Environmental safety construction programs optimization and reliability of their implementation

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Abstract. This paper shows a problem of creating the construction programs to ensure the environmental safety with regard to their reliability. The problem is in choosing the right projects for the program to achieve the required effect with minimum costs by restriction either the number of high-risk projects or their funding amount. This paper suggests algorithms of solving the problems using Branch and Bound method and Cost-effectiveness analysis.

1 Introduction

In recent years, the problems of creating safety programs of different types, i.e. reforming of enterprises, regional development, traffic safety, destruction of chemical weapons, safety in emergency situations have developed considerably in terms of applications of mathematical methods. In formal terms the problems, in general, have the following statements. There are a lot of construction projects that are candidates for participation in the program. Each project is characterized by effects – project contribution to a program, costs on implementation and reliability.

Reliability is estimated either by the probability of a project realization or qualitative characteristics: high, medium or low reliability, respectively, there is a high, medium or low risk project. Reliability of a program is defined as either the probability of a successful program implementation or a qualitative characteristic, i.e. a number of projects with high and medium risk or the funding amount of such projects. Accordingly, the problem is to choose projects that ensure the implementation of the program objectives with minimum costs by restriction either the probability of successful implementation of the program or a number of projects with a high and medium risk, or funding amount of such projects.

The aim of this paper is to develop a methodological approach for creating environmental safety programs taking into account the reliability of their implementation. To that end, the authors suggest to use a well-known Branch and Bound method and its modification based on the Cost-effectiveness analysis to simplify the algorithm of achieving the goal. Branch

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and Bound method is based on the idea of a sequential partitioning of the set of feasible solutions. At each stage of the method, the elements of the partition (subset) are analyzed if this subset has an optimal solution or not. This method is often used in solving optimization problems in operations research and allows to obtain the exact solution of the problem for a finite number of steps.

Branch and Bound method is used for a wide range of problems and has a variety of modifications. So, AitZai, Benmedjdoub and Boudhar [1] used it for the creation of the algorithms for no-wait job shop scheduling; Ardakan, Hakimian and Rezvan [2] used it to minimize the number of tardy jobs in a two-machine flow-shop problem; Neveu, Trombettoni and Araya [3] developed node selection strategies in interval Branch and Bound algorithms; Anjos, Chang and Ku [4] used Branch and Bound method for solving the integer least squares problems; Merakeb, Messine and Aidène [5] minimized the energy consumption of an electrical vehicle; Oberdieck, Wittmann-Hohlbein and Pistikopoulos [6] used it for the solution of multiparametric mixed integer linear programming problems; Costa, Prata, Ramos and Castro [7] applied it for pump scheduling in water distribution networks.

Branch and Bound method modification based on the Cost-effectiveness analysis was suggested by the Russian scientists Barkalov, Burkov, Polovinkina and others [8-13] which named it as a Cost-effect method. This method is the evaluation of the effectiveness of each project that claims to be in the program and of the projects selection in the order of decreasing efficiencies in the context of limited financial resources or before the achievement of the desired overall effect. It is assumed that for each direction (criterion) the program has numerous projects, and these sets do not overlap, at the same time at each stage it's necessary to solve the so-called knapsack problem. The variants of knapsack problem and methods of their solutions were reviewed by Buhrman [14]; Mastin and Jaillet [15]; Wolsey and Yaman [16]; Abdi and Fukasawa [17]; Goerigk [18]; Choi and Banerjee [19].

This paper has the following structure. Section 2 formulates the research problem of the paper. Sections 3 – 5 describe the use of Branch and Bound and Cost-effect methods. Algorithms for the optimization of the environmental safety programs taking into account the reliability of their implementation are suggested, examples of implementation of these algorithms are given. Section 6 concludes the paper.

2 Problem statement

The program is estimated in m criteria. The status of each criterion is usually scored on the scale: bad – 1, satisfactory – 2, good – 3, excellent – 4. In recent years, creating of an integrated assessment program based on the matrix package has become popular. Accordingly, the objective of a program is to achieve either the required value of scores on directions or the required integrated assessment.

There are n projects, i.e. candidates for the participation in a program. Each project is characterized by a_{ij} effects (contributions) that it has in one or more directions of the program $i = \overline{1, n}$, $j \in Q_i$ where Q_i is a set of directions to which an i -type project contributes to. As a rule, it is assumed that effects are added.

Each project can be implemented in two options – with low risk or high risk. Let's denote b_i as the costs of implementing a project with low risk and c_i as the costs of implementing a project with high risk – it is clear that $b_i > c_i$, $i = \overline{1, n}$.

To define the number of projects that ensures the achievement of objectives in all program directions (or the required values of the integrated assessment) with minimum costs by restriction either the financing amount of high-risk projects or their number.

Let's consider the example when each program has its set of projects, i.e. all projects are single-purpose – effect only in one direction. In this case, the problem is solved separately for each direction.

In formal statement of the problem let $x_i = 1$ if the i -type project was included in the program with low risk and $x_i = 0$ to the contrary. Accordingly, let $y_i = 1$ if the i -type project was included in the program with high risk and $y_i = 0$ to the contrary. It is obvious, that

$$x_i + y_i \leq 1, \quad i = \overline{1, n}. \quad (1)$$

At given $x_i, y_i, i = \overline{1, n}$, the effect for the appropriate direction of the program will be

$$R(x) = \sum_{i=1}^n (x_i + y_i) a_i. \quad (2)$$

Assume that the set of boundary values of the effect $A_j, j = \overline{1, 4}$ are such that if

$$A_j \leq R(x) < A_{j+1}, \quad (3)$$

the score is equal to j .

To account risk limits denote C as the maximum financing amount of risky projects and p as the maximum number of high-risk projects, then the appropriate limits have the form

$$\sum_i y_i c_i \leq C \quad (4)$$

or

$$\sum_i y_i \leq p. \quad (5)$$

The problem is to find $x_i, y_i, i = \overline{1, n}$ which minimize

$$W = \sum_{i=1}^n (b_i x_i + c_i y_i) \quad (6)$$

with delimitations of (2), (4) or (5) and delimitation of

$$\sum_i (x_i + y_i) a_i \geq A,$$

where A is equal to one of A_j values depending on the set objective of the direction.

3 Branch and Bound method

To apply the Branch and Bound method it is necessary to have means of estimating from below the subsets of solutions [20]. Consider the algorithm for obtaining the lower bounds.

To that end, let's first turn to new variables $z_i = x_i + y_i, i = \overline{1, n}$. In the new variables the problem will be to minimize

$$W(z, y) = \sum_{i=1}^n (b_i z_i + \Delta_i y_i), \quad (7)$$

where $\Delta_i = b_i - c_i$ with delimitations of (4), (8) and (9):

$$\sum_{i=1}^n z_i a_i \geq A, \quad (8)$$

$$y_i \leq z_i. \quad (9)$$

Consider two evaluation problems.

Problem 1. To minimize

$$F_1\left(z\right) =\sum_i b_i z_i \tag{10}$$

with delimitation of (8).

Problem 2. To minimize

$$F_2\left(y\right) =\sum_{i=1}^n y_i \Delta_i \tag{11}$$

with delimitation of (4).

Denote W_1 as a value of $F_1(z)$ in the optimal solution of the Problem 1 and W_2 as a value of $F_2(y)$ in the optimal solution of the Problem 2.

Theorem 1. *Value*

$$W_1-W_2 \tag{12}$$

is the lower bound for the objective function of problem (4), (7), (8), (9).

Proof. $\min \Phi\left(z,y\right) \geq \min F_1\left(z\right) +\min \left(-F_2\left(y\right) \right) =\min F_1\left(z\right) -\max F_2\left(y\right) .\quad \square$

Use estimation (12) in the Branch and Bound method.

Lemma 1. *If there are optimal solutions z and y of the Problems 1 and 2 so that $y < z$, $(y_i \leq z_i, i = 1, n)$, the pair (z, y) finds the optimal solution of the problem. The proof is obvious as this pair (z, y) is a feasible solution.*

4 Branch algorithm description

Step 1. Solve the Problems 1 and 2. Denote M_1 as the set of optimal solutions of the Problem 1 and M_2 as the set of optimal solutions of the Problem 2. If there is a pair (z, y) , $z \in M_1, y \in M_2$ and $y < z$, Lemma (z, y) is the optimal solution. Otherwise, let’s choose the j -type project so that $y_j = 1$ and $z_j = 0$, and partition the set of all solutions into two subsets. In the first one $y_j = 1, z_j = 1$, and in the second one $y_j = 0$.

Next, for these subsets let’s solve the evaluation problems; choose the subset with the best evaluation, etc. in accordance with the scheme of the Branch and Bound method.

Example 1. There are 7 projects which details are shown in Table 1.

Table 1. Data on projects.

i	1	2	3	4	5	6	7
a_i	10	20	30	25	50	60	75
b_i	20	35	40	50	70	90	65
Δ_i	10	23	25	20	10	40	25
c_i	10	12	15	30	60	50	40

Take $A = 140$ and $C = 70$. Solve the Problem 1. This is a *knapsack problem*. Let’s solve it by the dichotomic programming method. The structure of the network representation of the problem is shown in Fig. 1.

Fig. 1. Structure of the network representation of the problem.

Table 2. Resulting table.

Projects (5, 6, 7)	145; 85*	215; 135*	235; 144	—	—	—
	125; 75*	195; 125*	215; 135	—	—	—
	110; 65*	180; 115*	200; 125*	—	—	—
	95; 60*	165; 110*	185; 120*	—	—	—
	90; 55*	160; 105*	180; 115*	—	—	—
	75; 50*	145; 100*	165; 110	—	—	—
	60; 40	130; 90*	150; 100	—	—	—
	40; 30	110; 80*	130; 90	175; 155	—	—
	35; 20	105; 70*	125; 80	170; 145	—	—
	20; 10	90; 60	110; 70	155; 135	175; 145*	---
	0; 0	70; 50	90; 60	135; 125	155; 135*	225; 185

Projects (1, 2, 3, 4)

The optimal option corresponds to a cell (170; 145). Get solution:

$$z_1 = z_3 = z_4 = z_6 = 0, \quad z_2 = z_5 = z_7 = 1.$$

Solve the Problem 2 that is also a knapsack problem. Take the structure of a dichotomic representation from Fig. 1.

Table 3. Resulting table.

Projects (5, 6, 7)	57; 68	---	---
	37; 58	---	---
	27; 48	67; 73	---
	25; 35	65; 60	---
	22; 33	62; 58	---
	15; 25	55; 50	65; 65
	12; 23	52; 48	62; 63
	10; 10	50; 35	60; 50
	0; 0	40; 25	50; 40

Projects (1, 2, 3, 4)

The optimal solution is determined by the cell (67; 73). Applying the "reverse order" method, define the optimal solution:

$$y_1 = y_4 = y_5 = y_6 = 0, \quad y_2 = y_3 = y_7 = 1$$

with the costs 67 and effect 73. $W_1 = 170$, $W_2 = 73$. Obtain the lower bound that is $W_1 - W_2 = 97$. Since $y_3 = 1$ and $z_3 = 0$, this solution is not a feasible one for the initial problem.

Carry out branching of the project 3. In the first subset $y_3 = 1$, $z_3 = 1$, and in the second one $y_3 = 0$.

Estimate the first subset. Since $y_3 = 1$, the solution of the second problem does not change. Exclude all options when $z_3 = 0$ in the first problem. It is obvious that the resulting table will have the same table without the first three lines. The optimal solution is determined by the cell (175; 155) with the costs 175. Let's estimate from below the first subset

$$F(y_3 = 1) = 175 - 73 = 102.$$

Estimate the second subset. Since $y_3 = 0$, the solution to the first problem does not change. Consider the second problem by setting $y_3 = 0$. Perform the summary Table 4. The optimal solution is determined by the cell (62; 63) with the costs 62 and effect $W_2(y_3 = 0) = 63$. Let's estimate from below the second subset

$$W_2(y_3 = 0) = 170 - 63 = 107.$$

Choose the first subset with a lower bound.

Table 4. Calculation results.

Projects (1, 2, 3, 4)	50; 40	60; 50	62; 63	–	–	–
	40; 25	50; 35	52; 48	62; 58	–	–
	0; 0	10; 10	12; 23	22; 33	42; 43	52; 53

Projects (5, 6, 7)

The corresponding solution has the form of

$$z_1 = z_2 = z_4 = z_6 = 0, \quad z_3 = z_5 = z_7 = 1.$$

Comparing with the solution of the second problem

$$y_1 = y_4 = y_5 = y_6 = 0, \quad y_2 = y_3 = y_7 = 1,$$

we see that $y_2 = 1$ and $z_2 = 0$. Let's carry out branching of the second project, partitioning it into two subsets ($y_2 = 1$). The solution of the second problem does not change.

Consider the first problem if $z_2 = 1$ and $z_3 = 1$. We get

$$F(z_2 = 1, z_3 = 1) = 210.$$

Estimation of the first subset is

$$W(z_2 = 1, z_3 = 1) = 210 - 73 = 137.$$

Estimate the second subset when ($y_2 = 0$). In this case, the solution of the first problem does not change. Consider the second problem when $y_2 = 0$ and $y_3 = 1$. We have

$$W_2(y_2 = 0, y_3 = 1) = 65.$$

Estimation of subset ($y_2 = 0$, $y_3 = 1$) is equal to

$$W(y_2 = 0, y_3 = 1) = 175 - 65 = 110.$$

Take the subset ($y_3 = 0$) with minimal value $W(y_3 = 0) = 107$. The corresponding solution of the first problem is

$$z_1 = z_3 = z_4 = z_6 = 0, \quad z_2 = z_5 = z_7 = 1.$$

and of the second problem is

$$y_1 = y_3 = y_4 = y_5 = y_7 = 0, \quad y_2 = y_6 = 1.$$

Since $y_6 = 1$ and $z_6 = 0$, let's carry out branching of the sixth project, partitioning the set $y_3 = 0$ into two subsets ($y_3 = 0$, $y_6 = 1$) and ($y_3 = 0$, $y_6 = 0$). In the first subset the solution of the second problem does not change. The first problem solution is

$$z_2 = z_3 = z_4 = z_5 = 0, \quad z_1 = z_6 = z_7 = 1,$$

$$W_1(z_6 = 1) = 175.$$

Estimation of the first subset is $W(z_6 = 1) = 112$.

In the second subset ($y_3 = 0, y_6 = 0$) the solution to the first problem does not change. The second problem solution is

$$y_3 = y_4 = y_5 = y_6 = 0, \quad y_1 = y_2 = y_7 = 1,$$

$$W_2(y_2 = 0, y_3 = 0) = 58.$$

Estimation of the second subset is

$$W(y_2 = 0, y_3 = 0) = 170 - 58 = 112.$$

Take the subset ($y_2 = 0, y_3 = 1$) with minimal value 110. The corresponding solution is

$$z_1 = z_2 = z_4 = z_6 = 0, \quad z_3 = z_5 = z_7 = 1,$$

$$y_1 = y_2 = y_4 = y_5 = y_7 = 0, \quad y_3 = y_6 = 1.$$

Let's carry out branching of the sixth project.

Estimate the subset ($y_2 = 0, y_3 = 1, y_6 = 1$). The second problem solution does not change, $W_2(y_2 = 0, y_3 = 1, y_6 = 1) = 65$.

The first problem solution is

$$z_1 = z_2 = z_4 = z_5 = 0, \quad z_3 = z_6 = z_7 = 1,$$

$$W_1(z_3 = 1, z_6 = 1) = 195.$$

Estimation of the subset ($y_2 = 0, y_3 = 1, y_6 = 1$) is equal to

$$W(y_2 = 0, y_3 = 1, y_6 = 1) = 195 - 65 = 130.$$

Estimate the subset ($y_2 = 0, y_3 = 1, y_6 = 0$). The first problem solution does not change. The solution to the second problem is

$$y_2 = y_4 = y_5 = y_6 = 0, \quad y_1 = y_3 = y_7 = 1,$$

$$W_2(y_2 = 0, y_3 = 1, y_6 = 0) = 60.$$

Estimation of the subset ($y_2 = 0, y_3 = 1, y_6 = 0$) is equal to

$$W(y_2 = 0, y_3 = 1, y_6 = 0) = 175 - 60 = 115.$$

Take one of the subsets with the value 112, for example, the subset ($y_3 = 0, y_6 = 1$). Partition it into two subsets – the first one is ($y_3 = 0, y_6 = 1, y_2 = 1$) and the second one is ($y_3 = 0, y_6 = 1, y_2 = 0$). In the first subset of the solution of the Problem 2 does not change:

$$W_2(y_3 = 0, y_6 = 1, y_2 = 1) = 63.$$

The first problem solution is

$$z_1 = z_3 = z_4 = z_5 = 0, \quad z_2 = z_6 = z_7 = 1,$$

$$W_1(z_2 = 1, z_6 = 1) = 190.$$

$$W(y_3 = 0, y_1 = 1, y_6 = 1) = 190 - 63 = 127.$$

In the second subset the solution of the Problem 1 does not change. The solution of the Problem 2 is

$$y_2 = y_3 = y_4 = y_5 = y_7 = 0, \quad y_1 = y_6 = 1.$$

$$W_2(y_3 = 0, y_2 = 0, y_6 = 1) = 35.$$

$$W(y_3 = 0, y_2 = 0, y_6 = 1) = 175 - 35 = 140.$$

Take the subset ($y_3 = 0, y_6 = 0$) with a minimum value 112. The corresponding solution of the problem is

$$z_1 = z_3 = z_4 = z_6 = 0, \quad z_2 = z_5 = z_7 = 1,$$

$$y_3 = y_4 = y_5 = y_6 = 0, \quad y_1 = y_2 = y_7 = 1,$$

$$W_1 = 170, \quad W_2 = 58.$$

Carry out branching of the project 1. In the first subset $y_1 = 1$ and in the second one $y_1 = 0$.

Estimation of the first subset is ($y_1 = 1$). The first problem solution is

$$z_2 = z_3 = z_4 = z_5 = 0, \quad z_1 = z_6 = z_7 = 1,$$

$$W_1(z_1 = 1) = 175.$$

$$W(y_3 = 0, y_6 = 0, y_1 = 1) = 175 - 58 = 117.$$

Estimation of the second subset is ($y_1 = 0$). The second problem solution is

$$y_1 = y_3 = y_4 = y_5 = y_6 = 0, \quad y_2 = y_7 = 1.$$

$$W_2 = 48.$$

Estimation of the second subset is

$$W(y_1 = 0, y_3 = 0, y_6 = 0) = 170 - 48 = 122.$$

Take the subset ($y_3 = 1, y_2 = 0, y_6 = 0$) with a minimum value 115. The corresponding solution of the problem is

$$z_1 = z_2 = z_4 = z_6 = 0, \quad z_3 = z_5 = z_7 = 1,$$

$$W_1 = 175.$$

$$y_2 = y_4 = y_5 = y_6 = 0, \quad y_1 = y_3 = y_7 = 1.$$

$$W_2 = 60.$$

Carry out branching of the project 1.

Estimation of the first subset is ($y_1 = 1$). The first problem solution ($z_1 = 1, z_3 = 1$) is

$$z_2 = z_5 = z_6 = 0, \quad z_1 = z_3 = z_4 = z_7 = 1,$$

$$W_1 = 175.$$

$$W(y_3 = 1, y_2 = 0, y_6 = 0, y_1 = 1) = 175 - 60 = 115.$$

The obtained solution is feasible. Therefore, due to Lemma 1 the solution

$$z_2 = z_5 = z_6 = 0, \quad z_1 = z_3 = z_4 = z_7 = 1,$$

$$y_2 = y_4 = y_5 = y_6 = 0, \quad y_1 = y_3 = y_7 = 1.$$

is feasible with the objective function

$$F(z, y) = 175 - 60 = 115.$$

In optimal solution the projects 1, 3 and 7 are carried out with high risk, and the project 4 – with low risk. The tree branches are shown in Fig. 2.

5 Cost-effect method

The described method of obtaining the lower bounds requires at each step of branching solving a knapsack problem. There is an efficient approximate algorithm to solve the knapsack problem that is called the Cost-effect method; its brief description was given in Introduction. Apply this method to obtain the approximate lower bounds in the method of Branch and Bound. To that end, let's define the effectiveness of projects contribution to the direction development $q_i = a_i / b_i$ when they are carried out with low risk and the efficiency of the transition from high-risk to low-risk is $p_i = A_i / c_i, i = \overline{1, n}$. The table of efficiency is given in Table 5.

Fig. 2. Tree branches.

Table 5. Projects efficiency for the directions development.

<i>i</i>	1	2	3	4	5	6	7
<i>q_i</i>	1/2	4/7	3/4	1/2	5/7	2/3	15/13
<i>p_i</i>	1	23/12	5/3	2/3	1/6	4/5	5/8

Arrange the projects in descending order of the efficiencies for Problems 1 and 2 respectively – see Tables 6 and 7.

Table 6. Projects efficiency for Problems 1, 2.

Projects efficiency for Problem 1							
<i>i</i>	7	3	5	6	2	1	4
<i>a_i</i>	75	30	50	60	20	10	25
<i>b_i</i>	65	40	70	90	35	20	50
Projects efficiency for Problem 2							
<i>i</i>	2	3	1	6	4	7	5
<i>Δ_i</i>	23	25	10	40	20	25	10
<i>c_i</i>	12	15	10	50	30	40	60

Under the Cost-effect method the projects are chosen in accordance with the indicated orderings. Apply this method to the solution of Example 1.

Example 2. Solve the Problem 1. The solution contains projects 7, 3, and 5 with the costs 175. Solve the Problem 2. The solution contains projects 2, 3, 1, 4 with the effect 78. Obtain the lower bound that is $W = 175 - 78 = 97$.

Carry out branching of the project 2.

Estimate the subset ($y_2 = 1, z_2 = 1$). The solution of the Problem 1 includes projects 2, 7, 3, and 5 with the costs 210.

The result can be improved if in the last step of selection to look through all the projects the inclusion of which provides the desired effect, and then choose among them a project with minimal costs. This technique insignificantly increases the volume of calculation but allows in some cases to improve the result. So, if in the last step to take project 4 instead of project 5, the costs will decrease by 20.

With this comment in mind, take the solution with projects 2, 7, 3, and 4. The costs are equal to 190. Estimation of the subset is equal to

$$W(y_2 = 1) = 190 - 78 = 112.$$

Estimate the subset ($y_2 = 0$). The solution of the Problem 2 includes projects 1, 3 and 7. Project 7 is also included there because the last step of this project is better than the one of the project 4. Estimate the subset

$$W(y_2 = 0) = 175 - 60 = 115.$$

Choose the subset ($y_2 = 1$). Since $y_1 = 1$ and $z_1 = 0$ let's carry out branching of the project 1.

Estimate the subset ($y_2 = 1, y_1 = 1$). The solution of the Problem 1 includes projects 1, 2, 7, 3, 4 with the costs 210. Estimation of the subset is equal to

$$W(y_2 = 1, y_1 = 1) = 210 - 60 = 150.$$

Estimate the subset ($y_2 = 1, y_1 = 0$). The solution of the Problem 2 includes projects 2, 3 and 7 with the effect 73. Estimation of the subset is equal to

$$W(y_2 = 1, y_1 = 0) = 190 - 73 = 117.$$

Choose the subset ($y_2 = 1$).

The solution of the Problem 1 includes projects 7, 3, 5 with the costs 175 and the solution of the Problem 2 includes projects 1, 3 and 7 with the effect 60. Since $y_1 = 1$ and $z_1 = 0$ let's carry out branching of the project 1.

Estimate the subset ($y_2 = 0, y_1 = 1$). The solution of the Problem 1 includes projects 1, 7, 3, and 4 with the costs 175. Estimation of the subset is equal to

$$W(y_2 = 0, y_1 = 1) = 175 - 60 = 115.$$

Note that the condition of Lemma 1 has been carried out, therefore, the valid solution is obtained:

$$\begin{aligned} z_2 = z_5 = z_6 = 0, \quad z_1 = z_3 = z_4 = z_7 = 1, \\ y_2 = y_4 = y_5 = y_6 = 0, \quad y_1 = y_3 = y_7 = 1. \end{aligned}$$

Estimate the subset ($y_2 = 0, y_1 = 0$). The solution of the Problem 2 includes projects 3 and 6 with the effect 65. Estimation of the subset is equal to

$$W(y_2 = 0, y_1 = 0) = 175 - 65 = 110.$$

Choose the subset ($y_2 = 0, y_1 = 0$). Since $y_6 = 1, z_6 = 0$ let's carry out branching of the project 6.

Estimate the subset ($y_2 = 0, y_1 = 0, y_6 = 1$). The solution of the Problem 1 includes projects 1, 6 and 7 with the costs 175. Estimation of the subset is equal to

$$W(y_2 = 0, y_1 = 0, y_6 = 1) = 175 - 65 = 110.$$

Estimate the subset ($y_2 = 0, y_1 = 0, y_6 = 0$). The solution of the Problem 2 includes projects 3 and 7 with the effect 50. Estimation of the subset is equal to

$$W(y_2 = 0, y_1 = 0, y_6 = 0) = 175 - 50 = 125.$$

Choose the subset ($y_2 = 0, y_1 = 0, y_6 = 1, y_3 = 1$). Since $y_3 = 1$ and $z_3 = 0$ let's carry out branching of the project 3.

Estimate the subset ($y_2 = 0, y_1 = 0, y_6 = 1, y_3 = 1$). The solution of the Problem 1 includes projects 3, 6 and 7 with the costs 195. Estimation of the subset is equal to

$$W(y_2 = 0, y_1 = 0, y_6 = 1, y_3 = 1) = 195 - 50 = 145.$$

Estimate the subset ($y_2 = 0, y_1 = 0, y_6 = 1, y_3 = 0$). The solution of the Problem 2 includes project 6 with the effect 40. Estimation of the subset is equal to

$$W(y_2 = 0, y_1 = 0, y_6 = 1, y_3 = 0) = 175 - 40 = 135.$$

Choose the subset ($y_2 = 0, y_1 = 1$) with a minimum value 115. As the corresponding solution

$$z_2 = z_5 = z_6 = 0, \quad z_1 = z_3 = z_4 = z_7 = 1,$$

$$y_2 = y_4 = y_5 = y_6 = 0, \quad y_1 = y_3 = y_7 = 1$$

is acceptable, it is optimal.

6 Conclusion

In this paper a methodological approach for creating environmental safety programs was developed with regard to the reliability of their implementation on the assumption that each project included in the program has two variants of implementation. Generalization of this approach is also possible in case when the project has three options of implementation: with low, medium and high risks. To solve this problem the authors have developed Branch and Bound algorithm and its modification on the basis of Cost-effectiveness analysis – Cost-effect method, the examples of their implementation are also given.

The given examples show that with a small number of projects included in the program, these methods are almost equally effective. Therefore, for small programs it's more efficient to apply Cost-effect method because of its lower complexity, since at each stage of the method the knapsack problem is solved by a simple way of dichotomous programming.

The next stage is the development of a program implementation plan, i.e. the distribution of projects over time, given the limited means for periods. As a rule, loss of profit is the criterion of efficient allocation of resources. To that end, tools of project management can be applied.

The application of scheduling methods such as critical path method is also possible, and in case of large uncertainties in terms of performance of works and provision of resources PERT method can be used. For effective management of programs and projects the use of modern information technologies, implemented by software systems Microsoft Project, Primavera, Spider, Open Plan can be recommended.

Thus, all suggested measures allow to increase the efficiency of the management of environmental safety programs and to guarantee high reliability of their implementation. In addition, this approach can be successfully applied to programs implemented in other areas, for example, in the field of information security, occupational safety, etc.

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