Calculation of thin isotropic shells beyond the elastic limit by the method of elastic solutions

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Abstract. The paper focuses on the model of calculation of thin isotropic shells beyond the elastic limit. The determination of the stress-strain state of thin shells is based on the small elastic-plastic deformations theory and the elastic solutions method. In the present work the building of the solution based on the equilibrium equations and geometric relations of linear theory of thin shells in curved coordinate system α and β, and the relations between deformations and forces based on the Hirchhoff-Lave hypothesis and the small elastic-plastic deformations theory are presented. Internal forces tensor is presented in the form of its expansion to the elasticity tensor and the additional terms tensor expressed the physical nonlinearity of the problem. The functions expressed the physical nonlinearity of the material are determined. The relations that allow to determine the range of elastic-plastic deformations on the surface of the present shell and their changing in shell thickness are presented. The examples of the calculation demonstrate the convergence of elastic-plastic deformations method and the range of elastic-plastic deformations in thickness in the spherical shell. Spherical shells with the angle of half-life regarding 90 degree vertical symmetry axis under the action of equally distributed ring loads are observed.

1 State of the problem

The constructions in the form of shells take a special place among the spatial structures. The theory of the calculation of stress-stain state of shells is presented in the works [1, 4] and in many other scientific works. The practical necessity of the calculation beyond the elastic material work is caused by the fact that the high tensions exceeded the limit of material proportionality appear in thin-walled constructions in the places of local influence. In the present work the model of the numerical implementation of the calculation of thin isotropic shells the material of which works beyond the elastic limit is observed. The definition of stress-stain state of thin shells is based on the small elastic-plastic deformations theory and the elastic solutions method of A.A. Ilyushin [2].

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In the works of I.S. Tsurkov [3] the method got a theoretical justification and development for the calculation of thin shells. The usage of computers in the further calculation [4-8] allows to obtain the solutions not only distributed just on the surface of the loads, but also distributed

2 The method of solution

At the building of the solution, the main ratios are the equations of equilibrium and geometric ratios of the linear theory of thin shells in the curved coordinate system α and β [3, 8], Figure 1**.**

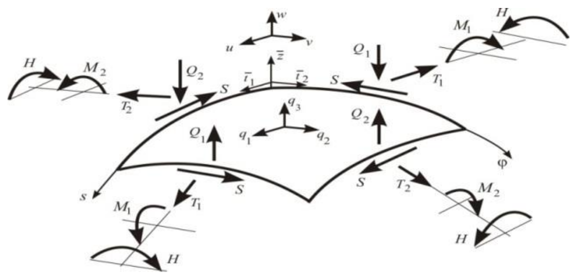

Fig. 1. Positive directions of outer load projections and of internal forces components.

Equilibrium equalities are considered in the following form:

$$\frac{\partial(T_1B)}{\partial\alpha}+\frac{1}{A}\frac{\partial\left(SA^2\right)}{\partial\beta}-\frac{\partial B}{\partial\alpha}T_2+\frac{2}{R_2}\frac{\partial A}{\partial\beta}H+\frac{1}{R_1}\left[\frac{\partial(M_1B)}{\partial\alpha}-\frac{\partial B}{\partial\alpha}M_2+2\left(\frac{\partial HA}{\partial\beta}\right)\right]+ABq_1=0$$

$$\frac{\partial(T_2A)}{\partial\beta}+\frac{1}{B}\frac{\partial\left(SB^2\right)}{\partial\alpha}-\frac{\partial A}{\partial\beta}T_1+\frac{2}{R_1}\frac{\partial B}{\partial a}H+\frac{1}{R_2}\left[\frac{\partial(M_2A)}{\partial b}-\frac{\partial A}{\partial b}M_1+2\left(\frac{\partial HB}{\partial a}\right)\right]+ABq_2=0$$

$$\frac{1}{AB}\frac{\partial}{\partial\alpha}\left\{\frac{1}{A}\left[\frac{\partial(M_1B)}{\partial\alpha}-\frac{\partial B}{\partial\alpha}M_2+\frac{1}{A}\frac{\partial\left(HA^2\right)}{\partial\beta}\right]\right\}+$$

$$+\frac{1}{AB}\frac{\partial}{\partial\beta}\left\{\frac{1}{B}\left[\frac{\partial(M_2A)}{\partial\beta}-\frac{\partial A}{\partial\beta}M_1+\frac{1}{B}\frac{\partial\left(HB^2\right)}{\partial\alpha}\right]\right\}-\frac{T_1}{R_1}-\frac{T_2}{R_2}+ABq_3=0 \qquad (1)$$

$$Q_1=\frac{1}{AB}\left[\frac{\partial(M_1B)}{\partial\alpha}-\frac{\partial B}{\partial\alpha}M_2+\frac{1}{A}\frac{\partial\left(HA^2\right)}{\partial\beta}\right],$$

$$Q_2=\frac{1}{AB}\left[\frac{\partial(M_2A)}{\partial\beta}-\frac{\partial A}{\partial\beta}M_1+\frac{1}{B}\frac{\partial\left(HB^2\right)}{\partial\alpha}\right],$$

T_1, T_2 – longitudinal forces, S – shifting force, Q_1, Q_2 – transverse forces, M_1, M_2 – bending moments, H – torque moments and q_1, q_2, q_3 – outer load projections distributed on the surface to the directions of the unit vectors $\bar{t}_1, \bar{t}_2, \bar{z}$. They operate at the edges of the infinitely small element the median surface figure 1.

The following equalities represent the ratios between deformations functions and displacements functions (2):

$$\varepsilon_1 = \frac{1}{A}\frac{\partial u}{\partial \alpha} + \frac{1}{AB}\frac{\partial A}{\partial \beta}\upsilon + \frac{w}{R_1},$$

$$\varepsilon_2 = \frac{1}{B}\frac{\partial \upsilon}{\partial \beta} + \frac{1}{AB}\frac{\partial B}{\partial \alpha}u + \frac{w}{R_2},$$

$$\gamma_{12} = \frac{B}{A}\frac{\partial}{\partial \alpha}\left(\frac{\upsilon}{B}\right) + \frac{A}{B}\frac{\partial}{\partial \beta}\left(\frac{u}{A}\right), \tag{2}$$

$$\kappa_1 = -\frac{1}{A}\frac{\partial}{\partial \alpha}\left(\frac{1}{A}\frac{\partial w}{\partial \alpha} - \frac{u}{R_1}\right) - \frac{1}{AB}\frac{\partial A}{\partial \beta}\left(\frac{1}{B}\frac{\partial w}{\partial \beta} - \frac{\upsilon}{R_2}\right),$$

$$\kappa_2 = -\frac{1}{B}\frac{\partial}{\partial \beta}\left(\frac{1}{B}\frac{\partial w}{\partial \beta} - \frac{\upsilon}{R_2}\right) - \frac{1}{AB}\frac{\partial B}{\partial \alpha}\left(\frac{1}{A}\frac{\partial w}{\partial \alpha} - \frac{u}{R_1}\right),$$

$$\kappa_{12} = -\frac{1}{AB}\left(\frac{\partial^2 w}{\partial \alpha \partial \beta} - \frac{1}{A}\frac{\partial A}{\partial \beta}\frac{\partial w}{\partial \alpha} - \frac{1}{B}\frac{\partial B}{\partial \alpha}\frac{\partial w}{\partial \alpha}\right) + \frac{1}{R_1}\frac{A}{B}\frac{\partial}{\partial \beta}\left(\frac{u}{A}\right) + \frac{1}{R_2}\frac{B}{A}\frac{\partial}{\partial \alpha}\left(\frac{\upsilon}{B}\right),$$

where A,B – Lame coefficients, u, υ, w – full vector displacement projections, ε_1, ε_2, γ_{12} – relative linear and angular deformation, κ_1, κ_2 – bending deformations, κ_{12} – torsion deformations, R_1, R_2 – radiuses of the principal curvatures.

The complete equation system is received by the connection to the equation system (1) and (2) of the ratios between deformations and forces. According to the Hirchhoff-Lave hypothesis the isotropic shells deformations ε_α, ε_β $\gamma_{\alpha\beta}$, in the layer located at the distance z from median surface can be presented in the form of the first two terms of the expansion in powers z (3):

$$\varepsilon_\alpha = \varepsilon_1 + \kappa_1 z,\ \varepsilon_\beta = \varepsilon_2 + \kappa_2 z,\ \gamma_{\alpha\beta} = \gamma_{12} + 2\kappa_{12} z \tag{3}$$

In small elastic-plastic deformations theory [6] deformations and tensions are connected with the ratios:

$$\varepsilon_\alpha = \frac{1}{\psi}\left(\sigma_\alpha - \frac{1}{2}\sigma_\beta\right),\ \varepsilon_\beta = \frac{1}{\psi}\left(\sigma_\beta - \frac{1}{2}\sigma_\alpha\right),\ \gamma_{\alpha\beta} = \frac{3}{\psi}\tau_{\alpha\beta}, \tag{4}$$

where the function ψ is equal to the ratio between the tension intensity σ_i and the deformation intensity ε_i (5):

$$\psi = \frac{\sigma_i}{\varepsilon_i} \tag{5}$$

As a result the relations between tensions and deformations take the forms (6), (7):

$$\sigma_\alpha = \frac{4}{3}\psi\left[\varepsilon_1 + \frac{1}{2}\varepsilon_2 + \left(\kappa_1 + \frac{1}{2}\kappa_2\right)z\right], \sigma_\beta = \frac{4}{3}\psi\left[\varepsilon_2 + \frac{1}{2}\varepsilon_1 + \left(\kappa_2 + \frac{1}{2}\kappa_1\right)z\right], \quad (6)$$

$$\tau_{\alpha\beta} = \frac{1}{3}\psi\left(\gamma_{12} + 2\kappa_{12}z\right) \quad (7)$$

In equilibrium equalities of thin shells theory (1) the integral characteristics are introduced instead of tensions: longitudinal and shifting forces, bending and twisting moments. The ratios (6), (7) allow to express the forces by means of the deformations in the following form:

$$T_1 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_\alpha dz = \frac{4}{3}\left[\left(\varepsilon_1 + \frac{1}{2}\varepsilon_2\right)J_1 + \left(\kappa_1 + \frac{1}{2}\kappa_2\right)J_2\right],$$

$$T_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_\beta dz = \frac{4}{3}\left[\left(\varepsilon_2 + \frac{1}{2}\varepsilon_1\right)J_1 + \left(\kappa_2 + \frac{1}{2}\kappa_1\right)J_2\right],$$

$$S = S_1 = -S_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{\alpha\beta} dz = \frac{1}{3}\left(\gamma_{12}J_1 + 2\kappa_{12}J_2\right), \quad (8)$$

$$M_1 = -\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_\alpha z dz = -\frac{4}{3}\left[\left(\varepsilon_1 + \frac{1}{2}\varepsilon_2\right)J_2 + \left(\kappa_1 + \frac{1}{2}\kappa_2\right)J_3\right],$$

$$M_2 = -\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_\beta z dz = -\frac{4}{3}\left[\left(\varepsilon_2 + \frac{1}{2}\varepsilon_1\right)J_2 + \left(\kappa_2 + \frac{1}{2}\kappa_1\right)J_3\right],$$

$$H = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{\alpha\beta} z dz = \frac{1}{3}\left(\gamma_{12}J_2 + 2\kappa_{12}J_3\right)$$

The following values are indicated in the formulas (8): h – thickness of the shell, z – distance from the median surface along the normal to the median surface, the functions J_1, J_2, J_3 – the variables along the median surface of the rigidity of the shell.

$$J_1 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi dz, \quad J_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi z dz, \quad J_3 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi z^2 dz \quad (9)$$

It should be noticed that the function ψ is equal to the material elasticity module $\psi = E$, and the rigidities of the shell are $J_1 = Eh$, $J_2 = 0$, $J_3 = Eh^3/12$ for the elastic area. Meanwhile internal forces according to the ratios (8) move to the elasticity ratios with Puasson coefficient equal to ½ that should be indicated by t_1, t_2, s_1, m_1,m_2, h_1:

$$t_1 = \frac{4}{3} Eh\left(\varepsilon_1 + \frac{1}{2}\varepsilon_2\right), \quad t_2 = \frac{4}{3} Eh\left(\varepsilon_2 + \frac{1}{2}\varepsilon_1\right), \quad s_1 = \frac{1}{3} Eh\gamma_{12}, \tag{10}$$

$$m_1 = -\frac{1}{9} Eh^3\left(\kappa_1 + \frac{1}{2}\kappa_2\right), \quad m_2 = -\frac{1}{9} Eh^3\left(\kappa_2 + \frac{1}{2}\kappa_1\right), \quad h_1 = \frac{1}{18} Eh^3\kappa_{12} \tag{11}$$

If the relation between tension intensity σ_i and deformation intensity ε_i is known, according to [6] the deformation intensity is defined by the formulas (12), (13):

$$\sigma_i = \Phi\left(\varepsilon_i\right), \quad \varepsilon_i = \frac{2}{\sqrt{3}}\sqrt{P_\varepsilon + 2zP_{\varepsilon\kappa} + z^2 P_\kappa}, \quad P_\varepsilon = \varepsilon_1^2 + \varepsilon_1\varepsilon_2 + \varepsilon_2^2 + \frac{1}{4}\gamma_{12}^2, \tag{12}$$

$$P_{\varepsilon\kappa} = \varepsilon_1\kappa_1 + \varepsilon_2\kappa_2 + \frac{1}{2}(\varepsilon_1\kappa_2 + \varepsilon_2\kappa_1) + \frac{1}{2}\gamma_{12}\kappa_{12}, \; P_\kappa = \kappa_1^2 + \kappa_1\kappa_2 + \kappa_2^2 + \gamma_{12}^2 \tag{13}$$

Meanwhile the rigidities (9) express in the form of nonlinearity functions from the deformations. As a result, elastic and plastic components obtained from the formulas (8), (10) and (11) are assigned in elastic solutions method in the internal forces ratios:

$$T_1 = t_1 - \Delta T_1, \quad T_2 = t_2 - \Delta T_2, \quad \mathrm{S}_1 = -S_2 = s_1 - \Delta S_1, \tag{14}$$

$$M_1 = m_1 - \Delta M_1, \quad \mathrm{M}_2 = m_2 - \Delta M_2, \quad \mathrm{H} = h - \Delta H, \tag{15}$$

The functions expressed physical nonlinearity of the material are defined by the formulas (16), (17),(18):

$$\Delta T_1 = \alpha_1 t_1 + \alpha_2 m_1, \quad \Delta T_2 = \alpha_1 t_2 + \alpha_2 m_2, \quad \Delta S_1 = \alpha_1 s_1 - \alpha_2 h, \tag{16}$$

$$\Delta M_1 = \alpha_3 m_1 + \alpha_4 t_1, \quad \Delta M_2 = \alpha_4 m_2 + \alpha_4 t_2, \quad \Delta H = \alpha_3 h - \alpha_4 s_1, \tag{17}$$

$$\alpha_1 = 1 - \frac{J_1}{Eh}, \quad \alpha_1 = \frac{12J_2}{Eh^3}, \quad \alpha_3 = 1 - \frac{12J_3}{Eh^3}, \quad \alpha_4 = \frac{J_2}{Eh} \tag{18}$$

Summarizing the ratios (16), (17) we get the result where the common internal stress tensor (T) can be presented in the form of its decomposition to elastic (t) and additional tensor (ΔT) expressing physical nonlinearity problems:

$$(T) = (t) - (\Delta T) \tag{19}$$

The right part of the system is obtained from the component in the form of two terms during the substitution of decomposition (19) in equilibrium equalities. Thus, the full equation system is a sum of the following equation systems in the problem solution by elastic solutions method. Substituting internal forces functions in the form (19) in the equilibrium equalities the terms that express physical nonlinearity of thin shell material appear in the right part of the equilibrium equalities system. In each approximation the system (1) has the form (20):

$$\frac{\partial\left(T_{1n}B\right)}{\partial\alpha}+\frac{1}{A}\frac{\partial\left(S_nA^2\right)}{\partial\beta}-\frac{\partial B}{\partial\alpha}T_{2n}+\frac{2}{R_2}\frac{\partial A}{\partial\beta}H_n+$$

$$+\frac{1}{R_1}\left[\frac{\partial(M_{1n}B)}{\partial\alpha}-\frac{\partial B}{\partial\alpha}M_{2n}+2\left(\frac{\partial H_nA}{\partial\beta}\right)\right]=-ABq_{1n}+L_1\left(\Delta T\right)_n,$$

$$\frac{\partial\left(T_{2n}A\right)}{\partial\beta}+\frac{1}{B}\frac{\partial\left(S_nB^2\right)}{\partial\alpha}-\frac{\partial A}{\partial\beta}T_{1n}+\frac{2}{R_1}\frac{\partial B}{\partial\alpha}H_n+ \tag{20}$$

$$+\frac{1}{R_2}\left[\frac{\partial\left(M_{2n}A\right)}{\partial\beta}-\frac{\partial A}{\partial\beta}M_{1n}+2\left(\frac{\partial H_nB}{\partial\alpha}\right)\right]=-ABq_{2n}+L_2\left(\Delta T\right)_n,$$

$$\frac{1}{AB}\frac{\partial}{\partial\alpha}\left\{\frac{1}{A}\left[\frac{\partial\left(M_{1n}B\right)}{\partial\alpha}-\frac{\partial B}{\partial\alpha}M_{2n}+\frac{1}{A}\frac{\partial\left(H_nA^2\right)}{\partial\beta}\right]\right\}+$$

$$+\frac{1}{AB}\frac{\partial}{\partial\beta}\left\{\frac{1}{B}\left[\frac{\partial\left(M_{2n}A\right)}{\partial\beta}-\frac{\partial A}{\partial\beta}M_{1n}+\frac{1}{B}\frac{\partial(H_nB^2)}{\partial\alpha}\right]\right\}-\frac{T_{1n}}{R_1}-\frac{T_{2n}}{R_2}=-ABq_{3n}+L_3\left(\Delta T\right)_n,$$

where: q_1, q_2, q_3 – the projections of the components of outer load distributed on the surface, $L_1\left(\Delta T\right)_n$, $L_2\left(\Delta T\right)_n$, $L_3\left(\Delta T\right)_n$ – the components expressed physical nonlinearity.

The ratios between deformations functions and displacements functions in each approximation are determined by the following formulas (21):

$$\varepsilon_{1n}=\frac{1}{A}\frac{\partial u_n}{\partial\alpha}+\frac{1}{AB}\frac{\partial A}{\partial\beta}\upsilon_n+\frac{w_n}{R_1},\quad \varepsilon_{2n}=\frac{1}{B}\frac{\partial\upsilon_n}{\partial\beta}+\frac{1}{AB}\frac{\partial B}{\partial\alpha}u_n+\frac{w_n}{R_2},$$

$$\gamma_{12n}=\frac{B}{A}\frac{\partial}{\partial\alpha}\left(\frac{\upsilon_n}{B}\right)+\frac{A}{B}\frac{\partial}{\partial\beta}\left(\frac{u_n}{A}\right),$$

$$\kappa_{1n}=-\frac{1}{A}\frac{\partial}{\partial\alpha}\left(\frac{1}{A}\frac{\partial w_n}{\partial\alpha}-\frac{u_n}{R_1}\right)-\frac{1}{AB}\frac{\partial A}{\partial\beta}\left(\frac{1}{B}\frac{\partial w_n}{\partial\beta}-\frac{\upsilon_n}{R_2}\right), \tag{21}$$

$$\kappa_{2n}=-\frac{1}{B}\frac{\partial}{\partial\beta}\left(\frac{1}{B}\frac{\partial w_n}{\partial\beta}-\frac{\upsilon_n}{R_2}\right)-\frac{1}{AB}\frac{\partial B}{\partial\alpha}\left(\frac{1}{A}\frac{\partial w_n}{\partial\alpha}-\frac{u_n}{R_1}\right),$$

$$\kappa_{12n}=-\frac{1}{AB}\left(\frac{\partial^2 w_n}{\partial\alpha\partial\beta}-\frac{1}{A}\frac{\partial A}{\partial\beta}\frac{\partial w_n}{\partial\alpha}-\frac{1}{B}\frac{\partial B}{\partial\alpha}\frac{\partial w_n}{\partial\alpha}\right)+\frac{1}{R_1}\frac{A}{B}\frac{\partial}{\partial\beta}\left(\frac{u_n}{A}\right)+\frac{1}{R_2}\frac{B}{A}\frac{\partial}{\partial\alpha}\left(\frac{\upsilon_n}{B}\right)$$

Physical ratios take the following form (22), (23):

$$T_{1n}=t_{1n}-\Delta T_{1n},\ \ T_{2n}=t_{2n}-\Delta T_{2n},\ \ \mathrm{S}_{1n}=-S_{2n}=s_{1n}-\Delta S_{1n}, \tag{22}$$

$$M_{1n}=m_{1n}-\Delta M_{1n},\ \ M_{2n}=m_{2n}-\Delta M_{2n},\ \ \mathrm{H}=h_n-\Delta H_n \tag{23}$$

Here t_{1n}, t_{2n}, s_{1n}, m_{1n}, m_{2n}, h_n are internal loads and the moments for elastic shell that has the same deformations as the present elastic-plastic shell (22), (23):

$$t_{1n} = \frac{4}{3}Eh\left(\varepsilon_{1n} + \frac{1}{2}\varepsilon_{2n}\right),\ t_{2n} = \frac{4}{3}Eh\left(\varepsilon_{2n} + \frac{1}{2}\varepsilon_{1n}\right),\ s_{1n} = \frac{1}{3}Eh\gamma_{12n}, \tag{24}$$

$$m_{1n} = -\frac{1}{9}Eh^3\left(\kappa_{1n} + \frac{1}{2}\kappa_{2n}\right),\ m_{2n} = -\frac{1}{9}Eh^3\left(\kappa_{2n} + \frac{1}{2}\kappa_{1n}\right),\ h_n = \frac{1}{18}Eh^3\kappa_{12n} \tag{25}$$

The functions expressed physical nonlinearity of the material are determined by the formulas (26)-(32):

$$\begin{aligned} \Delta T_{1n} &= \alpha_{1n}t_{1,n\text{-}1} + \alpha_{2n}m_{1,n\text{-}1}, & \Delta M_{1n} &= \alpha_{3n}m_{1,n-1} + \alpha_{4n}t_{1,n-1}, \\ \Delta T_{2n} &= \alpha_{1n}t_{2,n\text{-}1} + \alpha_{2n}m_{2,n\text{-}1}, & \Delta M_{2n} &= \alpha_{3n}m_{2,n-1} + \alpha_{4n}t_{2,n-1}, \\ \Delta S_{1n} &= \alpha_{1n}s_{1,n\text{-}1} - \alpha_{2n}h_{n\text{-}1}, & \Delta H_n &= \alpha_{3n}h_{n-1} - \alpha_{4n}s_{1,n-1}, \end{aligned} \tag{26}$$

$$\alpha_{1n} = 1 - \frac{J_{1n}}{Eh},\ \alpha_{1n} = \frac{12J_{2n}}{Eh^3},\ \alpha_{3n} = 1 - \frac{12J_{3n}}{Eh^3},\ \alpha_{4n} = \frac{J_{2n}}{Eh} \tag{27}$$

$$J_{1n} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_n dz,\quad J_{2n} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_n z dz,\quad J_{3n} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi_n z^2 dz, \tag{28}$$

$$\psi_n = \frac{\sigma_{i,n-1}}{\varepsilon_{i,n-1}},\quad \varepsilon_{i,n-1} = \frac{2}{\sqrt{3}}\sqrt{P_{\varepsilon,n-1} + 2zP_{\varepsilon\kappa,n-1} + z^2 P_{\kappa,n-1}}, \tag{29}$$

$$P_{\varepsilon,n-1} = \varepsilon_{1,n-1}^2 + \varepsilon_{1,n-1}\varepsilon_{2,n-1} + \varepsilon_{2,n-1}^2 + \frac{1}{4}\gamma_{12,n-1}^2, \tag{30}$$

$$P_{\varepsilon\kappa,n-1} = \varepsilon_{1,n-1}\kappa_{1,n-1} + \varepsilon_{2,n-1}\kappa_{2,n-1} + \frac{1}{2}(\varepsilon_{1,n-1}\kappa_{2,n-1} + \varepsilon_{2,n-1}\kappa_{1,n-1}) + \tag{31}$$

$$+\frac{1}{2}\gamma_{12,n-1}\kappa_{12,n-1},\quad P_{\kappa,n-1} = \kappa_{1,n-1}^2 + \kappa_{1,n-1}\kappa_{2,n-1} + \kappa_{2,n-1}^2 + \kappa_{12,n-1}^2 \tag{32}$$

Using elastic solutions method where the components (26) are supposed to be equal to zero, we get the values of stress-strain state components relevant to the phase of elastic material work in the first approximation with $n = 1$. The deformation values of the first approximation allow to define the value of the functions (26) for the second approximation, the deformation values of the second approximation serve for the defining the adding functions for the third approximation, etc. The achievement of the demanding the accuracy for the values of the stress-strain state components of thin isotropic shell demands the performing n number of approximations. The functions expressed physical nonlinearity problems (26) are not equal to zero at the points of the formation of plastic deformations. The ratios (26) - (32) allow to determine the area of plastic deformations distribution in the shell thickness.

3 Results

The study of the convergence of elastic solutions method was conducted by numerical method [5-7] where each tenth approximations for all the resolving functions was compared.

a) b)

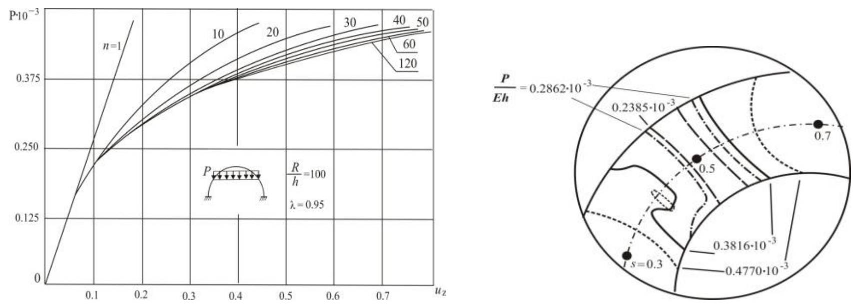

Fig. 2. Rotational shell under the action of the rotation load: a) the diagram P u_z (the load – vertical displacement), b) the area of plastic deformations distribution in the shell thickness

The results of the study for the spherical shell with the angle of half-life $\alpha_0 = 90^0$ and with the relation of the shell curve radius R/h =100 are given under the action of the ring load applied to the middle of the Meridian. The diagram $P - u_z$ (the load – vertical displacement) depending on the number of approximations is presented in the figure 2a). The section below the point of load application is considered. The area of plastic deformations distribution in the shell thickness (figure 2b)) is presented. The material is steel, the module of the material elasticity E = $2.1{\cdot}10^5$ MПa, the diagram with linear strengthening where $\lambda = 0.95$. Under the obtaining the relations, it is occured that elastic solution is useful for the load before $P/Eh \approx 0.15{\cdot}10^{-3}$. It takes 10 approximations, about 20-30 approximations for the loads $P/Eh \approx 0.25{\cdot}10^{-3}$, and about 100 approximations in the case of high loads $P/Eh > 0.35{\cdot}10^{-3}$ for the achieving the accuracy $\mu = 0.1$ % with the loads before $P/Eh \approx 0.2{\cdot}10^{-3}$.

4 Conclusions

The calculations demonstrate that, with the growing of the load, the necessary number of approximations for the achieving the necessary accuracy is increasing, the plastic deformations area is increasing too. The elastic solutions method converges too slowly for the range of loads, where small elastic-plastic deformations theory becomes unjust.

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