Guidelines on calculation of the concrete thermal treatment modes

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Abstract. The article provides guidelines on calculation of the cast-in-place reinforced concrete thermal treatment modes: the problem of thermal treatment mode by setting the mathematical temperature field model in the hardening concrete has been solved; model equation which allows calculating the temperature field in the hardening concrete structure using computer software has been solved numerically. One of the most pressing social problems in the world is the problem of housing for citizens, towards whom the state has obligations in accordance with applicable law. Currently, the most efficient way to solve this problem directly is a mechanism of provision of housing at the expense of state funds. Successful completion of such programs is also achieved due to the formation of an affordable economy class housing market that meets the requirements of energy efficiency and environmental friendliness. In turn, the energy efficiency criterion is achieved by rational choice of technology and organization of construction operations. An important criterion for energy efficiency of cast reinforced concrete technology utilization is the rational temperature condition of concrete hardening.

1 Introduction

1.1 The energy efficiency problem of economy class residential buildings

One of the most pressing social problems in the world is the problem of housing for citizens, towards whom the state has obligations in accordance with applicable law. Currently, the most efficient way to solve this problem directly is a mechanism of provision of housing at the expense of state funds. Successful completion of such programs is also achieved due to the formation of an affordable economy class housing market that meets the requirements of energy efficiency and environmental friendliness. In turn, the energy efficiency criterion is achieved by rational choice of technology and organization of construction operations.

1.2 Energy efficient concrete temperature conditions

Currently, the use of technology and organization of construction by cast-in-place method, which has a significantly accelerated level of adoption, has been increasingly recognized in

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the construction industry. An important criterion for energy efficiency of cast reinforced concrete technology utilization is the rational temperature condition of concrete hardening. It is necessary to have a common method of the temperature field calculation in the hardening concrete structure for a satisfactory resolution of this problem. The implementation of this calculation method by using computer modeling is currently possible and relevant due to the widespread use of computer technology.

1.3 The task of searching for the concrete structures thermal treatment modes

Under the influence of non-homogeneous and non-stationary temperature field on the concrete hardening the thermo-stress state arises, which can lead to the ultimate structural strength reduction. Therefore, it is necessary to know the temperature fields dynamics in concrete structures for different heat treatment methods. Choosing the most efficient mode of concrete structures thermal treatment, it is possible to provide high quality of concrete at the minimum duration of thermal treatment and maximum reduction of energy costs. During the concrete structures thermal treatment the temperature field in the structure can be controlled by changing the initial concrete temperature, heating elements power and heat-exchange conditions on the structure surface. There appears the problem of searching of such thermal treatment mode, under which the temperature field has the required characteristics. These characteristics include temperature, rate of its rise and temperature gradient.

1.4 Basic steps of the thermal treatment problem solving

The first step of the thermal treatment problem solving is the creation of accurate mathematical model of temperature field in the hardening concrete. The second step should be devoted to the numerical solution of model equations which allow calculating the temperature field in the hardening concrete structure using the computer software. With this method, it is possible to investigate the temperature field dynamics under different thermal treatment modes and develop the most rational modes, without recourse to a large series of scientific experiments by using the software.

2 Literature review

B.V. Zhadanovsky, S.A. Sinenko Prospects for increasing the technical level of production of concrete works in modern construction.

B.V. Zhadanovsky, D.G. Dragan, S.A. Sinenko Energy-efficient way keeping fresh concrete in the construction of monolithic structures.

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A. Behnood, H. Ziari Effect of silica fume addition and water cement ratio on properties of high-strength concrete after exposure to high temperatures.

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P.K. Mehta, P.J. Monteiro Concrete Microstructure.
M.S. Morsy, S.H. Alsayed, M. Aqel Effect of elevated temperature on mechanical properties and microstructure of silica flour concrete.
A.A. Volkov, A.V. Sedov, P.D. Chelyshkov Modelling the thermal comfort of internal building spaces in social buildings.

3 Materials and methods

The proposed method of temperature field calculation can be applied in various ways of concrete thermal treatment, during choosing the capacity and heat insulation of the designed heating formwork, as well as for the calculation of the thermal conditions of buildings and constructions.

3.1 Mathematical model of the temperature field

The numerical solution of model equations defined by locally one-dimensional scheme [LOS] of the total approximation method is adopted with the above mathematical model of the temperature field in the hardening concrete structure of arbitrary shape with different heat exchange conditions on the surface.

3.1.1 Warm water equation

Heat transfer in the concrete in the absence of intense sources [drains] of water and steam is mainly determined by the thermal conductivity [8]. The thermal conductivity equation for three-dimensional area G is written as:

$$C \frac{\partial u}{\partial t} = Lu + f, \quad Lu = \sum_{p=1}^{3} L_p u$$

where

- $C$ – specific heat per unit volume, $J \cdot m^3 \cdot ^\circ C$,
- $\lambda_p$ – thermal conductivity coefficient, $W \cdot m^{-1} \cdot ^\circ C$,
- $x_p [P = 1; 2; 3]$ – Cartesian coordinates, $m$,
- $t$ – time, $s$,
- $u$ – temperature, $^\circ C$,
- $f$ – internal heat sources intensity, $W / m^3$.

3.1.2 Domain task

Area G must include not only the fresh concrete but the temperature field of which is under consideration, but also solid parts in contact with it [for example, previously poured concrete, foundation soil, etc.]. It is necessary to establish conditions on the part of boundaries $G$ of $G$ area in a solid:

$$\frac{u}{r} = \text{const or } \frac{du}{dn} = 0$$

(2)
Values C, L and f in Equation 1 are not the same in different zones of G area, i.e. they depend on coordinates due to the non-heterogeneity.

3.1.3 Reinforcement influence

Steel reinforcement in concrete typically occupies a small relative volume, therefore its influence on values c and j is not significant. However, since the steel thermal conductivity $[\lambda_p]$ is greater than $\lambda_p$ of concrete, then the heat flows spreading over the reinforcement bars have to be taken into account. Thus, $\lambda_p$ also depends on the location and direction of reinforcement [anisotropy of medium].

3.1.4 Internal heat sources

Intensity of internal heat sources consists of the energy inflow rate from outside [electric curing etc.] and intensity of concrete heat release [5]. The first term is the control action and is given as coordinate and time function. Concrete heat release intensity depends on the temperature and concrete state [21]. It should be noted that the thermal conductivity of hardening concrete also depends on temperature and its state [18]. Currently, this issue needs shall be experimentally studied.

3.1.5 Simulation of heat propagation in the frozen foundation

When laying the concrete on the partially frozen earth foundation during the concrete curing process, the boundary between melt and frozen zones moves in the soil. The temperature at the boundary line is constant and equal to the phase transition temperature $a_{cr}$. Temperature in each zone satisfies the equation 1, and the heat capacity and thermal conductivity coefficient in the frozen and melt zones are different [4]. Each point $g = [G, gG, g_3] E$ moves along the normal to the boundary line at this point. Values related to the frozen zone are denoted by index 1, and in the melt zone - by index 2. The unit normal vector to the boundary line from melt in the frozen zone. Then the boundary line velocity is determined by the heat balance equation:

$$q_f \left( \frac{d \tilde{c}}{dt} * \tilde{n} \right) = \lambda_1 \frac{\partial u}{\partial n|_1} - \lambda_2 \frac{\partial u}{\partial n|_2}$$

(3)

where $q$ – specific volumetric heat of phase transition, $J/m^3$.

Instead of the two thermal conductivity equations for frozen and melt zones and bonding heat balance equation one type equation, where the heat of phase transition is taken into account by artificial change in medium heat can be used.

In the real soil the boundary line between frozen and melt zones is not surface $GL = GL = C_{tt}$, but layer bounded by surfaces $I = I - L$ and $I = 1$, because the water freezing point in the thin soil capillaries is below °C, and heat capacity and thermal conductivity coefficient are continuously smoothed in the interval $[Q, C_f]$ in the separation layer, and expressed as continuous functions of temperature $G[u]$ and $G [CC]$, common to the frozen and melt zones [12]. In order to take into account the heat of phase transition $\delta$-shaped continuous function $\delta[u - a_{cr}, \Delta]$ different from zero only in the interval $[u - \Delta, a_{cr}]$ that satisfies the normalization condition is introduced.

$$\int_{a_{cr}-\Delta}^{u_{cr}} \delta[u - \Delta, a_{cr}] du = 1 - \frac{1}{q_f} \int_{u_{cr}-\Delta}^{u_{cr}} \tilde{c}[u]du$$

(4)

Then the heat propagation process in the frozen and melt zones is simulated by a single equation with heat capacity (Equation 5) and thermal conductivity coefficient (Equation 6):
\[ c = \tilde{c}(u) + q_f \delta(u - \Delta, u_{cr}) \]  \hspace{1cm} (5) \\
\[ \lambda = \tilde{\lambda}(u) \]  \hspace{1cm} (6)

### 3.2 Concrete heat release kinetics equation

The heat release intensity of hardening concrete at any given time is determined by the temperature and concrete state at this point [13].

#### 3.2.1 Concrete state parameter

Fundamentally it is possible to determine the state of hardening concrete completely by a certain set of physical parameters, which may vary in the hardening process. If the concrete has no destructive changes its state is determined by the concrete mix composition and a certain parameter monotonically changing in the concrete hardening process. Thus it may be assumed that the physical parameters are uniquely expressed in terms of their initial values and this generic parameter. As such it is convenient to assume the concrete specific heat release \( Q \) [J/m³] or relative heat release where \( Q_{\text{max}} \) is the maximum concrete specific heat release [J/m³].

#### 3.2.2 General kinetics equation of the concrete heat release

According to the above, the concrete heat release intensity is a temperature function and relative heat release or

\[ \frac{dq}{dt} = Q_{\text{max}}E(u, \omega) \]  \hspace{1cm} (7)

\[ \frac{d\omega}{dt} = E(u, \omega) \]  \hspace{1cm} (8)

Where \( E[u, \omega] \) – heat release function coefficients determined by the concrete mix composition.

This equation is the concrete state equation and, simultaneously, heat release kinetics equation, because the relative concrete heat release \([\omega]\) is selected as the state parameter. The heat release function can be presented as the product of temperature function \( \varphi \) and state function \( \psi \):

\[ E(u, \omega) = \varphi(u)\psi(\omega) \]  \hspace{1cm} (9)

For isothermal solidification modes such representation is equivalent to the relation of temporarily equal heat releases, that aligns very well with the experimental data in the region \( w_{cr} < w < 1 \), where \( w_{cr} \) is a critical value of relative heat release under which the heat release rate in isothermal mode reaches its maximum.

In the region \( 0 < w < w_{cr} \) relation of equal heat release time is violated, therefore in this region function can not be represented as a product. However, the difference between \( w_{cr} \) and \( w_0 \) [relative heat release of concrete placed in the formwork] is small, therefore for practical calculations the heat release function in the kinetics equation as product in the region is permissible \( \omega_0 < \omega < 1 \).

#### 3.3.3 Concrete heat release function

Function of the state given as:
\[
\psi(\omega) = (1 - \omega)^\nu
\]

Where \(\nu\) – constant coefficient.

Two functions provide the best approximation of experimental data for concrete heat release:

\[
\tilde{\psi}(u) = K \exp\left(\frac{u - u_0}{\xi}\right)
\]

\[ (11) \]

\[
\bar{\psi}(u) = K \left(\frac{u - u_3}{20 - u_3}\right)^\zeta
\]

\[ (12) \]

Where \(u_3\) – concrete freezing point, °C; \(K, \xi, \zeta\) – coefficients that depend on the the concrete mix composition.

Function [11] leads to the known relationship:

\[
\frac{\tilde{\psi}[u_2]}{\tilde{\psi}[u_1]} = 2^{\frac{u_2 - u_1}{\epsilon}}, \quad \epsilon = \xi \ln 2
\]

\[ (13) \]

Thus, the concrete heat release function is as follows:

\[
E(u, \omega) = \bar{\psi}(u)\psi(\omega) = K \left(\frac{u - u_3}{20 - u_3}\right)^\zeta [1 - \omega]^{\nu}
\]

\[ (14) \]

Where function \(\tilde{\psi}(u)\) can be replaced with \(\tilde{\psi}[u]\), and the heat release function coefficients are determined experimentally.

3.3 Boundary conditions

Thermal conductivity and concrete heat release kinetics equations must be supplemented by initial and boundary conditions.

3.3.1 Initial conditions

The initial condition for the thermal conductivity equation is the temperature distribution in the area \(0\) at the initial time of thermal treatment process of concrete mix:

\[
U l_{t=0} = u_0(x)
\]

\[ (15) \]

For the area occupied by the concrete mix function \(u_0(x)\) may be assumed constant. In areas occupied by previously poured concrete or earth foundation a real temperature distribution should be adopted. If this distribution is not known, it must first be pre-computed according to the procedure specified in the article. The initial condition for concrete heat release kinetics with the state parameter distribution in the area \(O\) in the initial moment of thermal treatment process:

\[
\omega l_{t=0} = \omega_0(x)
\]

\[ (16) \]

For the area occupied by the concrete mix function \(\omega_0[x]\) may be assumed constant. If the area has a zone of previously poured concrete, where the heat release process is not finished yet, then the state parameter distribution in this area should be obtained by the preliminary calculation. In the rest of the area state parameter is equal to one.

3.3.2 Boundary conditions

On the part of \(G\) boundary of area \(G\) in the soil or in old concrete, the constant temperature condition is made. It should be understood that, if the boundary is chosen too close to the
hardening concrete area, the design temperature in the adjacent points to the boundary will grow significantly. In this case, the area boundary should be moved aside. The boundary condition for the rest of boundaries is the heat balance equation:

$$\left[ \lambda \frac{\partial u}{\partial t} + \frac{1}{R} (u - v) + c_n \frac{\partial u}{\partial t} \right]_{\partial \Omega} = q_n$$  \hspace{1cm} (17)

Where

- $R$ – thermal resistivity, $[m^2\cdot°C/W]$;
- $\frac{1}{R}$ – heat transfer coefficient, $[W/(m^2°C)]$;
- $V$ – ambient air temperature, °C;
- $c_n$ – specific heat capacity of the formwork, $[J/(m^2°C)]$;
- $q_n$ – specific heat capacity of the heating formwork, $W/m^2$.

Values $R$, $c_n$ and $q_n$ may depend on time and coordinates.

For a plane boundary the thermal resistivity is:

$$R = \frac{d}{\lambda} + \frac{1}{\alpha}$$  \hspace{1cm} (18)

Where

- $\alpha$ – heat transfer coefficient of the outside structure surface, depending on the wind speed, temperature, emissivity and boundary orientation, $[W/(m^2°C)]$;
- $d$ – insulation layer thickness, $m$;
- $\lambda$ – thermal conductivity coefficient of heat insulating material, $[W/m°C]$.

In some areas where there is the metal connection of deck with the outer surface of formwork, the heat transfer resistance can be significantly reduced. In this case, the corresponding value $\alpha$ is introduced in the boundary condition. In case of steel deck the heat flow spreading over the deck must be taken into account. In this case it is more convenient to withdraw a member $c_n \frac{\partial u}{\partial t}$ from the boundary conditions and introduce concentrated heat capacity at the area boundary in the thermal conductivity equation, taking into account the heat flow that spreads over the deck, also the heat flow along the reinforcement bar is taken into account by the corresponding thermal conductivity coefficient increase in directions parallel to the boundary in the narrow near boundary layer. Specific capacity of heating formwork is the main insulated influence on the concrete temperature field [14]. It can be changed during hardening by any technologically acceptable law in order to select the most efficient thermal treatment mode.

3.4 The closed system of thermal conductivity and concrete heat release kinetics equations

Outlined basic elements of mathematical model of the temperature field in the hardening concrete structure are included in the model equation.

Heat release kinetics equation

$$\frac{\partial \omega}{\partial t} = E[u, \omega]$$  \hspace{1cm} (19)

With the initial condition

$$\omega|_{t=0} = \omega_0[\vec{x}]$$  \hspace{1cm} (20)

In the inert zones, where heat release is absent, $\omega_0[\vec{x}]=1$.

Thermal conductivity equation

$$c(\vec{x}, u) \frac{\partial u}{\partial t} = Lu + f(\vec{x}, t, u, \omega)$$  \hspace{1cm} (21)
\[ Lu = \sum_{p=1}^{3} L_p u, \quad L_p u = \frac{\partial [\lambda_p(\bar{x}, u, \omega) \frac{\partial u}{\partial x_p}]}{\partial x_p} \]  
(22)

\[ f = Q_{max} E(u, \omega) + f^{(e)} f(\bar{x}, t) \]  
(23)

Where \( f^{(e)} \) – energy inflow rate from outside, \( W/m^3 \).

Initial condition for temperature:
\[ u_{t=0} = u_{0}(\bar{x}) \]  
(24)

Boundary condition:
\[ \left[ \frac{\lambda_p(\bar{x}, u, \omega)}{\partial n} + \frac{1}{R(\bar{x}, u, t)}(u + v) + c_n(\bar{x}, t) \frac{\partial u}{\partial t} \right]_{\bar{x} \in G} = q_n(\bar{x}, t) \]  
(25)

or, if the boundary lays in the solid,
\[ u_{L,G} = v_{0}(\bar{x}) \]  
(26)

Under this model the most of problems on the calculation of temperature fields in the hardening concrete can be formalized so that the general method for solving such problems can be obtained. The model is quite complex, but its equations are solved by effective numerical method. It is necessary to pay attention to some aspects of the model application. When solving specific problems significant simplifications can be found in the model. For example, when heating the concrete in the heating formwork \( f^{(e)} = 0 \) or when heat flows in the reinforcement can be disregarded, the thermal conductivity coefficient does not depend on the direction. The dependence of thermal conductivity coefficient on the concrete state parameter on the interval \( 0 \leq \omega \leq 1 \) can be taken into account by the Equation:
\[ \lambda(\omega) = \lambda(0) - \beta \omega [\lambda(0) - \lambda(1)] \]  
(27)

Where \( \beta \) – constant coefficient.

For the initial temperature distribution \( u_0 = [x] \) in the old concrete or soil at its pre-heating the same model is applied, but without heat release kinetics equation: \( \omega_0[x] = 1 \).

### 3.5 Numerical method of model equation solution of temperature field

For the numerical solution of model equations locally one-dimensional scheme (LOS) of total approximation method is applied [9]. This scheme is economic, relatively simple for programing [10], does not require large volume of RAM, allowing for calculations on a personal computer [19].

#### 3.5.1 Replacement of three-dimensional thermal conductivity equation with a chain of one-dimensional equations

Let's introduce in G area the bound grid spaced at \( h_p \), formed by three families of \( G_p \) lines, parallel to the coordinate axes \( x_p \), division points
\[ t_j = (j - 1)\tau; \quad j = 1, \ldots, j_0, \tau = \frac{t_0}{j_0} \]  
(28)

At each interval \([t_j; t_{j+1}]\) three-dimensional thermal conductivity equation shall be replaced with the chain of one-dimensional equations:
\[ \frac{1}{3} c \frac{\partial u_p}{\partial t} = L_p u_p + f_p \]  
when \( t_j + \frac{p - 1}{3} \tau < t \leq t_j + \frac{p}{3} \tau \]  
(29)

along all lines
\[ G_p, P = 1, 2, 3, \sum_{p=1}^{3} f_p = f \]  

(30)

with the initial conditions

\[ u_1(\bar{x}, t_j) = u(\bar{x}, t_j); u_2(\bar{x}, t_j + \frac{1}{3} \tau) = u_1(\bar{x}, t_j + \frac{1}{3} \tau); u_3(\bar{x}, t_j + \frac{2}{3} \tau) = u_2(\bar{x}, t_j + \frac{2}{3} \tau) \]  

(31)

The boundary conditions for each equation of the chains are set at the intersection of corresponding line of \( G_p \) family with boundary area.

The heat release kinetics equations are similarly replaced with a chain of equations:

\[ \frac{1}{3} \frac{\partial \omega_p}{\partial t} = E_p, \quad \sum_{p=1}^{3} E_p = E \]  

(32)

It isn't necessary, though care should be taken to ensure that the contribution of heat release in \( f[\tau] \) in a time \( \tau \) will be equal to \([Q_{\text{max}}, E, \tau]\) at each point of the area.

Physically the replacement of three-dimensional thermal conductivity equation with a chain of one-dimensional equations means that the thermal conductivity process occurring in the space is replaced by a sequence of one-dimensional processes by coordinate directions.

Suppose that at the time \( t_j \) the heat-proof partitions in \( X_1 \) and \( X_3 \) directions are installed i.e. heat spreads only in the direction of \( X_1 \).

At time \( t = t_j + \tau \) in directions \( X_1 \) and \( X_3 \) exchange their parts, and at time \( t = t_j + 2\tau \) heat spreads only in the direction \( X_2 \). As a result, at time \( t_j + 3\tau \) the temperature distribution will be the same as at time \( t = t_j + \tau \) at three-dimensional conductivity.

For numerical solution of model equations on the interval \([0, t_0]\) it is necessary to obtain solutions on half-intervals \( (t_j, t_{j+1}, j = 1, \ldots, J) \) and the problem solution in the previous half-interval is the initial condition for the problem in the next half-interval. In turn, the problem solution in each half-interval \([t_j, t_{j+1}]\), is carried out successively in three directions.

### 3.5.2 Approximation of one-dimensional differential equations of the linear difference equation system

Along each line of \( C \) family, the differential equation system

\[ \frac{1}{3} \frac{\partial u_p}{\partial t} = L_p u_p + f_p \]  

(33)

\[ \frac{1}{3} \frac{\partial \omega_p}{\partial t} = E_p \]  

(34)

is approximated by the linear difference equation system

\[ c_i \frac{u_i - u_{i-1}}{\tau} = \frac{1}{h_p} \left( a_{i+1} \frac{u_{i+1} - u_i}{h_p} - a_i \frac{u_i - u_{i-1}}{h_p} \right) + f_{p_i}; \quad i = 2, \ldots, N - 1 \]  

(35)

\[ \frac{\omega_i - \omega_{i-1}}{\tau} = E_{p_i}; \quad i = 1, \ldots, N \]  

(36)

\[ a_{i+1} = \frac{1}{2} \left[ \lambda_{p_{i+1}} + \lambda_{p_i} \right] \]  

(37)

where function arguments are taken in the \( i \)-th node on \( G_p \) line at time point \([t_j + \frac{p-1}{3} \tau]\), if symbol ^ is absent, and at time point \([t_j + \frac{p-1}{3} \tau]\), if symbol ^ is present.

The resulting system comprises \([2N - 2]\) linear equations and \(2N\) unknowns \(- \widehat{u_i} \) and \( \widehat{\omega_i} \). Values \( u_t \) and \( \omega_t \) known from the previous solution. This system is complemented by two equations obtained from the boundary conditions at the intersection points of \( G_p \) line with the area boundary.
\[ \hat{u}_i = x_i \hat{u}_2 + \mu_1 \]  
(38)

\[ \hat{u}_N = x_2 \hat{u}_{N-1} + \mu_2 \]  
(39)

with the following symbols:

\[ x_1 = \frac{\tau a_2}{h_p^2 \Delta_1}, \Delta_1 = \frac{\tau}{h_p^2} a_2 + \frac{1}{2} c_1 + \frac{1}{h_p} \left( \frac{c_{n_1}}{R_1} + \mu_1 \right) = \frac{1}{\Delta_1} \left( \frac{1}{h_p} \left[ c_{n_1} u_1 + \tau \left( \frac{V_1}{R_1} + q_{n_1} \right) \right] + \frac{1}{2} \left[ c_1 u_1 + \tau f_{p_1} \right] \right), \]

\[ x_2 = \frac{\tau a_N}{h_p^2 \Delta_2}, \Delta_2 = \frac{\tau}{h_p^2} a_N + \frac{1}{2} c_N + \frac{1}{h_p} \left( \frac{c_{n_N}}{R_N} + \mu_2 \right) = \frac{1}{\Delta_2} \left( \frac{1}{h_p} \left[ c_{n_N} u_N + \tau \left( \frac{V_N}{R_N} + q_{n_N} \right) \right] + \frac{1}{2} \left[ c_N u_N + \tau f_{p_N} \right] \right) \]  
(40)

This equation system is solved by known sweep method, and stream option of this method can be applied as more reliable that gives the best results for the system with strongly varying coefficients.

Dependence of the concrete thermal conductivity coefficient on coordinates and direction, caused by the presence of steel reinforcement, can not be directly considered in the difference equations, because the grid pitch is greater than the reinforcement bar diameter [17]. Assume that the reinforcement bar is artificially allocated in concrete so that the total thermal resistivity of the bar and surrounding concrete is not changed. Suppose, for example, that the reinforcement bar is located along G family line, and \( f \) has effective cross-section. Then the effective thermal conductivity coefficient is determined from the equation

\[ \lambda_B (S' - S_A) + \lambda_A S_A = \lambda' S' \]  
(41)

Where \( \lambda_B \) and \( \lambda_A \) – concrete and steel thermal conductivity coefficients, \( W/[m^2\cdot{^\circ}C] \); \( S' = h_2h_3 \) – concrete pier area with conditionally distributed reinforcement, \( m^2 \); \( \lambda' \) – thermal conductivity coefficient of this concrete, \( W/[m^2\cdot{^\circ}C] \). Therefore,

\[ \lambda' = \lambda_B + (\lambda_A - \lambda_B) \frac{S_A}{S} \]  
(42)

This value must be used in the corresponding difference equation.

Recommended LOS default is absolutely stable and converges uniformly at a rate of \( 0[\tau+|h|^2] \). We do not recommend the use of explicit schemes, because three-dimensional explicit scheme is stable only if \( \tau < h^2/6\alpha \) (\( \alpha \) – temperature-conductivity coefficient, \( m^2/s \)), which leads to low pitch in time.

In order to solve the specific problem for the given method the calculation program for computer facilities, after which, varying the control actions and carrying out calculations of temperature field, the most efficient thermal treatment mode is selected.

3.6 Calculation method of heat release function coefficients from experimental data

According to the Equations 7, 8, 9 we have

\[ \frac{dQ}{dt} = Q_{max} K \left[ \frac{u-u_3}{20-u_3} \right]^S \left( 1 - \frac{Q}{Q_{max}} \right)^\nu \]  
(43)

Where temperature function \( \bar{\phi} \) can be replaced \( \bar{\phi} \) with [see. Subsection 3.2].

Record the experimental data on the concrete heat release in the table, where:

- \( i \) - sequence number of measurement;
- \( t_i \) - time point of \( i \)-th measurement from the beginning of hydration, \( h \);
According to the data, the most efficient thermal treatment mode is selected. If the reinforcement bar is located along the G family line, and the thermal resistivity of the bar and surrounding concrete is not changed. Suppose, for example, difference equations, because the grid pitch is greater than the reinforcement bar diameter in the direction, caused by the presence of steel reinforcement, cannot be directly considered in the varying coefficients. The method can be applied as more reliable that gives the best results for the system with strongly varying coefficients.

\[ h_i = \text{record time point of } t_i - \text{measurement from the beginning of hydration, } h; \]

\[ W \]

\[ \lambda \]

\[ ℎ \]

\[ \alpha \]

\[ \mu \]

\[ \nu \]

\[ \Delta \]

\[ \omega \]

\[ \xi \]

\[ \zeta \]

Table 1. Experimental data values on the concrete heat release.

<table>
<thead>
<tr>
<th>No.</th>
<th>t</th>
<th>u</th>
<th>Q</th>
<th>( \frac{dQ}{dt} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Note. Only \( Q_i \) or \( \left( \frac{dQ}{dt} \right)_i \) can be measured in the experiment, then the other value is obtained by known recalculation.

To determine the coefficients \( Q_{max}, K, u_3, \zeta, \nu \) it is recommended to use the following mathematical treatment method of experimental data. Using the heat release kinetics equation, experimental the function is drawn up:

\[
\sigma[Q_{max}, K, u_3, \zeta, \nu] = \sum_{i=1}^{N} \left[ \left( \frac{dQ}{dt} \right)_i - Q_{max}K \left( \frac{u_i - u_3}{20 - u_3} \right)^S \left( 1 - \frac{Q_i}{Q_{max}} \right)^\nu \right]^2 \quad (43)
\]

Function arguments 43 are the maximum specific heat release \( Q_{max} \) and heat release function coefficients. Minimum of function \( \sigma \) is achieved at the desired values of these coefficients. There are a number of searching methods for the minimum non-linear functions of several variables. Libraries of standard programs usually contain a program implementing one of these methods. For the independent preparation of such program, the most simple method of descent by coordinates can be recommended.

The above calculation method of heat release function coefficients provides a function that approximates in the best way the experimental dependence data of the heat release rate \( \left( \frac{dQ}{dt} \right) \) from temperature \( (u) \) and concrete heat release \( [Q] \), with the minimum data available. It is enough to have the results of only one experiment on the concrete heat release in random temperature conditions covering the entire temperature range of interest. This is achieved by use of the recommended mathematical treatment method of experimental results.

4 Methodical example of the temperature field calculation

Let's consider the example of temperature field calculation for the parallelepiped with dimensions 0.48 x 0.48 x 2.7 m [column fragment]. The lower part of 0.48 x 0.48 x 1.7 m area is occupied by previously poured concrete with relative heat release coefficient \( \omega_0 = 0.9 \), and the upper part of 0.48 x 0.48 x 1 m occupies the concrete mixture \( \omega_0 = 0.01 \). On top concrete is covered with mat of \( c = 0.05 \) m thick with thermal conductivity coefficient \( d = 0.05 \) [W/m²·°C], outside surface heat transfer coefficient of the structure \( \alpha = 25 \) [W/m²·°C], ambient air temperature \( U_v = -10^\circ C \).

Let's place the lower parallelepiped vertex in the origin of coordinates and direct axis J up to the edge, and axes X1 and X2 horizontally on the foundation edges. The heating formwork is adjacent to the side faces, starting from a height \( X3 = 1.5 \) m. There are four reinforcement bars with a diameter of 0.02 m at a distance of 0.06 m from the side faces in the concrete in vertical direction along side edges. Because the column is symmetric to the vertical axis passing through the parallelepiped center of the box it is enough to make calculations for a quarter of the area, cut-out by planes \( X1 = 0.24 \) m and \( X2 = 0.24 \) m. The heat flow through these planes will be equal to zero. Let's introduce in the grid spaced at \( h_1 = 0.03 \) m, \( h_2 = 0.03 \) m and \( h_3 = 0.06 \) m respectively along the axes X1, X2 and X3. The total
number of grid nodes $9 \times 9 \times 46 = 3726$. The reinforcement bar passes the nodes $[3, 3, N]$ at $N = 1, \ldots, 46$, therefore the thermal conductivity coefficient in the direction $X_3$ in these nodes should be calculated according to the Equation:

$$\lambda' = \lambda_B \left[ 1 + \left( \frac{\lambda_A}{\lambda_B} - 1 \right) \frac{SA}{S'} \right]$$  (44)

where $S_A = 3.14 \times (0.01)^2$ $[m^2]$; $S' = h_1 h_2 = [0.03]^2$ $[m^2]$.

Assume $\lambda_A = 40$ $W/[m \cdot ^\circ C]$; $\lambda_B = \lambda_0 - \omega(\lambda_0 - \lambda_1)$; $\lambda_0 = 3$ $W/[m \cdot ^\circ C]$; $\lambda_1 = 2.6 W/[m \cdot ^\circ C]$.

Coefficients in the concrete heat release kinetics equation: $Q_{max} = 1.25 \times 10^5$ $kJ/m^3$; $K = 4.77 \times 10^{-6}$ $[c^{-1}]$; $u_3 = -6.7$ $[^\circ C]$; $\zeta = 2.3$; $\nu = 2.2$.

The specific volume heat capacity of concrete $c_B = 2000$ $kJ/[m^3 \cdot ^\circ C]$.

The initial value of relative heat release assume $\omega_0=0.9$ in nodes $[I,J,N]$ at $1 \leq N \leq 29$ $[0 \leq x_3 \leq 1.7$ m – previously poured concrete] and $\omega_0=0.1$ in nodes $[I,J,N]$ at $29 < N \leq 46$ $[1.7 < x_3 < 2.7$ m – concrete mix]. Assume that the initial temperature of concrete mix $u_0 = +50^\circ C$ in nodes $[I,J,N]$, $I \leq N \leq 29$.

Specific capacity of heating formwork $q_n = 1$ $kW/m^2$, the deck insulation layer thickness $d=0.04$ m, thermal conductivity coefficient of heat insulating material $\lambda_{u3} = 0.04$ $W/[m \cdot ^\circ C]$, and specific deck heat $c_n = 20$ $kJ/[m^2 \cdot ^\circ C]$.

Let at the lower face of parallelepiped $1/R = 0$, $c_n = 0$, $q_n = 0$.

The heat flow is zero in planes $x_1=0.24$ m and $x_2=0.24$ m.

Heat exchange takes place on the open side faces: $1/R = 25$ $kJ/[m^2 \cdot ^\circ C]$, $c_n = 20$ $kJ/m^2$, $q_{n} = 1$ $kW/m^2$.

At the upper face: $R=1.04$ $[m^2 \cdot ^\circ C]/W$, $c_n = 0$, $q_n = 0$.

The temperature field calculation for this problem is performed on a personal computer by the program compiled in FORTRAN. Part of the calculation results is shown in Tables 2, 3, 4.

**Table 2.** The temperature distribution $[^\circ C]$ in grid nodes in 1 hour after the start of heating in section $X_3=2.7$ m.

<table>
<thead>
<tr>
<th>I /J</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>23.13</td>
<td>17.34</td>
<td>15.91</td>
<td>15.88</td>
</tr>
<tr>
<td>4</td>
<td>17.34</td>
<td>12.97</td>
<td>10.43</td>
<td>10.41</td>
</tr>
<tr>
<td>6</td>
<td>15.91</td>
<td>10.43</td>
<td>5.70</td>
<td>4.69</td>
</tr>
<tr>
<td>8</td>
<td>15.88</td>
<td>10.41</td>
<td>4.69</td>
<td>4.61</td>
</tr>
</tbody>
</table>

**Table 3.** The temperature distribution $[^\circ C]$ in grid nodes in 1 hour after the start of heating in section $X_3=2.7$ m.

<table>
<thead>
<tr>
<th>I /J</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
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<td>24.71</td>
<td>22.77</td>
<td>21.89</td>
</tr>
<tr>
<td>4</td>
<td>24.71</td>
<td>18.21</td>
<td>15.55</td>
<td>13.08</td>
</tr>
<tr>
<td>6</td>
<td>22.77</td>
<td>15.55</td>
<td>11.61</td>
<td>9.37</td>
</tr>
<tr>
<td>8</td>
<td>21.89</td>
<td>13.08</td>
<td>9.37</td>
<td>9.22</td>
</tr>
</tbody>
</table>

**Table 4.** The temperature distribution $[^\circ C]$ in grid nodes in 1 hour after the start of heating in section $X_3=2.2$ m.

<table>
<thead>
<tr>
<th>I /J</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>29.17</td>
<td>23.66</td>
<td>21.32</td>
<td>21.05</td>
</tr>
<tr>
<td>4</td>
<td>23.66</td>
<td>17.01</td>
<td>14.72</td>
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<tr>
<td>6</td>
<td>21.32</td>
<td>14.72</td>
<td>10.20</td>
<td>9.14</td>
</tr>
<tr>
<td>8</td>
<td>21.05</td>
<td>11.85</td>
<td>9.14</td>
<td>9.10</td>
</tr>
</tbody>
</table>
3 Conclusion

Thus the dynamics of temperature field is investigated using the example of special case with the preset variables, that determine the concrete thermal treatment mode. It is evident that the boundary conditions and input data are always set again, based on the tasks provided for the solution that in turn indicates the applicability of proposed method, under different concrete thermal treatment modes.

During calculations with the use of software the limitation at the Equations set through a functional approach was observed, including the preset parameters passed to the subroutines as well as the need for duplication of variables that significantly complicated the set of program in Fortran language. This indicates the possibility of facilitating of the writing the program by selecting another software language, such as C++. However, it should be noted that the calculating speed will not change much depending on the different software compilers selection [7].

As one can see, the numerical solution of model equations which allows calculating the temperature field in the hardening concrete structure by using computer software directly indicate the applicability of model in the development area of the most efficient modes without the need for volume calculations, which can significantly reduce the scientific experiment time.

Theoretical and practical significance of study results is the possibility to select the most efficient mode of thermal treatment of concrete structures, saving time for calculations, and, therefore, high quality of concrete with minimum thermal treatment duration and maximum reduction of energy costs is provided [20]. Directly affecting the construction and installation works schedule and improving the construction quality of buildings and structures of reinforced concrete, basic requirements of energy efficiency and environmental friendliness of residential buildings in the implementation of the government program of the affordable housing market development are met.

References

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