar, one should also analyze the interactive relationship connecting compression with bending the bar. This article also presents the theory of the problem under consideration and based on it examples of calculations for a compression and bending channel with a constant length and constant support conditions losing thickness.

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2 Problem formulation and theoretical problem solution

Let us consider a straight eccentrically compressed elastic rod with deflections and torsions restrained by simply supported ends, with a "C" cross-section, with one symmetry axis (Fig. 1) and a constant wall thickness g . It has been assumed that buckling of the bar is elastic.

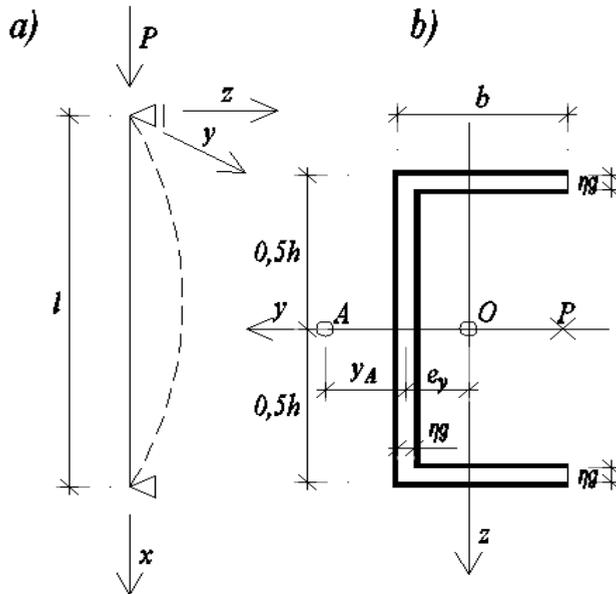


Fig. 1. (a) Rod under compression and bend; (b) model of inner and outer surface corrosion on "C" cross-section; A -shear center, O -centroid.

It was assumed that the rod along the entire length is subject to uniformly distributed side corrosion on thin-walled open cross-section. The model of corrosion of the outer and inner surface of the rod with a cross-section "C" is illustrated in Fig. 1b. The walls of the cross-section of the bar with the initial thickness g (before corroding) after corrosion have a smaller thickness. The thickness of the "C" section walls after corrosion is ηg where $0 < \eta g < 1$. Only the walls of the class section are considered: 1, 2, 3. Class 4 is not considered, due to local instability. A linear corrosion rate is assumed at time t . It is assumed that the compressive force P acts on the eccentric $e = e_y$, e.g. 1b, i.e. it is applied at the point on the opposite side of the cross section than the shear center A .

In the further part of the work, calculations made according to the theory of stability of thin-walled bars [5, 6, 7] will be presented, the results of which will be needed in the analysis of interactive dependence. The aim of the work is to determine the P_{kr} value of eccentrically compressive rods corroded from the interaction formula [7] and to discuss the results obtained:

$$\frac{P_{kr}}{P_{kro}} + \frac{P_{kr}e}{M_T} = 1 \tag{1}$$

where:

P_{kr} – flexural-torsional buckling load for eccentric load having an eccentricity of e ,

P_{kro} – flexural-torsional buckling load for concentric load,

e – eccentricity,

M_T – critical moment causing tension on shear center side centroid.

The system of differential equations of equilibrium for thin-walled bars with one symmetry axis in the cross-section is [5, 6, 7]:

$$EJ_z v^{IV} + P v^{II} = 0 \tag{2.1}$$

$$EJ_y w^{IV} + P w^{II} - P y_A \theta^{II} = 0 \tag{2.2}$$

$$EJ_\omega \theta^{IV} - (GJ_O - P r_O^2) \theta^{II} - P y_A w^{II} = 0 \tag{2.3}$$

where:

P – compression force,

y, z – principal centroidal axes,

O, A – designations of the centroid and of the shear centre,

v, w – displacement along the principal centroidal axes y, z ,

θ - angle of torsion about the shear centre A ,

J_y, J_z – second moments of area about axes y, z ,

J_ω - warping constant of torsion about point A ,

y_A – position of point A ; $z_A=0$,

E, G – elasticity and shear modulus respectively,

$J_O = K_s$ – torsional modulus of elasticity,

r_O – polar radius of gyration of cross-section;

$$r_O^2 = \frac{J_y + J_z}{F} + y_A^2 + z_A^2,$$

F – cross-sectional area.; $dF = \eta g ds$.

The boundary conditions for the rod, which is shown in Fig. 1a. are following:

$$\begin{aligned} w(0) = v(0) = \theta(0) = 0, \\ w(l) = v(l) = \theta(l) = 0 \\ w^{II}(0) = v^{II}(0) = \theta^{II}(0) = w^{II}(l) = v^{II}(l) = \theta^{II}(l) = 0 \end{aligned} \tag{3}$$

and own functions in the form of:

$$\begin{aligned} w &= C_1 \sin(\alpha x), \\ v &= C_2 \sin(\alpha x), \\ \theta &= C_3 \sin(\alpha x) \end{aligned} \tag{4}$$

where:

$$\alpha = \frac{\pi}{l}.$$

As a result of the solution of equation (2.1), the critical force of flexural buckling is obtained, and from the system of equations (2.2) and (2.3), critical buckling-torsional buckling forces are obtained.

The solution of the system of stability equations has been taken from work [7]. It is as follows:

$$P_1 = P_z = \frac{\pi^2 EJ_z}{l^2} \tag{5.1}$$

$$P_2 = \frac{1}{2\beta} \left[(P_y + P_x) + \sqrt{(P_y + P_x)^2 - 4\beta P_y P_x} \right] \tag{5.2}$$

$$P_3 = \frac{1}{2\beta} \left[(P_y + P_x) - \sqrt{(P_y + P_x)^2 - 4\beta P_y P_x} \right] \tag{5.3}$$

where:

P_l - critical buckling load,

P_2, P_3 – critical torsional buckling loads,

$$\beta = 1 - \left(\frac{y_A}{r_0} \right)^2,$$

$$P_y = \frac{\pi^2 EJ_y}{l^2},$$

$$P_x = \left[EJ_\omega \frac{\pi^2}{l^2} + GJ_0 \right] \frac{1}{r_0^2},$$

$J_0 = (1/3) \Sigma(b_i g_i^3)$ – St. Venant torsion constant of cross section.

The force P_3 is always smaller than P_2 , but it may be smaller or larger than P_l . The critical force P_{kr} selected from three is the smallest and will be marked as P_{kro} . The formula on M_T [6] for an articulated support rod is:

$$M_T = -P_y \left[j - \sqrt{j^2 + r_0^2 \left(\frac{P_x}{P_y} \right)} \right] \tag{6}$$

where;

$$j = \frac{\beta_z}{2} = \frac{1}{2J_z} \left[\int y^3 dF + \int yz^2 dF \right] - y_A \tag{7}$$

The bending moment M_T is the critical moment of lateral torsional buckling. The sizes P_{kro} and M_T are inserted into the interactive formula (1). Assuming different values of the eccentricity e , one can determine the values of P_{kr} from the equation (1) according to the dependence:

$$P_{kr} = \frac{1}{\left(\frac{1}{P_{kro}} + \frac{e}{M_T} \right)} \tag{8}$$

In works [8, 9] it was proved that the coordinates of points A, O after uniform corrosion of both shelves and web of this cross-section have not changed. Values of inertia moments of the cross-section decreased η - times, according to the following relationships:

$$I_y = \sum_{i=1}^n \eta g_i \int_0^i z^2 ds, \tag{9}$$

$$I_x = \sum_{i=1}^n \eta g_i \int_0^i y^2 ds,$$

$$I_{\omega} = \sum_{i=1}^n \eta g_i \int_0^l \omega^2 ds,$$

In the same way, η -times, the values of integrals found in formula (7) decreased:

$$\sum_{i=1}^n \eta g_i \int_0^l y^3 ds, \tag{10}$$

$$\sum_{i=1}^n \eta g_i \int_0^l yz^2 ds,$$

The torsion constant decreased η^3 -times:

$$I_0 = \frac{1}{3} \sum_{i=1}^n b_i (\eta g_i)^3 \tag{11}$$

Thus, in the corroded section, all geometrical characteristics used to determine P_{kro} and M_T are smaller than in the non-corroded section. Therefore, the critical force P_{kro}^{kor} operating on the eccentricity e is determined from the equation (1) taking into account, respectively, the reduced values of geometric characteristics from the formula:

$$P_{kr}^{kor} = \frac{1}{\left(\frac{1}{P_{kro}^{kor}} + \frac{e}{M_T^{kor}} \right)} \tag{12}$$

where:

P_{kro} i M_T - adequate critical force at the axial load of the corroded rod, the critical moment of lateral buckling of the corroded rod.

3 Sample calculation

The bar, as in Fig. 1, with the "C" cross-section with the dimensions of the shelf and web respectively $b = 4$ cm, $h = 8$ cm and wall thickness $g = 4$ mm was used for calculations. The buckling length of the rod is $l = 200$ cm, which corresponds to the slenderness of the bar equal to $\lambda = 155$. The calculations were made on the basis of equation (12). The critical force P_{kro}^{kor} (the critical force of flexural buckling) is calculated for eccentricity values: $e = 0, 1, 2, 3, 4$ and 5 cm. In the calculations, the influence of corrosion on the cross-section of the section was taken into account. It was assumed that the beam-column is made of S235JR steel and is not protected by anti-corrosion coating. This rod was corroded in the atmosphere of an industrial city. Based on data from the literature [10, 11], it was assumed that the corrosion rate is linear and amounts to $v_p = 0.09$ [mm / year]. Given such corrosion progress, it was calculated that the rod lost 10% of g wall thickness after 4.4 years, 20% g thickness after 8.9 years and 30% g thickness after 13.3 years. As a result of the calculations, the graph presented in Fig. 2 was made. The four curves illustrated in this graph correspond to respectively: 100%, 90%, 80% and 70% of wall thickness g "C", i.e. $\eta = 1; 0.9; 0.8; 0.7$. These curves are non-linear and decreasing.

4 Conclusions

The analysis of the stability problem of a thin-walled steel beam – column bar proved that reducing the thickness of the cross-sectional walls by corrosion significantly affected the decrease of the critical force value. A computational example was made, on the basis of

which the diagrams of dependence of critical force - eccentricity were obtained. These dependencies are non-linear. If the eccentricity increases, then the critical force will decrease very much. These dependencies allow to evaluate the work safety of the corroded eccentrically compressed rod.

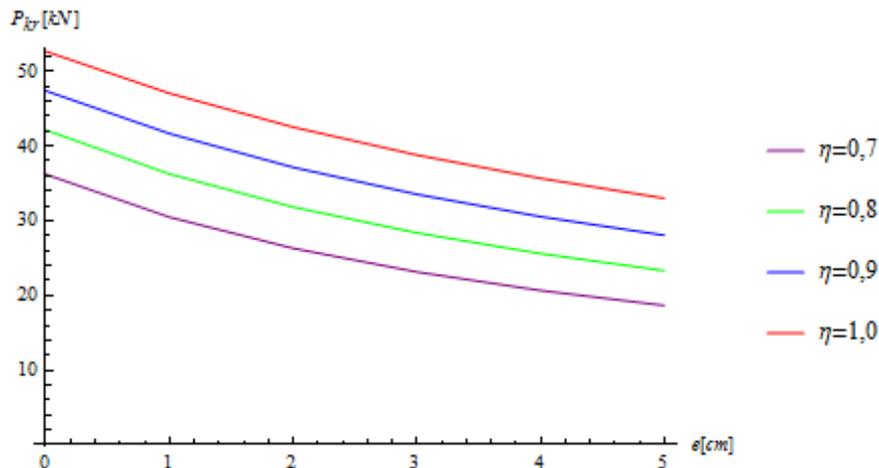


Fig. 2. Critical force of beam – column with the "C" cross-section versus eccentricity; the wall thickness defects are $\eta = 1,0; 0,9; 0,8; 0,7$.

References

1. R. Rahgozar, *Remining capacity assesment of corrosion damaged beams using minimum curves*, Journal of Constructional Steel Research, **65**, 299-307, (2009)
2. N. Tatsuro, M. Hisao, Y. Norio, A. Hironori, *Effect of pitting corrosion on local strength of hold frames of bulk carriers (1 st report)*. Marine Structure **17**, 403 – 432, (2004)
3. A. Glinicka, S. Imielowski: *Wpływ zmiany przekroju poprzecznego skorodowanych prętów ściskanych na nośność*. Logistyka **4**, 3411 – 3416, (2015)
4. P.R. Roberge, Corrosion Engineering. Principles and Practice. McGraw-Hill, USA, (2008).
5. P.Jastrzębski, J.Mutermilch, W.Orłowski, *Wyttrzymałość Materiałów 2*, Arkady, (1986)
6. C.F. Kollbrunner, N. Hajdin, *Dünnwandige Stäbe*. Band 1. Springer – Verlag, Berlin Heidelberg, New York (1972)
7. Wei-Wen Yu, R. A. LaBoube, *Cold-Formed Steel Design*, Fourth Edition, Wiley, (2010)
8. A. Glinicka, S. Imielowski, C. Ajdukiewicz: *Influence of uniformly distributed corrosion on the compressive capacity of selected thin walled metal columns*. Procedia Engineering , Vol/issue **111C**, 262 – 268, (2015)
9. A. Glinicka, C. Ajdukiewicz, S. Imielowski: *Effects of uniformly distributed side corrosion on thin-walled open cross-section steel columns*. Skutki równomiernej korozji słupów stalowych o przekrojach cienkościennych otwartych. Roads and Bridges – Drogi i Mosty, **15**, 257 – 270, (2016)
10. ISO 9224 Corrosion of metals and alloy – Corrosivity of atmospheres – Guiding values of the corrosivity categories, (1992)
11. S. D. Cramer, B. S. Covino, Corrosion. Environments and industries, Handbook, **13**, ASM International (2006)