Calculation of multilayer enclosing structures with middle layer of polystyrene concrete

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Abstract. The strength and strain relations of characteristics of the layers in multilayer wall structures made of concrete of different strength are presented, the dependence of the stress values in the layers on the operating conditions of the structure is revealed. On the basis of the principle of strain compatibility of concrete and reinforcement, the estimation of the stress-strain state of multilayer wall panels is done taking into account the conditional concentrated shears in the joint between concretes. A calculation method is proposed, which is based on strains during the combined action of torque and bending moments of three-layer reinforced concrete elements with rectangular section in the stage of working with cracks.

1 Introduction

The constant rise in the cost of energy resources and the growing requirements for increasing technical and economic indicators for enclosing structures have made it necessary to further develop calculation methods for multilayer enclosing structures, which rightfully occupy one of the leading places in major construction. In the calculation of strength of such structures, the most common method is to reduce the inhomogeneous section to the T-type or the I-type, when the hypothesis of plane sections is adopted, i.e. the cross sections remain plane during deformation and perpendicular to the longitudinal axis of the element, and it is also assumed that there is no shear along the section thickness.

In multilayer structures of reinforced concrete, the differences between the shear modulus of layers are less significant than in composite structures using polymeric materials. This is explained by the fact that the shear modulus of concrete is higher than the effective heat insulation. In spite of the fact that the cross-section of multilayer structures is usually made of materials of different strength, the element can be calculated as monolithic in the case of ensuring rigid connections between its constituent parts.

Thus, for example, generalizing the experimental studies of Russian scientists [2-4] and comparing them with the results of the theoretical study on the choice of a calculation method [5], similar features as for I-elements are adopted for the multilayer slabs of roof and floor as an analytic model according to [6].

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2 Results section

Replacing the multilayer section by the reduced homogeneous one and using the plane section hypothesis make it possible to considerably simplify the calculation of multilayer structures. The introduction of additional working hypotheses in the calculation of such structures will allow taking into account the features of deformation of a multilayer structure. Determination of the curvature of three-layer elements is adopted as for plane bending using the experimental data on the values of empirical coefficients. The reduction of the actual compression zone in the form of a trapezoid and a triangle to a rectangular one allowed using the formulas of the design standards in the calculations [6].
The magnitude of the bending of the three-layer reinforced concrete element under the combined action of vertical and horizontal loads is characterized by medium strains of the concrete of the most compressed rib and the most tensioned reinforcement:

$$\varepsilon_{s,m} = \frac{\psi_s \sigma_s}{E_s}; \varepsilon_{b,m} = \frac{\psi_b \sigma_b}{E_b}. \tag{1}$$

In this case, the bending curvature is defined as the tangent of the slope angle of the medium strains diagram, then:

$$\frac{1}{r} = \frac{\varepsilon_{s,m}}{h_0 - X_m} = \frac{\varepsilon_{s,m}+\varepsilon_{b,m}}{h_0}, \tag{2}$$

where ψ_b - the coefficient that takes into account the unevenness of strains of the concrete of the outermost compression fiber. To determine the curvature of a multilayer wall panel, taking into account the medium strains, substituting formula (1) in (2), we obtain the following equation:

$$\frac{1}{r} = \frac{\psi_s \sigma_s}{E_s (h_0 - X_m)} = \frac{\psi_s \sigma_b}{v E_b X_m} = \frac{\psi_s \sigma_s}{E_s h_0} + \frac{\psi_b \sigma_b}{v E_b h_0}, \tag{3}$$

When the multilayer structure is used in a section with a crack, the stresses in the reinforcement and concrete will be equal to:

$$\sigma_s = \frac{M_\gamma}{W_s} = \frac{M_\gamma}{A_s z}, \tag{4}$$

$$\sigma_b = \frac{M_\gamma}{W_c} = \frac{M_\gamma}{\left(A_b + \alpha \frac{A_s}{2v}\right) z}, \tag{5}$$

where M_γ - the effective bending moment in the section;
W_c, W_s- elastic-plastic moments of resistance along the compressed and tensioned part of the section respectively;
Z – the distance from the center of gravity of the sectional area of the tensioned reinforcement to the point of application of the resultant forces in the compression zone of the section above the crack;
A_b - the area of the compression zone of the concrete taking into account the compression reinforcement for rectangular elements is calculated by the formula:

$$A_b = bx = bh_0 \varepsilon, \tag{6}$$

Taking into account the medium strains and the acting stresses in the reinforcement and concrete (2-5), the formula for determining the curvature of the element in the areas with cracks in the tensioned zone will have the form:

$$\frac{1}{r} = \frac{M_y}{h_0 z}\left[\frac{\psi_s}{A_s E_s} + \frac{\psi_s}{(\varepsilon+y_f) b h_0 E_b v}\right], \quad (7)$$

where y_f - the coefficient that takes into account the presence of compression reinforcement in the section of the element;
v – the coefficient that characterizes the elastic-plastic state of concrete in the compression zone.
The angle of action of external forces is determined by the slope of the neutral line in the section. On the basis of the results of experimental studies with the combined action of the torque and bending moments, the position of the neutral line in the oblique bending tests of the wall panels indicates that the slope of the neutral line varies little with variation in the load level [4]. The shape of the compression zone of the concrete will have a different outline and the coefficient of completeness of the stress diagram can be found as the ratio of the volume of the actual stress diagram in the compression zone to the volume of the reduced rectangular stress diagram.
With the compression zone in the form of a triangle intersecting three layers of the section of the element, the coefficient of elastoplasticity will be equal to

$$v = \frac{1{,}8 sin^2\gamma\left[\frac{X^3}{3sin^2\gamma} + \frac{X^2 b_2(n-1)}{sin\gamma} - \frac{C(n-1)sin\gamma}{3} - XA(n-1)\right]}{X\left\{\left(Xsin\gamma + \frac{X}{sin\gamma} - b_1\right)b_1 + b_2 n\left[2\left(\frac{X}{sin\gamma} - b_1\right) + b_2\right] + \left[\frac{X}{sin\gamma} - (b_1+b_2)\right]^2\right\}}, \quad (8)$$

where $A = (b_1+b_2)^2 - b_1^2; \quad C = (b_1+b_2)^3 - b_1^2.$

Taking into account the value of the coefficient of duration of the load for heavy concrete and deformed reinforcement, the formula for the case under consideration is as follows:
where M_{crc}- the moment in the formation of cracks.
The results of tests of three-layer elements of rectangular section confirm the possibility of using formulas and dependencies of design standards for plane bending and for oblique bending without regard to the shape of the compression zone of the concrete.
The coefficient $\psi_b = \varepsilon_{b,m}^{exp} / \varepsilon_{b,max}^{exp}$ takes into account the irregularity in the distribution of strains of the outermost compression concrete fiber along the cracked zone and is determined directly from the experiments by measuring during testing.
Numerous studies of bending eccentric compression elements of various cross-sectional shapes made of various concretes with arbitrary reinforcement did not reveal certain regularities in the variation of the coefficient ψ_b, whose variation for heavy concrete in most cases is within the range of 0.8-0.9.
Depending on the slope angle of the force plane, different outlines of the compression zone are possible. In this case, the actual compression zone of the concrete is replaced by a rectangular one, while maintaining its true depth and ensuring the strains of the outermost compression fiber. Therefore, the reduced width of the rectangular section is found from the equation of the elastic-plastic moments of resistance over the compression zone in the form of a rectangle and the actual compression zone with oblique bending in the form of a triangle or trapezium.
The distance from the most compressed concrete rib to the center of gravity of the real compression zone is:

$$X_{sb} = \frac{X_b(1-sin^2\gamma)}{cos\gamma} + y_b sin\gamma = X_b cos\gamma + y_b sin\gamma, \quad (10)$$

where: X_b, y_b - the coordinates of the center of gravity of the actual diagram of the compressive stresses, depending on the shape of the compression zone.
The elastic-plastic moment of resistance of the compression zone for its real outline is

$$W_b = (h_0 - X_{sb})\left[\frac{X^2}{sin2\gamma} + \frac{Xb_2(n-1)}{cos\gamma} - \frac{Atg\gamma(n-1)}{2}\right]; \tag{11}$$

and the equivalent rectangular section is

$$W_{b,red} = b_{red}X\left(h_0 - \frac{X}{2}\right). \tag{12}$$

Let's determine the reduced width of an equivalent rectangular section. For $X = \varepsilon \cdot h_0$, the following notation is obtained:

$$b_{red} = \frac{(h_0 - x_{sb})\left[\frac{\varepsilon^2 h_0^2}{sin2\gamma} + \frac{\varepsilon b_2 h_0}{cos\gamma}(n-1) - \frac{A}{2}(n-1)tg\gamma\right]}{\varepsilon\left(1-\frac{\varepsilon}{2}\right)h_0^2}. \tag{13}$$

The coefficient δ taking into account the load level:

$$\delta = \frac{M}{b_{red}h_0^2 R_{b,ser}} = \frac{M\xi\left(1-\frac{\xi}{2}\right)}{R_{b,ser}(h_0 - x_{sb})\left[\frac{\xi^2 h_0^2}{sin2\gamma} + \frac{b_2 h_0}{cos\gamma}\xi(\mathrm{n}-1) - \frac{A}{2}(n-1)tg\gamma\right]}. \tag{14}$$

The reinforcement ratio taking into account b_{red} will be equal to:

$$\mu = \frac{A_s}{h_0 b_{red}} = \frac{A_s h_0\left(1-\frac{\xi}{2}\right)}{(h_0 - x_{sb})\left[\frac{\xi^2 h_0^2}{sin2\gamma} + \frac{b_2 h\xi(\mathrm{n}-1)}{cos\gamma} - \frac{A(n-1)}{2}tg\gamma\right]}. \tag{15}$$

For λ = 0, the formula for determining the relative depth of the compression zone is obtained:

$$\xi = \frac{1}{1{,}8+\frac{1+5\delta}{10\mu\alpha}} = \frac{1}{1+\frac{5M\xi(1-\frac{\xi}{2})}{R_{b,ser1}(h_0 - x_{sb})\left[\frac{\xi^2 h_0^2}{sin2\gamma} + \frac{(n-1)}{cos\gamma}\left(b_2 h_0\xi - \frac{Asin\gamma}{2}\right)\right]}} \cdot 1{,}8 +$$

$$\frac{10\alpha A_s h_0\left(1-\frac{\xi}{2}\right)}{(h_0 - x_{sb})\left[\frac{\xi^2 h_0^2}{sin2\gamma} + \frac{(n-1)}{cos\gamma}(b_2 h_0\xi - \mathrm{Asin}\gamma)\right]}, \tag{16}$$

After the transformation, we have:

$$\xi\left[\frac{R_{b,ser1}h_0^2}{sin2\gamma}(h_0 - x_{sb}) - 2{,}5M\right] + \xi^2\left[5M - 9R_{b,ser}\alpha A_s h_0 + \frac{R_{b,ser1}(\mathrm{n}-1)}{cos\gamma}b_2 h_0(h_0 - x_{sb})\right] +$$

$$\xi\left[28R_{b,ser1}\alpha A_s h_0 - \frac{A}{2}(\mathrm{n}-1)tg\gamma R_{b,ser1}(h_0 - x_{sb})\right] - 10\alpha A_s h_0 R_{b,ser1} = 0. \tag{17}$$

The relative depth of the compression concrete zone can be found from the solution of the reduced cubic equation:

$$\xi^3 + \xi^2 r + \xi \mathrm{s} + \mathrm{t} = 0, \tag{18}$$

where:

$$r = \frac{5M-\left[9\alpha A_s-\frac{b_2}{cos\gamma}(h_0-x_{sb})\right]R_{b,ser}h_0}{\frac{R_{b,ser1}{h_0}^2(h_0-x_{sb})}{sin2\gamma}-2{,}5M};$$

$$s = \frac{R_{b,ser1}\left[28\alpha A_s h_0-\frac{A}{2}(\mathrm{n}-1)tg\gamma(h_0-x_{sb})\right]}{\frac{R_{b,ser}{h_0}^2}{sin2\gamma}(h_0-x_{sb})-2{,}5M}. \quad (19)$$

In the case of multilane arrangement of the reinforcement, the curvature of the reinforced concrete elements is determined from the solution of the system of physical dependences in the form:

$$\begin{cases} \frac{1}{r} = b_{11}M + B_{12}N \\ \varepsilon_0 = B_{12}M + B_{22}N \end{cases}, \quad (20)$$

where:

$$\begin{cases} M = D_{12}\frac{1}{r} + D_{22}\varepsilon_0 \\ N = D_{12}\frac{1}{r} + D_{22}\varepsilon_0 \end{cases}; \quad (21)$$

$$D_{12} = \sum_{i=1}^{n}\frac{E_{si}A_{si}}{\psi_s}Z_{si}^2 + \sum_{j=1}^{K}E_{sj}A'_{sj}Z_{sj}^2 + b\left(y_f + \xi_1\right)\frac{h_0\bar{v}z_b}{\psi_b}; \quad (22)$$

$$D_{12} = \sum_{i=1}^{n}\frac{E_{si}}{\psi_s}A_{si}Z_{si} + \sum_{j=1}^{K}E_{sj}A'_{sj}Z_{si} + \left(y_f + \xi_1\right)\frac{bh_0\bar{v}Z_b}{\psi_b}; \quad (23)$$

$$D_{22} = \sum_{i=1}^{n}\frac{E_{si}}{\psi_s}A_{si} + \sum_{j=1}^{K}E_{sj}A'_{sj} + \left(y_f + \xi_1\right)\frac{bh_0E_b\bar{v}}{\psi_b}. \quad (24)$$

Here
i - ordinal number of the rod of the longitudinal tensioned reinforcement;
j - the same for compression reinforcement;
ξ - relative depth of the compression zone of the section, equal to $\frac{X}{h_{01}}$ and is determined depending on the position of the neutral line;
Z_{si}, Z_{sj} - the distance from the center of gravity of the i-th and j-th reinforcement to the y axis.
The calculation is performed by the iteration method with the slope angles of the resultant of external forces to the vertical from 0 to 90 degrees and the discrete arrangement of the reinforcement with an arbitrary number of it (Fig. 1).

1) 2) 3)

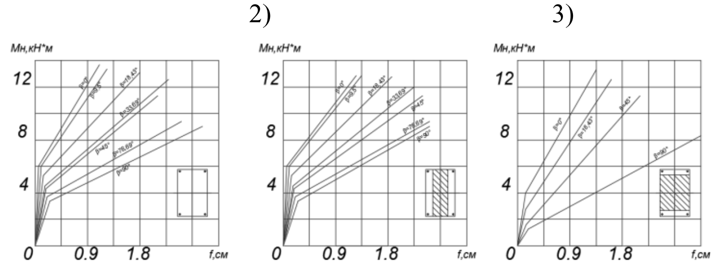

Fig.1 Displacements of beams: 1 - solid rectangular section; 2 - three-layer rectangular section with an inner low-strength layer located along the long side; 3 - three-layer rectangular section located along the short side of the section.

The choice of the calculated case is made using the boundary conditions:

$$\frac{x}{cos\gamma} < h; \frac{x}{cos\gamma} \geq h; \frac{x}{sin\gamma} > b_n; \frac{x}{sin\gamma} < b_n.$$

When calculating by strains, the slope angle of the neutral line is determined from the calculation by the formation of cracks. The reinforcement ratio can be found by a formula

$$\mu = \frac{\sum_{i=1}^{10} A_{si} \frac{h_{0i}-1,3x}{h_{01}-1,3x}}{bh_0}. \quad (25)$$

On the assumption, the *y* axis is located within the effective depth of the section at the distance of *1.3x* from the most compressed face of the section of the element.
In formulas (22-24), the values of Z_b and Z_{sj} are assumed to be negative, and Z_{si} is positive and located below the *y* axis.
When determining ξ_1, the effective depth of the section is assumed to be h_0.
The coefficient ψ_s can be found by the formula:

$$\psi_s = 1,25 - \frac{M_{crc}}{M} \cdot \frac{h_{01}}{h_{oi}} \cdot y_{ls}. \quad (26)$$

The approximate value of the depth of the compression zone is given by x = 0.25h.
After determining the cross-sectional area of the reinforcement by the formula

$$A_s = \sum_{i=1}^{n} A_{si} \frac{h_{oi}-1,3x}{h_{oi}-1,3x} \quad (27)$$

Conclusions

The graphs presented in Fig. 1 consist of two sections with different slope, which represent the work of the element in the elastic stage and after the formation of cracks.

The moment in the formation of cracks depends on the slope angle of the force plane, and its value lies between the values determined by the calculation for a plane bend at angles of 0 and 90 degrees. In all cases, there is an increase in displacements in a direction perpendicular to the neutral line with an increase in the slope angle of the force plane. With the same load level, the values of the displacements of solid and three-layer beams with an inner layer located along the larger side differ by up to 20%.

A similar difference was established when comparing the deflections of solid and three-layer section beams with an inner layer located along the short side of the section. In this case, the differences reach up to 24%.

The presented data of numerical analysis indicates that the significant decrease in rigidity occurs in beams of the three-layer section with an increase in the horizontal component of the total moment. A consequence of the established fact is the presence of the middle low-strength layer in the three-layer sections of the beams.

When adding a vertical moment, horizontal displacements increase. The largest of them will be observed at the small slope angles of the force plane to the vertical, when the compression zone has a trapezoid outline. In the quantitative estimation of strains, the effect of the vertical load with oblique bending on horizontal displacements has a significant influence.

References

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